The Investigation of Exact Traveling Wave Solutions of the (2+1)-Dimensional Burger Equation using the Generalized Kudryashov Methods

Mst. Razia Pervin[#1]

[#] Department of Mathematics, Mawlana Bhashani Science and Technology University, Tangail-1902, Bangladesh

Abstract *In this paper, using generalized kudryashov method, we present some new explicit formulas of exact traveling the (2+1)-Dimensional Burger equation. Three exact traveling wave solutions in terms exponential function are found from the investigation. It is shown that the generalized kudryashov method is a very effective and powerful mathematic tool for solving nonlinear evolution equations in mathematical physics and engineering*.

Keywords — *The generalized kudryashov method; the (2+1)-Dimensional Burger equation; traveling wave solutions.*

I. INTRODUCTION

Most scientific problems and physical phenomena occur nonlinearly. Except in a limited number of these problems, finding the exact analytical solutions of such problems are rather difficult. Recently, namy kinds of powerful methods have been proposed to find exact solutions of nonlinear partial differential equations, e.g., the homogeneous balance method [1], homotopy analysis method [2, 3], three-wave method [4], extended homoclinic test approach [5], the (G'/G)-expansion method [6, 10] and the exp-function method [11-13] and so on. Hellal and Mehanna [14] used a semi-analytical method, that is the Adomian decomposition and the tanh method to handle the foam drainage equation. Roshid et. al. [15-16] investigated solitary wave solutions for some nonlinear evolution equations via exp-function and Exp $(-\varphi(\xi))$-expansion method. Demiray et. al. [17] investigate exact solutions of nonlinear time fractional Klein –Gordon equation by using generalized Kudryashov. The modified simple equation method is performed in [18], multi-soliton solutions are found in [19].

In this paper, we will apply the the generalized Kudryashov method on the (2+1)-Dimensional Burger equation.

II. FUNDAMENTAL PROPERTIES OF THE GENERALIZED KUDRYASHOV METHOD

The basic properties of the generalized Kudryashov method is explained in this section. In order to apply the methods, we consider the nonlinear evolution equation of the following form:

$$\Omega(u, u_t, u_x, u_{xx}, u_{tt}, u_{xt}, \cdots \cdots) = 0\,, \qquad (1)$$

where Ω is a polynomial of the function $u = u(x,t)$ and its partial derivatives with respect to the spatial variable x, y and the time variable t. Let us combine the real variables x, y and t by a compound variable η as $u(x, y,t) = u(\eta);\ \ \eta = kx + ly \pm wt$ which convert the Eq.(1) into an ODE for $u = u(\eta)$:

$$\Psi(u, u', u'', u''', \cdots \cdots) = 0\ . \qquad (2)$$

Step 1: Suppose that the solution of Eq. (2) has the following form:

$$u(\eta) = \frac{\sum_{i=0}^{N} A_i \rho^i(\eta)}{\sum_{j=0}^{M} B_j \rho^j(\eta)} \qquad (3)$$

where $A_i\ (i = 0,1,2,\cdots\cdots, N)$ and $A_j\ (j = 0,1,2,\cdots\cdots, M)$ are constants to be determined afterward such that $A_N \neq 0$ and $B_M \neq 0$, and $\rho = \rho(\eta)$ satisfies the following auxiliary differential equation:

$$\frac{d\rho(\eta)}{d\eta} = \rho^2(\eta) - \rho(\eta). \qquad (4)$$

Equation Eq. (4) has the solution

$$\rho(\eta) = \frac{1}{1 + C\exp(\eta)}, \qquad (5)$$

where C is a constant of integration.

Step 2: The positive integers N and M appearing in Eq. (3) can be determined by define the degree of $u(\eta)$ as $D(u(\eta)) = N - M$ which gives rise to the degree of other expression as follows:

$$D\left(\frac{d^q u}{d\eta^q}\right) = N - M + q,$$

$$D\left(u^p\left(\frac{d^q u}{d\eta^q}\right)^s\right) = (N - M)p + (N - M + q)s,$$ where p, q, s are integer numbers. In this regard, we can find the value of N and M in Eq. (3).

Step 3: Inserting Eq.(3) with Eq.(4) into Eq.(2) provides a polynomial in $\rho(\eta)$. We equate the coefficients of all terms of the same powers of $\rho(\eta)$ to zero, this procedure yields a system of equations which can be solved to find unknown parameters in the trial solution.

III. The (2+1)-Dimensional Burger Equation

In this section, we apply the generalized Kudryashov method to construct the traveling wave solutions for the (2+1)-Dimensional Burger equation in the form:

$$u_t - uu_x - u_{xx} - u_{yy} = 0 \qquad (6)$$

Upon using the transformation

$$u = u(\eta);\ \eta = kx + ly - wt \qquad (7)$$

where k, l and w are constants, eq. (6) is transferred to

$$(k^2 + l^2)u'' + kuu' + wu' = 0. \qquad (8)$$

Integrating eq. (8) with respect to η, we have

$$(k^2 + l^2)u' + ku^2/2 + wu + C = 0. \qquad (9)$$

where the prime denotes differentiation with respect to η. Taking homogeneous balance between the highest order derivative term u' and highest nonlinear term u^2 in the Eq.(9), we get the relation . When $N = 1$, then $M = 2$. So the equation (3) has the following solution

$$u(\eta) = \frac{A_0 + A_1\rho(\eta) + A_2\rho^2(\eta)}{B_0 + B_1\rho(\eta)};\ A_2 \neq 0, B_1 \neq 0. \qquad (10)$$

Substitute (10) and (4) into (9), let the coefficient of $(\rho(\eta))^i, (i = 0,1,2,\cdots)$ be zero, yields a set of algebraic equations about A_i, B_i, w as follows:

$$\begin{cases} kA_2^2 + 2l^2A_2B_1 + 2k^2A_2B_1 = 0, \\ 2wA_2B_1 + 2kA_1A_2 - 2k^2A_2B_1 + 4l^2A_2B_0 + 4k^2A_2B_0 - 2l^2A_2B_1 = 0, \\ 2CB_1^2 - 4k^2A_2B_0 - 4l^2A_2B_0 - 2l^2B_1A_1 + 2wA_2B_0 + 2k^2A_1B_0 \\ + 2wA_1B_1 + 2kA_0A_2 + 2l^2A_1B_0 + kA_1^2 - 2k^2B_1A_1 = 0, \\ 2l^2B_1A_0 + 2k^2B_1A_0 - 2k^2A_1B_0 - 2l^2A_1B_0 + 2wA_1B_0 \\ + 2wA_0B_1 + 2kA_0A_1 + 4CB_0B_1 = 0, \\ kA_0^2 + 2CB_0^2 + 2wA_0B_0 = 0. \end{cases}$$

Then the solution of the system of equations, we achieve the solutions sets:

Set-1:

$$C = -\frac{A_0(2k^2B_0 + 2l^2B_0 - kA_0)}{2B_0^2},\ w = \frac{k^2B_0 + l^2B_0 - kA_0}{B_0},$$
$$A_1 = -\frac{2(kA_0 + l^2B_0 + k^2B_0)}{k}, A_2 = \frac{4(k^2 + l^2)B_0}{k}, B_1 = -2B_0,$$
$$A_0 = const., B_0 = const. \qquad (11)$$

Set-2:

$$C = \frac{A_1(8k^2B_0 + 8l^2B_0 + kA_1)}{8B_0^2},\ w = \frac{4k^2B_0 + 4l^2B_0 + kA_1}{2B_0},\ A_0 = -\frac{A_1}{2},$$
$$A_2 = \frac{4B_0(k^2 + l^2)}{k}, B_1 = -2B_0, A_1 = const., B_0 = const. \qquad (12)$$

Set-3:

$$C = \left[\frac{(4B_0^2 - 4B_0B_1)(k^4 + l^4) - 8k^2l^2B_0B_1 + 8k^2l^2B_0^2 + k^2A_1^2 - 2kA_1(B_1 - 2B_0)(k^2 + l^2)}{2kB_1^2}\right],$$
$$w = \frac{(B_1 - 2B_0)(k^2 + l^2) - kA_1}{B_0},\ A_0 = \frac{B_0(2l^2B_0 + 2k^3B_0 + kA_1)}{kB_1},$$
$$A_2 = -\frac{2(k^2 + l^2)B_1}{k}, A_1 = const., B_0 = const., B_1 = const. \qquad (13)$$

Substituting (11) with (5) into (10) , we have

$$u(\eta) = \left[\frac{kA_0(1 + c\exp(\eta))^2 - 2(k^2B_0 + l^2B_0 + kA_0)(1 + c\exp(\eta)) + 4B_0(l^2 + k^2)}{\{B_0(1 + c\exp(\eta)) - 2B_0\}(1 + c\exp(\eta))}\right], \qquad (14)$$

where $\eta = kx + ly - \frac{k^2B_0 + l^2B_0 - kA_0}{B_0}t.$

Substituting (12) with (5) into (10) , we have

$$u(\eta) = \frac{-kA_1(1 + c\exp(\eta))^2 + 2kA_1(1 + c\exp(\eta)) + 8B_0(l^2 + k^2)}{2\{B_0(1 + c\exp(\eta)) - 2B_0\}k(1 + c\exp(\eta))}, \qquad (15)$$

where $\eta = kx + ly - \frac{4k^2B_0 + 4l^2B_0 + kA_1}{2B_0}t.$

Substituting (13) with (5) into (10) , we have

$$u(\eta) = \left[\frac{B_0(2l^2B_0 + 2k^3B_0 + kA_1)(1 + c\exp(\eta))^2 + kA_1B_1(1 + c\exp(\eta)) - 2B_1^2(l^2 + k^2)}{kB_1\{B_0(1 + c\exp(\eta)) + B_1\}(1 + c\exp(\eta))}\right], \qquad (15)$$

where $\eta = kx + ly - \frac{(B_1 - 2B_0)(k^2 + l^2) - kA_1}{B_0}t.$

IV. Conclusions

We successfully implemented the generalized Kudryashov method to construct the traveling wave solutions for the (2+1)-Dimensional Burger equation. In fact, we have presented three new solutions for the nonlinear Burger equation. Results of the current work illustrates that the generalized Kudryashov method is indeed powerful analytical technique for most types of nonlinear problems and several such problems in scientific studies and engineering may be solved by this method.

References

[1] X.Q. Zhao, L. Wang and W. Sun, The repeated homogeneous balance method and its applications to nonlinear partial differential equations, Chaos, Solitons and Fractals, 28(2), 448-453, 2006.

[2] S.J. Liao, A new branch of solutions of boundary-layer flows over an impermeable stretched plate, Int. J. Heat Mass Transfer, 48, 2529-2539, 2005 .

[3] S.J. Liao, A general approach to get series solution of non-similarity boundary-layer flows, Commun. Nonlinear Sci. Numer. Simul., 14(5), 2144-2159, 2009.

[4] M.T. Darvishi and Mohammad Najafi, Some exact solutions of the (2+1)-dimensional breaking soliton equation using the three-wave method, World Academy of Science, En-gineering and Technology, 87, 31-34, 2012 .

[5] M.T. Darvishi and Mohammad Najafi, Some complexiton type solutions of the (3+1)-dimensional Jimbo-Miwa equation, World Academy of Science, Engineering and Technology, 87, 42-44, 2012.

[6] M.L. Wang., X.Z. Li and J.L. Zhang, The (G'/G) -expansion method and traveling wave solutions of nonlinear evolution equations in mathematical physics,, Physics Letters A, 372 ,417-423, 2008.

[7] M.T. Darvishi, Maliheh Najafi and Mohammad Najafi, Traveling wave solutions for the (3+1)-dimensional breaking soliton equation by (G'/G) -expansion method and modified F-expansion method, World Academy of Science, Engineering and Technology, 88, 37-42, 2012.

[8] H.O. Roshid, N. Rahman and M.A. Akbar, Traveling waves solutions of nonlinear Klein Gordon equation by extended (G′/G)-expasion method, Annals of Pure and Appl. Math., 3, 10-16, 2013.

[9] H.O. Roshid, M.N. Alam, M.F. Hoque and M.A. Akbar, A new extended (G′/G)-expansion method to find exact traveling wave solutions of nonlinear evolution equations, Mathematics and Statistics, 1(3), 162-166, 2013.

[10] M.N. Alam, M.A. Akbar and H.O. Roshid, Study of nonlinear evolution equations to construct traveling wave solutions via the new approach of generalized (G'/G) -expansion method, Mathematics and Statistics, 1(3), 102-112, 2013, DOI: 10.13189/ms.2013.010302.

[11] J.H. He and M.A. Abdou, New periodic solutions for nonlinear evolution equations using Exp-function method, Chaos, Solitons and Fractals, 34, 1421-1429, 2007.

[12] B.C. Shin, M.T. Darvishi and A. Barati, Some exact and new solutions of the Nizhnik-Novikov-Vesselov e5quation using the Exp-function method, Comput. Math. Appl., 58(11/12) (2009) 2147-2151, 2009.

[13] X.H. Wu and J.H. He, Exp-function method and its application to nonlinear equations, Chaos, Solitons and Fractals, 38(3), 903-910, 2008.

[14] M.A. Hellal and M.S. Mehanna, The tanh method and Adomian decomposition method for solving the foam drainage equation, Appl. Math. Comput., 190 (1), 599-609, 2007.

[15] H. O. Roshid, M. R. Kabir, R. C. Bhowmik and B. K. Datta, Investigation of Solitary wave solutions for Vakhnenko-Parkes equation via exp-function and Exp $(-\varphi(\xi))$ -expansion method. SpringerPlus 3:692, 2014.

[16] H. O. Roshid and M. A. Rahman, The exp $(-\varphi(\eta))$ -expansion method with application in the (1+1)-dimensional classical Boussinesq equations. Results in Physics, Vol. 4, 150-155, 2014.

[17] S. T. Demiray Y. Pandir and H. Bulut, The investigation of exact solutions of nonlinear time fractional Klein –Gordon equation by using generalized Kudryashov, AIP Conf. Proc. 1637(2014) 283, 2014.. doi. 10.1063/1.4904590.

[18] H. O. Roshid, M. M. Roshid, N. Rahman and M. R. Pervin, New solitary wave in shallow water, plasma and ion acoustic plasma via the GZK-BBM equation and the RLW equation, Propulsion and Power Research, 6(1), 49-57, 2017.

[19] H. O. Roshid and M. M. Rashidi, Multi-soliton fusion phenomenon of Burgers equation and fission, fusion phenomenon of Sharma–Tasso–Olver equation, J. of Ocean Eng. Sci., 2(2), , 2017. https://doi.org/10.1016/j.joes.2017.04.001.