

# SOME THEOREMS ON $T$ – ANTI-FUZZY IDEALS OF A $\ell$ –NEAR-RING

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**Abstract:** In this paper, we made an attempt to study the properties of  $T$  – anti-fuzzy ideal of a  $\ell$  – near-ring and family of  $T$  – anti-fuzzy ideals of  $\ell$  – near-ring, smallest  $T$  – anti-fuzzy ideal of  $\ell$  – near-ring in  $R$ . We introduce the product of two  $T$  – anti-fuzzy ideals of a  $\ell$  – near-ring in  $R$  and the product  $\Pi A_i$  of  $T$  – anti-fuzzy ideals of a  $\ell$  – near-ring in  $R$ .

**Keywords:** Fuzzy subset,  $T$  - anti-fuzzy ideal, join of  $T$  - anti-fuzzy ideal, product of two  $T$  - fuzzy ideals, and  $\Pi A_i$  of  $T$  - fuzzy ideal.

## I. INTRODUCTION

The concept of fuzzy sets was initiated by Zadeh, L.A., in 1965. Liu. W., has studied fuzzy ideals of a ring and many researchers are engaged in extending the concepts. Abou-Zaid, S., introduced the notion of a fuzzy subnear-ring, and studied fuzzy ideals of a near-ring, and many followers discussed further properties of fuzzy ideals in near-rings. In Biswas, R., introduced the concept of anti-fuzzy subgroups of groups, and Kim, K.H., and Jun, J.B., studied the notion of anti-fuzzy  $R$ -subgroups of near-ring. In Kim, K.H., Jun, Y.B., and Yon, Y.H., introduced anti-fuzzy ideals in Near-Rings. In Hohle, U., introduced into fuzzy set theory and suggested that the  $T$ -norm be used for the intersection and union of fuzzy sets. Deena, P., Mohanraj, G., and Akram, M., have studied several properties of  $T$  – fuzzy ideals of rings and  $T$  – fuzzy ideals of near-rings. We extended the results of Akram, M., to  $\Gamma$  – near-rings. In this paper we define, characterize and study the  $T$  – anti-fuzzy right and left ideals. Prakashmanimaran, J., Chellappa, B., and Jeyakumar, M., introduced  $T$  – anti-fuzzy right ideals of  $\ell$  – ring. We introduced  $T$  – anti-fuzzy right ideals of  $\ell$  – near-ring. We have shown that ring is regular if and only if union of any  $T$  – anti-fuzzy right ideal with  $T$  – anti-fuzzy left ideal is equal to its product. We discuss some of its properties. We have shown that product of two  $T$  – fuzzy ideals and  $\Pi A_i$  of  $T$  – fuzzy ideal.  $\ell$  – near-ring.

## II. DEFINITIONS AND EXAMPLES

### Definition: 1

A non-empty set  $R$  is called a near-ring with two binary operations “+” and “.” satisfying the following axioms:

- (i)  $(R, +)$  is a group
- (ii)  $(R, \cdot)$  is a semigroup
- (iii)  $(x + y) \cdot z = x \cdot z + y \cdot z$ , for all  $x, y, z$  in  $R$   
(i.e. Multiplicative is left distributive with respect to addition) We denote  $x \cdot y$  by  $xy$ .

### Definition: 2

A non-empty set  $R$  is called lattice ordered near-ring or  $\ell$  – near-ring if it has four binary operations “+”, “.”,  $\vee$ ,  $\wedge$  defined on it and satisfy the following

- (i)  $(R, +)$  is a group
- (ii)  $(R, \cdot)$  is a semigroup
- (iii)  $(R, \vee, \wedge)$  is a lattice
- (iv)  $x \cdot (y + z) = x \cdot y + x \cdot z$ , for all  $x, y, z$  in  $R$
- (v)  $x + (y \vee z) = (x + y) \vee (x + z)$ ;  
 $x + (y \wedge z) = (x + y) \wedge (x + z)$ ;  
 $(y \vee z) + x = (y + x) \vee (z + x)$ ;  
 $(y \wedge z) + x = (y + x) \wedge (z + x)$
- (vi)  $x \cdot (y \vee z) = (xy) \vee (xz)$ ;  
 $x \cdot (y \wedge z) = (xy) \wedge (xz)$ ;  
 $(y \vee z) \cdot x = (yx) \vee (zx)$ ;  
 $(y \wedge z) \cdot x = (yx) \wedge (zx)$ , for all  $x, y, z$  in  $R$  and  $x \geq 0$

### Example: 1

$(n\mathbb{Z}, +, \cdot, \vee, \wedge)$  is a  $\ell$  – near-ring, where  $\mathbb{Z}$  is the set of all integers and  $n \in \mathbb{Z}$

### Definition: 3

A mapping from a nonempty set  $X$  to  $[0, 1]$   $\mu: X \rightarrow [0, 1]$  is called a fuzzy subset of  $X$ .

**Definition: 4**

A fuzzy subset  $\mu$  of a lattice ordered ring (or  $\ell$ -ring)  $R$  is called an anti-fuzzy sub  $\ell$ -ring of  $R$ , if the following conditions are satisfied

- (i)  $\mu(x-y) \leq \max\{\mu(x), \mu(y)\}$
- (ii)  $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$
- (iii)  $\mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$
- (iv)  $\mu(x \wedge y) \leq \max\{\mu(x), \mu(y)\}, \forall x, y \in R$

**Example: 2**

Consider the anti-fuzzy subset  $\mu$  of the  $\ell$ -ring  $(Z, +, \cdot, \vee, \wedge)$

$$\mu(x) = \begin{cases} 0.4 & \text{if } x \in \langle 3 \rangle \\ 0.8 & \text{if } x \in Z - \langle 3 \rangle \end{cases}$$

Then  $\mu$  is an anti-fuzzy  $\ell$ -sub ring.

**Example: 3**

Consider a fuzzy subset  $\mu$  of the  $\ell$ -ring  $(Z, +, \cdot, \vee, \wedge)$

$$\mu(x) = \begin{cases} 0.3 & \text{if } x \in \langle 3 \rangle \\ 0.8 & \text{if } x \in Z - \langle 3 \rangle \end{cases}$$

Then  $\mu$  is not an anti-fuzzy  $\ell$ -sub ring

For example, let  $x = 3$  and  $y = 7$ ,

then  $x + y = 10$ .

Here  $\mu(x) = 0.8$  and  $\mu(y) = 0.8$

$\therefore \max\{\mu_A(x), \mu_B(x)\} = \max\{0.8, 0.8\} = 0.8$

But  $\mu_A(x+y) = 0.3$ .

Hence  $\mu_A(x+y) \leq \max\{\mu_A(x), \mu_A(y)\}$ .

Thus  $\mu$  is not an anti-fuzzy  $\ell$ -sub ring of  $R$

**Definition: 5**

Let  $R$  be a  $\ell$ -near-ring. A nonempty subset  $(I, +)$  of  $(R, +)$  is called

- (i) a left ideal if  $x \cdot (y+i) - x \cdot y \in I$ , for all  $x, y \in R$  and  $i \in I$
- (ii) a right ideal if  $i \cdot x \in I$ , for all  $x \in R$  and  $i \in I$
- (iii) an ideal if it is both a left ideal and a right ideal of  $R$

**Definition: 6**

A fuzzy set  $\mu$  in a near-ring  $R$  is said to be an anti-fuzzy ideal of  $R$ , if the following conditions are satisfied,

- (i)  $\mu(x-y) \leq \max\{\mu(x), \mu(y)\}$
- (ii)  $\mu(y+x-y) \leq \mu(x)$
- (iii)  $\mu(xy) \leq \mu(y); \mu(xy) \leq \mu(x)$
- (iv)  $\mu((x+z)y - xy) \leq \mu(z), \forall x, y, z \in R$

**Proposition: 1**

If  $\mu$  is an anti-fuzzy ideal of  $R$ , then  $\mu(0) \leq \mu(x)$ , for all  $x \in R$

**Definition: 7**

A fuzzy subset  $\mu$  of a  $\ell$ -near-ring  $R$  is called an anti-fuzzy ideal, if the following conditions are satisfied,

- (i)  $\mu(x-y) \leq \max\{\mu(x), \mu(y)\}$
- (ii)  $\mu(y+x-y) \leq \mu(x)$
- (iii)  $\mu(xy) \leq \mu(y); \mu(xy) \leq \mu(x)$
- (iv)  $\mu((x+z)y - xy) \leq \mu(z)$
- (v)  $\mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$
- (vi)  $\mu(x \wedge y) \leq \max\{\mu(x), \mu(y)\} \forall x, y, z \in R$

**Example: 4**

Consider a near-ring  $R = \{a, b, c, d\}$  with the following Cayley's tables:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

.	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	b

We define an anti-fuzzy subset  $\mu: R \rightarrow [0, 1]$  by

$\mu(a) < \mu(b) < \mu(d) = \mu(c)$

Then  $\mu$  is an anti-fuzzy right (resp. left) ideal of  $R$ .

**Definition: 8**

A mapping  $T: [0, 1] \times [0, 1] \rightarrow [0, 1]$  is called a triangular norm [ $t$ -norm] if and only if it satisfies the following conditions:

- (i).  $T(x, 1) = T(1, x) = x$ , for all  $x \in [0, 1]$
- (ii).  $T(x, y) = T(y, x)$ , for all  $x, y \in [0, 1]$ .
- (iii).  $T(x, T(y, z)) = T(T(x, y), z)$
- (iv).  $T(x, y) \leq T(x, z)$ , whenever  $y \leq z$

**Proposition: 2**

The minimum  $T$ -norm (min  $T$ -norm) is defined by  $T(a, b) = \min\{a, b\}$

**Definition: 9**

An anti-fuzzy subset  $\mu$  of a ring  $R$  is called  $T$ -anti-fuzzy right (resp. left) ideal if

- (i)  $\mu(x-y) \leq T(\mu(x), \mu(y))$
- (ii)  $\mu(xy) \leq (\mu(x))$  (resp. left  $\mu(xy) \leq (\mu(y))$ ), for all  $x, y$  in  $R$

**Definition: 10**

A fuzzy subset  $\mu$  of a ring  $R$  is called  $T$ -anti-fuzzy ideal if it is satisfied both right and left ideals.

**Proposition: 3**

Every anti-fuzzy right ideal of a ring  $R$  is an  $T$ -anti-fuzzy right ideal.

**Definition: 11**

A fuzzy subset  $\mu$  of a  $\ell$ -ring  $R$  is called an  $T$ -anti-fuzzy ideal, if the following conditions are satisfied,

- (i)  $\mu(x-y) \leq T(\mu(x), \mu(y))$
- (ii)  $\mu(xy) \leq \mu(x); \mu(xy) \leq \mu(y)$
- (iii)  $\mu(x \vee y) \leq T(\mu(x), \mu(y))$
- (iv)  $\mu(x \wedge y) \leq T(\mu(x), \mu(y))$ , for all  $x, y \in R$

**Definition: 12**

A fuzzy subset  $\mu$  of a near-ring  $R$  is called an  $T$ -anti-fuzzy ideal, if the following conditions are satisfied,

- (i)  $\mu(x-y) \leq T(\mu(x), \mu(y))$
- (ii)  $\mu(y+x-y) \leq (\mu(x))$
- (iii)  $\mu(xy) \leq \mu(y); \mu(xy) \leq \mu(x)$
- (iv)  $\mu((x+z)y-x y) \leq (\mu(z))$ ,  $\forall x, y, z \in R$

**Proposition: 4**

Every  $T$ -fuzzy ideal of a near-ring  $R$  is a  $T$ -fuzzy subnear-ring of  $R$ . Converse of Proposition 1 may not be true in general as seen in the following example.

**Example: 5**

From example: 4

Let  $T: [0,1] \times [0,1] \rightarrow [0,1]$  be a function defined by  $T(x, y) = \max(x+y-1, 0)$ , which is a  $t$ -norm for all  $x, y \in [0,1]$ . By routine calculations, it is easy to check that  $\mu$  is a  $T$ -anti-fuzzy subnear-ring of  $R$ . It is clear that  $\mu$  is also left  $T$ -anti-fuzzy ideal of  $R$ . But  $\mu$  is not  $T$ -anti-fuzzy right ideal of  $R$ , since  $\mu((c+d)d-cd) = \mu(d) < \mu(b)$

**Definition: 13**

A fuzzy subset  $\mu$  of a  $\ell$ -near-ring  $R$  is called an  $T$ -anti-fuzzy ideal, if the following conditions are satisfied,

- (i)  $\mu(x-y) \leq T(\mu(x), \mu(y))$
- (ii)  $\mu(y+x-y) \leq (\mu(x))$
- (iii)  $\mu(xy) \leq \mu(y); \mu(xy) \leq \mu(x)$
- (iv)  $\mu((x+z)y-x y) \leq (\mu(z))$

$$(v) \quad \mu(x \vee y) \leq T(\mu(x), \mu(y))$$

$$(vi) \quad \mu(x \wedge y) \leq T(\mu(x), \mu(y)), \quad \forall x, y, z \in R$$

**Example: 6**

Now  $(R = \{a, b, c\}, +, \cdot, \vee, \wedge)$  is a  $\ell$ -near-ring.

Consider an anti-fuzzy subset  $\mu$  of the  $\ell$ -ring  $R$

$$\mu(x) = \begin{cases} 0.2 & \text{if } x = a \\ 0.5 & \text{if } x = b \\ 0.8 & \text{if } x = c \end{cases}$$

Then  $\mu$  is an  $T$ -anti-fuzzy ideal of  $\ell$ -near-ring  $R$

**Definition: 14**

Let  $\mu$  and  $\lambda$  be  $T$ -anti-fuzzy ideals of a  $\ell$ -near-ring  $R$ . Then the direct product of  $T$ -anti-fuzzy ideal is defined by  $(\mu \times \lambda)(x, y) = T(\mu(x), \lambda(y))$ , for all  $x, y$  in  $R$

### III. THEOREMS

**Theorem: 1**

Every fuzzy ideal of a  $\ell$ -near-ring  $R$  is an  $T$ -anti-fuzzy ideal in  $\ell$ -near-ring  $R$ .

**Proof:**

Let  $\mu$  be a fuzzy right ideal of  $R$ . Then

- (i)  $\mu(x-y) \leq T(\mu(x), \mu(y)), \quad \forall x, y \in R$
- (ii)  $\mu(y+x-y) \leq \mu(x), \quad \forall x, y \in R$
- (iii)  $\mu(xy) \leq \mu(y), \quad \forall x, y \in R$
- (iv)  $\mu((x+z)y-x y) \leq \mu(z), \quad \forall x, y, z \in R$
- (v)  $\mu(x \vee y) \leq T(\mu(x), \mu(y)), \quad \forall x, y \in R$
- (vi)  $\mu(x \wedge y) \leq T(\mu(x), \mu(y)), \quad \forall x, y \in R$

Hence  $\mu$  is an  $T$ -anti-fuzzy ideal in  $\ell$ -near-ring  $R$

**Theorem: 2**

If  $\mu$  and  $\lambda$  are any two  $T$ -anti-fuzzy ideal of  $\ell$ -near-rings  $R_1$  and  $R_2$  then the product of  $\mu \times \lambda$  is also  $T$ -anti-fuzzy ideal of  $\ell$ -near-ring  $R_1 \times R_2$ .

**Proof:**

Given  $\mu$  and  $\lambda$  are any two  $T$ -anti-fuzzy ideal of  $\ell$ -near-rings  $R_1$  and  $R_2$  respectively.

Let  $x, y, z \in R$

$$\begin{aligned} (i). \quad (\mu \times \lambda)(x-y) &= T(\mu(x-y), \lambda(x-y)) \\ &\leq T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y))) \\ &= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y)) \\ &= T(T(T((\mu \times \lambda)(x)), \mu(y)), \lambda(y)) \\ &= T(T((\mu \times \lambda)(x)), T((\mu \times \lambda)(y))) \end{aligned}$$

$$\begin{aligned}
 &= T((\mu \times \lambda)(x), (\mu \times \lambda)(y)) \\
 \therefore (\mu \times \lambda)(x - y) &\leq T((\mu \times \lambda)(x), (\mu \times \lambda)(y)), \\
 &\quad \forall x, y \in R \\
 \text{(ii). Since } \mu(y + x - y) &\leq \mu(x) \text{ and} \\
 \lambda(y + x - y) &\leq \lambda(x) \\
 (\mu \times \lambda)(y + x - y) &= T(\mu(y + x - y), \lambda(y + x - y)) \\
 &\leq T(\mu(x), \lambda(x)) \\
 &= (\mu \times \lambda)(x) \\
 \therefore (\mu \times \lambda)(y + x - y) &\leq (\mu \times \lambda)(x), \forall x, y \in R \\
 \text{(iii). Since } \mu(xy) &\leq \mu(x) \text{ and } \lambda(xy) \leq \lambda(x) \\
 (\mu \times \lambda)(xy) &\leq T(\mu(xy), \lambda(xy)) \\
 &\leq T(\mu(x), \lambda(x)) = (\mu \times \lambda)(x) \\
 \therefore (\mu \times \lambda)(xy) &\leq (\mu \times \lambda)(x), \forall x, y \in R \\
 \text{(iv) Since } \mu((x + z)y - xy) &\leq \mu(z) \text{ and} \\
 \lambda((x + z)y - xy) &\geq \lambda(z) \\
 (\mu \times \lambda)((x + z)y - xy) &= T(\mu((x + z)y - xy), \lambda((x + z)y - xy)) \\
 &\leq T(\mu(z), \lambda(z)) \\
 &\leq (\mu \times \lambda)(z) \\
 \therefore (\mu \times \lambda)((x + z)y - xy) &\leq (\mu \times \lambda)(z), \forall x, y, z \in R \\
 \text{(v). } (\mu \times \lambda)(x \vee y) &= T(\mu(x \vee y), \lambda(x \vee y)) \\
 &\leq T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y))) \\
 &= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y)) \\
 &= T(T(T((\mu \times \lambda)(x)), \mu(y)), \lambda(y)) \\
 &= T(T((\mu \times \lambda)(x)), T((\mu \times \lambda)(y))) \\
 &= T((\mu \times \lambda)(x), (\mu \times \lambda)(y)) \\
 \therefore (\mu \times \lambda)(x \vee y) &\leq T((\mu \times \lambda)(x), (\mu \times \lambda)(y)), \\
 &\quad \forall x, y \in R \\
 \text{(vi). } (\mu \times \lambda)(x \wedge y) &= T(\mu(x \wedge y), \lambda(x \wedge y)) \\
 &\leq T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y))) \\
 &= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y)) \\
 &= T(T(T((\mu \times \lambda)(x)), \mu(y)), \lambda(y)) \\
 &= T(T((\mu \times \lambda)(x)), T((\mu \times \lambda)(y))) \\
 &= T((\mu \times \lambda)(x), (\mu \times \lambda)(y)) \\
 \therefore (\mu \times \lambda)(x \wedge y) &\leq T((\mu \times \lambda)(x), (\mu \times \lambda)(y)), \\
 &\quad \forall x, y \in R
 \end{aligned}$$

Thus  $(\mu \times \lambda)$  is an  $T$ -anti-fuzzy right ideal of  $\ell$ -near-ring  $R_1 \times R_2$ .

**Theorem: 3**

If  $\mu_i$  are  $T$ -anti-fuzzy ideal of  $\ell$ -near-rings  $R_i$ , then  $\Pi \mu_i$  is  $T$ -anti-fuzzy ideal of  $\ell$ -near-ring  $\Pi R_i$

**Proof:**

If  $\mu_i$  are  $T$ -anti-fuzzy ideal of  $\ell$ -near-rings  $R_i$ ,

Let  $x, y, z \in R$  and

Let  $\mu_i \in R_i, i=1, 2, 3, \dots, n$

Let  $\mu_1 \times \mu_2 \times \dots \times \mu_n$  in  $R_1 \times R_2 \times \dots \times R_n$

Let  $x = (x_1, x_2, \dots, x_n)$  and  $y = (y_1, y_2, \dots, y_n) \in R_i$

Let  $R = \prod_{i=1}^n R_i$

$$\begin{aligned}
 \text{(i). } (\mu_1 \times \mu_2 \times \dots \times \mu_n)(x - y) &= T(\mu_1(x - y), \mu_2(x - y), \dots, \mu_n(x - y)) \\
 &\leq T(T(\mu_1(x), \mu_1(y)), \dots, T(\mu_n(x), \mu_n(y))) \\
 &= T(T((\mu_1 \times \dots \times \mu_n)(x)), T((\mu_1 \times \dots \times \mu_n)(y))) \\
 &= T((\mu_1 \times \dots \times \mu_n)(x), (\mu_1 \times \dots \times \mu_n)(y)) \\
 \therefore (\mu_1 \times \dots \times \mu_n)(x - y) &\leq T((\mu_1 \times \dots \times \mu_n)(x), (\mu_1 \times \dots \times \mu_n)(y)) \quad \forall x, y \in R \\
 \text{(ii). Since } \mu_i(y + x - y) &\leq \mu_i(x) \\
 (\mu_1 \times \mu_2 \times \dots \times \mu_n)(y + x - y) &= T(\mu_1(y_1 + x_1 - y_1), \dots, \mu_n(y_n + x_n - y_n)) \\
 &\geq T(\mu_1(x), \mu_2(x), \dots, \mu_n(x)) \\
 &= (\mu_1 \times \mu_2 \times \dots \times \mu_n)(x) \\
 \therefore (\mu_1 \times \dots \times \mu_n)(y + x - y) &\leq (\mu_1 \times \dots \times \mu_n)(x), \\
 &\quad \forall x, y \in R \\
 \text{(iii). Since } \mu_i(xy) &\leq \mu_i(x) \text{ and } \lambda_i(xy) \leq \lambda_i(x) \\
 (\mu_1 \times \mu_2 \times \dots \times \mu_n)(xy) &\leq T(\mu_1(xy), \mu_2(xy), \dots, \mu_n(xy)) \\
 &\leq T(\mu_1(x), \mu_2(x), \dots, \mu_n(x)) \\
 &\leq (\mu_1 \times \mu_2 \times \dots \times \mu_n)(x) \\
 \therefore (\mu_1 \times \mu_2 \times \dots \times \mu_n)(xy) &\leq (\mu_1 \times \mu_2 \times \dots \times \mu_n)(x), \\
 &\quad \forall x, y \in R \\
 \text{(iv). Since } \mu_i((x + z)y - xy) &\leq \mu_i(z) \\
 (\mu_1 \times \mu_2 \times \dots \times \mu_n)((x + z)y - xy) &= T(\mu_1((x_1 + z_1)y_1 - x_1y_1), \dots, \mu_n((x_n + z_n)y_n - x_ny_n)) \\
 &= T(\mu_1(x_1y_1 + z_1y_1 - x_1y_1), \dots, \mu_n(x_ny_n + z_ny_n - x_ny_n)) \\
 &= T(\mu_1(z_1y_1), \mu_2(z_2y_2), \dots, \mu_n(z_ny_n))
 \end{aligned}$$

$$\begin{aligned}
 &\leq T(\mu_1(z_1), \mu_2(z_2), \dots, \mu_n(z_n)) \\
 &\leq (\mu_1 \times \mu_2 \times \dots \times \mu_n)(z) \\
 &\therefore (\mu_1 \times \dots \times \mu_n)((x+z)y - xy) \leq (\mu_1 \times \dots \times \mu_n)(z), \\
 &\forall x, y \in R \\
 (v) &((\mu_1 \times \dots \times \mu_n))(x \vee y) = T(\mu_1(x \vee y), \dots, \mu_n(x \vee y)) \\
 &\leq T(T(\mu_1(x), \mu_1(y)), \dots, T(\mu_n(x), \mu_n(y))) \\
 &= T(T((\mu_1 \times \dots \times \mu_n)(x)), T((\mu_1 \times \dots \times \mu_n)(y))) \\
 &= T(((\mu_1 \times \dots \times \mu_n)(x)), ((\mu_1 \times \dots \times \mu_n)(y))) \\
 &\therefore (\mu_1 \times \mu_2 \times \dots \times \mu_n)(x \vee y) \\
 &\leq T((\mu_1 \times \dots \times \mu_n)(x), (\mu_1 \times \dots \times \mu_n)(y)) \quad \forall x, y \in R \\
 (vi) &(\mu_1 \times \dots \times \mu_n)(x \wedge y) = T(\mu_1(x \wedge y), \dots, \mu_n(x \wedge y)) \\
 &\leq T(T(\mu_1(x), \mu_1(y)), \dots, T(\mu_n(x), \mu_n(y))) \\
 &= T(T((\mu_1 \times \dots \times \mu_n)(x)), T((\mu_1 \times \dots \times \mu_n)(y))) \\
 &= T((\mu_1 \times \dots \times \mu_n)(x), (\mu_1 \times \dots \times \mu_n)(y)) \\
 &\therefore (\mu_1 \times \dots \times \mu_n)(x \wedge y) \\
 &\leq T((\mu_1 \times \dots \times \mu_n)(x), (\mu_1 \times \dots \times \mu_n)(y)), \quad \forall x, y \in R
 \end{aligned}$$

Thus  $\mu_1 \times \mu_2 \times \dots \times \mu_n$  is an  $T$ -anti-fuzzy ideal of  $\ell$ -near-ring  $R_i$ .

Hence  $\Pi \mu_i$  is an  $T$ -anti-fuzzy ideal of  $\ell$ -near-ring  $R_i$

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