Deformation of Poroelastic Half-Space due to Centre of Dilatation

Heena[#1], Ram Swaroop[*2], Vinay Kumar[#3]

[#]Department of Mathematics, RPIIT, Karnal (HR), India
**Department of Mathematics, NIMS University, Jaipur, India*
[#] Department of CSE, GNIOT, Greater Noida (UP), India

Abstract-*The closed-form analytical expressions for the displacements and strains due to centre of dilatation located in a homogeneous, isotropic poroelastic half-space are obtained. The variation of the radial displacement and strains with epicentral distance for the various materials Ruhr Sandstone, Tennessee Marble, Charcoal Granite, Berea Sandstone, Westerly Granite are discussed.*

Keywords-*Porous media, deformation, centre of dilatation*

I. INTRODUCTION

Analytic expressions for various nuclei of strain in an elastic half-space, including a centre of dilatation as in [1]. Reference [2] shows the problem of strains in an elastic half-space under the action of concentrated force. Using the Galerkin vector, they obtained the formula for calculating the displacement vector and the stresses for various dipolar sources. Reference [3] shows a centre of dilatation in an elastic half-space to interpret the ground deformation produced in volcanic areas. This model is often called Mogi's model and has been used very extensively since then. A complete solution for the fundamental type of nuclei of strain as in [4] in which the two Lamé parameters λ and μ are equal (Poisson's ratio ν= 0.25). Reference [7] shows an analytic solution for the displacement and stress fields due to centre of dilatation and pressure source in a viscoelastic half-space. The results obtained were applied to the volcanic areas of Campi Flegrei. Reference [8] shows the deformation of two welded elastic half-spaces caused by point dislocation sources. Reference [11] shows the displacements and stresses due to a single force in an elastic half-space in welded contact with another half-space. Reference [12] shows the displacements and stress fields produced by a centre of explosion in a viscoelastic half-space in welded contact with another elastic half-space.

A poroelastic half space is an important model for its geophysical and engineering applications. Therefore, the response of a poroelastic half space to the external loads has been studied for a very long time both in the frequency domain and in the time domain. There are many kinds of analytical and numerical methods that can be used to solve the response of a poroelastic half-space. The problem of a point source in a poroelastic media has been discussed by several researchers (e.g. [9], [10], [5] & [6]).

Reference [13] shows the deformation of a homogeneous, isotropic poroelastic half-space due to a concentrated force. In that paper he has considered viz. (1) force normal to the boundary and (2) force parallel to the boundary. The displacement vector in terms of the Galerkin vector was presented. In the present paper, we study displacement and strains for poroelastic half-space due to centre of dilatation.

II. THEORY

A centre of dilatation is equivalent to three equal mutually orthogonal dipoles.

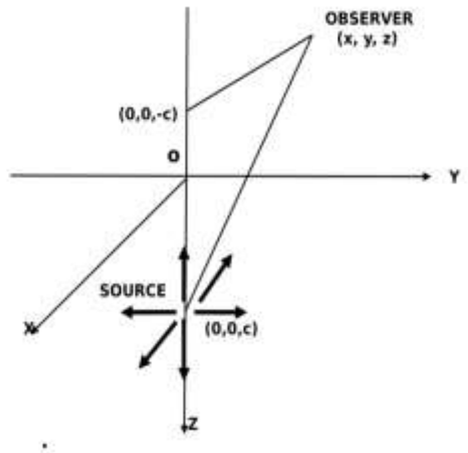

Fig. 1. Geometry of the centre of explosion in two Welded half-spaces

Using displacement field as in [13] we obtained the expressions for the displacement components due to centre of dilatation of magnitude P acting at the point (0, 0, c) in a poroelastic half-space.

$$u_i = \frac{\partial u_i^1}{\partial x_1} + \frac{\partial u_i^2}{\partial x_2} + \frac{\partial u_i^3}{\partial x_3} \quad where\ i = 1,2,3$$

$$u_1 = \frac{Px_1}{8\pi\mu}\left[\begin{array}{l}\hat{\sigma}\left(\dfrac{-1}{R_1^3} + \dfrac{1}{R_2^3} + \dfrac{6c(x_3+c)}{R_2^5}\right) \\ +6c\bar{\sigma}\left\{\dfrac{x_3}{R_2^5} + (x_3+c)\left(\dfrac{1}{R_2^5} - \dfrac{5x_3(x_3+c)}{R_2^7}\right)\right\}\end{array}\right] \quad (1)$$

u_2 can be obtained by replacing x_1 to x_2 in eqn. (1)

$$u_3 = \frac{P}{8\pi\mu}\left[\begin{array}{l} \hat{\sigma}\left\{\begin{array}{l}(x_3 - c)\left(\dfrac{-1}{R_1^3} + \dfrac{1}{R_2^3} + \dfrac{6c(x_3+c)}{R_2^5}\right) \\ +\dfrac{4c}{R_2^3}\end{array}\right\} \\ +6cx_3(x_3+c)\bar{\sigma}\left(\dfrac{(1+4\tilde{\sigma})}{R_2^5} - \dfrac{5(x_3+c)^2}{R_2^7}\right)\end{array}\right]$$

(2)

Where

$R_1^2 = x_1^2 + x_2^2 + (x_3 - c)^2$; $R_2^2 = x_1^2 + x_2^2 + (x_3 + c)^2$ (3)

$\kappa = \lambda + \dfrac{2}{3}\mu$ is bulk-modulus?

λ, μ are Lamé constants?

$\hat{\sigma} = \dfrac{1-2\tilde{\sigma}}{1-\tilde{\sigma}}; \bar{\sigma} = \dfrac{1}{1-\tilde{\sigma}}; \tilde{\sigma} = \dfrac{\tilde{\lambda}}{2(\tilde{\lambda}+\mu)}$ is Poisson ratio.

If the porosity disappears, then $\tilde{\lambda} \to \lambda$

Case 1: When $z \neq 0$

Using $x_1 = r\cos\theta;\quad x_2 = r\sin\theta;\quad x_3 = z$ in eqn (3) we get

$R_1^2 = r^2 + (z-c)^2, \qquad R_2^2 = r^2 + (z+c)^2$

The displacements components in cylindrical co-ordinates are given as

$u_r = u_1\cos\theta + u_2\sin\theta$; $u_\theta = u_1\sin\theta - u_2\cos\theta$;

$u_z = u_3$

$$u_r = \frac{\Pr}{8\pi\mu}\left[\begin{array}{l}\hat{\sigma}\left(\dfrac{-1}{R_1^3} + \dfrac{1}{R_2^3} + \dfrac{6c(z+c)}{R_2^5}\right) \\ +6c\bar{\sigma}\left\{\dfrac{z}{R_2^5} + (z+c)\left(\dfrac{1}{R_2^5} - \dfrac{5z(z+c)}{R_2^7}\right)\right\}\end{array}\right]$$

(4)

$u_\theta = 0$

(5)

$$u_z = \frac{P}{8\pi\mu}\left[\begin{array}{l}\hat{\sigma}\left\{\begin{array}{l}(z-c)\left(\dfrac{-1}{R_1^3} + \dfrac{1}{R_2^3} + \dfrac{6c(z+c)}{R_2^5}\right) \\ +\dfrac{4c}{R_2^3}\end{array}\right\} \\ +6cz(z+c)\bar{\sigma}\left\{\dfrac{(1+4\tilde{\sigma})}{R_2^5} - \dfrac{5(z+c)^2}{R_2^7}\right\}\end{array}\right]$$

(6)

strains in cylindrical co-ordinates can be calculated

Using $e_{ij} = \dfrac{1}{2}\left(\dfrac{\partial u_i}{\partial x_j} + \dfrac{\partial u_j}{\partial x_i}\right)$

$$e_{rr} = \frac{P}{8\pi\mu}\left[\begin{array}{l}\hat{\sigma}\left(\dfrac{-1}{R_1^3} + \dfrac{1}{R_2^3} + 3r^2\left(\dfrac{1}{R_1^5} - \dfrac{1}{R_2^5}\right)\right) \\ -\dfrac{24cz\bar{\sigma}}{R_2^5} \\ +6c(z+c)\left\{\begin{array}{l}2\left(\dfrac{1}{R_2^5} - \dfrac{5r^2}{R_2^7}\right) \\ +\dfrac{35zr^2(z+c)\vec{\sigma}}{R_2^9}\end{array}\right\}\end{array}\right]$$

(7)

$$e_{zz} = \frac{P}{8\pi\mu}\left[\begin{array}{l}\hat{\sigma}\left(\begin{array}{l}\dfrac{-1}{R_1^3} + \dfrac{1}{R_2^3} + \dfrac{3(z-c)^2}{R_1^5} \\ +\dfrac{3(z+c)(6c-(z+c))}{R_2^5}\end{array}\right) \\ \bar{\sigma}\left(\begin{array}{l}\dfrac{18c(4\tilde{\sigma}(z+c)-c)}{R_2^5} \\ +30c(z+c)^2\left\{\begin{array}{l}\dfrac{-2((z+c)\tilde{\sigma}+3z)}{R_2^7} + \\ \dfrac{7z(z+c)^2}{R_2^9}\end{array}\right\}\end{array}\right)\end{array}\right]$$

(8)

$$e_{rz} = \frac{3\Pr}{8\pi\mu}\left[\begin{array}{l}\hat{\sigma}\left(\dfrac{(z-c)}{R_1^5} - \dfrac{(z+c)}{R_2^5}\right) \\ +2c\bar{\sigma}\left\{\dfrac{1}{R_2^5} - 5(z+c)\left(\dfrac{(4z+c)}{R_2^7} - \dfrac{7z}{R_2^9}\right)\right\}\end{array}\right]$$

(9)

$$\begin{array}{l} e_{\theta\theta} = 0, e_{zr} = e_{rz,} \\ e_{z\theta} = 0, e_{\theta z} = 0 \\ e_{\theta r} = 0, e_{r\theta} = 0 \end{array} \qquad (10)$$

Case 2: When $z = 0$

Displacements components are given as

$$u_r = \frac{3\Pr c^2}{4\pi\mu R^5} \qquad (11)$$

$$u_z = \frac{Pc\hat{\sigma}}{4\pi\mu}\left(\frac{2}{R^3} - \frac{3c^2}{R^5}\right) \qquad (12)$$

where $R = \sqrt{r^2 + c^2}$

Strains components are given as

$$e_{rr} = \frac{3Pc^2}{2\pi\mu}\left(\frac{1}{R^5} - \frac{5r^2}{R^7}\right) \qquad (13)$$

$$e_{zz} = \frac{3P\tilde{\sigma}\bar{\sigma}c^2}{2\pi\mu}\left(\frac{3}{R^5} - \frac{5c^2}{R^7}\right) \qquad (14)$$

III. NUMERICAL RESULTS

We define dimensionless epicentral distance D, dimensionless radial displacement U, dimensionless vertical displacement (uplift) W and dimensionless radial strain E by the relations

$$D = \frac{r}{c}, U = \frac{Q}{c}u_r, W = \frac{Q}{c}u_z, E = Q\, e_{rr}$$

Where Q is a dimensionless constant for each source, chosen in such a manner that $W = 1\, at\; r = 0$

$$W = \frac{(1-2D^2)}{(1+D^2)^{5/2}} \qquad (16)$$

$$U = \frac{-6D}{\hat{\sigma}(1+D^2)^{5/2}} \qquad (17)$$

$$E = \frac{-6(1-4D^2)}{\hat{\sigma}(1+D^2)^{\frac{7}{2}}} \qquad (18)$$

$$Q = \frac{-4\pi c^3\mu}{P\hat{\sigma}} \qquad (19)$$

IV. DISCUSSION & CONCLUSION

Analytical expressions for the displacement and strain components for drained behaviour due to five materials, namely, Ruhr Sandstone, Tennessee Marble, Charcoal Granite, Berea Sandstone, and Westerly Granite for Centre of dilatation in a poroelastic medium have been obtained.

For numerical computations, we assume different values of c, r=1 in equations (16)-(18), and using Table 1.

TABLE 1.
MATERIAL PROPERTY

Materials	Poisson ratio ($\tilde{\sigma}$)
Ruhr Sandstone(RS)	0.12
Tennessee Marble(TM)	0.25
Charcoal Granite(CG)	0.27
Berea Sandstone(BS)	0.20
Westerly Granite(WG)	0.25

Numerical result presented shows the variation of the radial displacement, vertical displacement i.e. uplift and radial strain with epicentral distance. The source and material consider in this paper serve as useful models to describe various geophysical phenomenon. A centre of dilatation has been used very extensively to model spherical inflation of magma (Mogi [3]).

Fig (1) shows the variation of dimensionless radial displacement with epicentral distance for drained behaviour of five materials i.e. RS, CG, TM, BS, WG. For all these materials we observe that as we move away from epicentre the displacement decrease gradually. The rate of decrease is more in case of RS as compared to CG.

Fig (2) shows the variation of dimensionless vertical displacement (uplift) with epicentral distance. For all these materials, variation in vertical displacement (uplift) doesn't vary with the material i.e. response is quite similar for all materials. However vertical displacement shows variation near to epicentre and after that it shows constant behaviour.

Fig (3) shows the variation of dimensionless radial strain with epicentral distance for drained behaviour. We observe that radial strain first increases, reaches its maximum (2.62) after that it decreases. Rate of decrease is more in case of CG as compared to RS. Hence CG shows more variation as compared to RS .Value of poisson ratio ($\tilde{\sigma}$) is same for TM & WG.

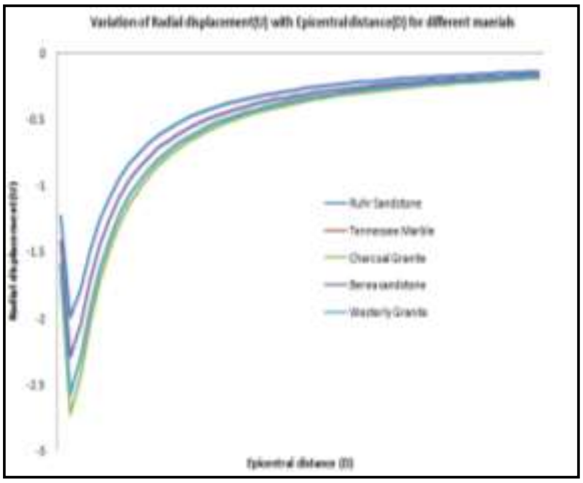

Fig. 1: Variation of radial displacement with epicentral distance

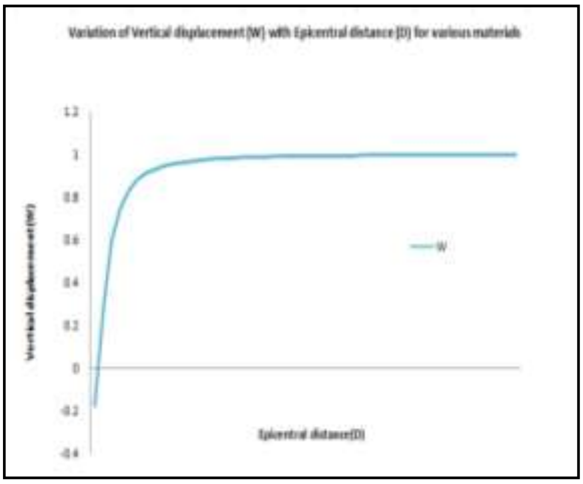

Fig. 2: Variation of vertical displacement with epicentral distance

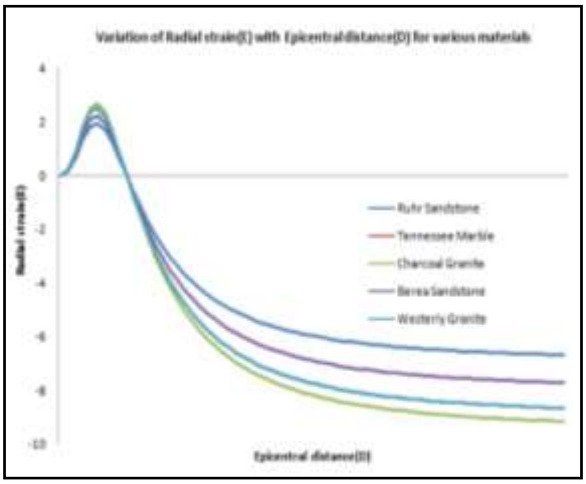

Fig. 3. Variation of radial strain with epicentral distance

REFERENCES

[1] R. D. Mindlin, D. H. Cheng, "Nuclei of strain in the semi-infinite solid," *J. Appl. Phys.*, Vol. 21, pp. 926-930, 1950.

[2] R. D. Mindlin, D. H. Cheng, "Nuclei of strain in the semi-infinite solid, "*J. Appl. Phys.*, Vol. 21, pp. 926-930, 1952.

[3] K. Mogi, "Relations of the eruptions of various volcanoes and the deformations of the ground surface around them," *Bull. Earthq. Res. Inst., Tokyo Univ.*, Vol. 36, pp. 99-134, 1958.

[4] T. Maruyama, "Statical elastic dislocations in an infinite and semi-infinite medium," *Bull Earthq. Res. Inst., Tokyo Univ*., Vol. 42, pp. 289-368, 1964.

[5] J. R. Rice, M. P. Clearly, "Some basic stress diffusion solutions for fluid saturated elastic porous media with compressible constituents," *Rev. Geophys. Space Phys.,* Vol. 14, pp. 227–241, 1976.

[6] J. W. Rudnicki, "Fluid mass sources and point forces in linear elastic diffusive solids," *Mech. Mater,* Vol. 5, pp. 383–393, 1986.

[7] M. Bonafede, M. Dragoni and F. Quareni, "Displacement and stress fields produced by a centre of dilatation and by a pressure source in a viscoelastic half-space application to the study of ground deformation and seismic activity at Campi Flegrei, Italy," *Geophys. J. R. Astr. Soc.,* Vol. 87, pp. 455-485, 1986.

[8] G. Kumari, S. J Singh and K. Singh, "Static deformation of two welded elastic half-spaces caused by point dislocation sources," *Phys. Earth Planet. Int*., Vol. 73, pp. 53-76, 1992.

[9] K. T. Chau, "Fluid point source and point forces in linear elastic diffusive half-spaces," *Mech. Mater*, Volume 23, pp. 241–253, 1996.

[10] E. Pan, "Green's functions in layered poroelastic half-spaces*,"* *Int. J. Numer. Anal. Meth. Geomech.*, Vol. 23, pp. 1631–1653, 1999.

[11] S. J. Singh, G. Kumari and K. Singh, "Displacement and stresses due to a single force in a half-space in welded contact with another half-space," *Geophys. J. Inter.*, Vol. 139, pp. 591-596, 1999.

[12] K. Singh, V. Kumar and R. K. Sehrawat, "Displacement and stress fields produced by a centre of explosion in a viscoelastic half-space in welded contact with another elastic half-space," *Far East J. Appl. Math.*, Vol. 27, No. 1, pp. 45-64, 2007.

[13] Kumar, K. Singh and M. K. Sharma, "Displacement and stress fields in a poroelastic half-space due to a concentrated force," *Int. J. Eng. Sci. Technol,* Vol. 6, No. 4 , pp. 2475-2484, 2012.