Occurrence and Nature of Singularities in (n+2) – Dimensional Monopole Vaidya Solution

K.D.Patil [#1], Lakshmi Madireddy [*2], S.S.Zade [#3]

[#]Professor, [*]Assistant Professor, Department of Applied Mathematics, RTM Nagpur University,
B.D. College of Engineering, Sevagram, (M.S), India

[#]Associate Professor, Department of Mathematics, RTM Nagpur University
J.B. College of Science, Wardha, (M.S), India

Abstract -- *In the present paper we investigate here the occurrence of naked singularities as well as their nature in the gravitational collapse of higher dimensional space times of Monopole Vaidya solution. In the final state of the collapse, Black holes and naked singularities are shown to be developed. The number of dimensions is not restricted. These results involving here might be important in the light of the recent proposal given by String theory, which states that initially our Universe may be of infinite dimensions at higher energy level, there after that it got settled to 4D case by dimensional reduction the lower energy level. Thus final outcome of (n+2) dimensional gravitational collapse becomes most important issue in the gravitational Physics.*

Keywords -- *Gravitational Collapse, Cosmic Censorship, Naked Singularity*

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I. INTRODUCTION

The investigation of final outcome of continual gravitational collapse of matter is one of the most important fields of research in general relativity ([4], [2]). The question of final outcome of endless collapse of a massive star under its own gravity is of central importance in gravitational theory and relativistic astrophysics now a day.

The massive star which is five times more than the size of the sun is undergoes continuous gravitational collapses which has exhausted its nuclear fuel could stabilize as white dwarf or neutron star. In earlier the singularity theorems in general relativity gave some hints referred and predicted that this develops into space time singularity, which is either visible to external universe or hidden within an event horizon of gravity [7]. The space time curvatures and densities get arbitrarily high and because of ultra strong gravity regions they can diverge. Their visibility to faraway observers is determined by the normal structure within the dynamically developed collapsing cloud, which is governed by the Einstein field equations when this internal dynamics of the gravitational collapse delays the formation of horizon, then they become visible and they could also communicate with the physical effects of the External Universe.

It is an assumption that the four dimensional space time of our Universe where we live in is obtained by reducing the higher dimensional space time. There has been the considerable interest in recent times to find the solutions of Einstein equation in more than four dimensional space time, is inspired by the work in String theory and other field theories. It is also believed that the space-time in large energy limit of Plank energy may have higher dimensions other than four. It has been also considered as possible avenues to unify the basic forces of nature in higher dimensional gravity theories.

However, in our experiment we cannot directly observe the extra dimensions. We can detect the extra dimensions indirectly with the help of string theory. Arkani, Hamid and Dimopolous had suggested all the possible effects of extra dimensions. The first inspiration of higher dimensional space time was observed from super gravity theory and super string theory.

The objective of this paper is to explore what is happening at last of higher dimensional space time. Here we are trying to show the results which are acquired in four dimensional space time are also authentic in (n+2) dimensional space time.

In this paper we discuss (n+2) dimensional solution to monopole vaidya space time in section 2 and 3. This is followed by nature of singularity in section 4. Finally we give the concluding remarks in section 5

II. VAIDYA SOLUTION IN (n+2) DIMENSIONAL SPACE TIME

The (n+2) dimensional space time of generalized vaidya metric is as follows [11]

$$ds^2 = -\left(1-\frac{m(w,r)}{r^{n-1}}\right)dw^2 + 2dwdr + r^2[d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_2^3 + \sin^2\theta_1 \sin^2\theta_2 ....\sin^2\theta_{n-1} d\theta_n^2]$$

(1)

Where w is advanced Eddington time coordinate and r is radial coordinate

with the condition $0 < r < \infty$

where m(w,r) is gravitational mass which will present in the sphere of radius r and

$$d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_2^3 + + \sin^2\theta_1 \sin^2\theta_2 ....\sin^2\theta_{n-1} d\theta_n^2$$

(2)

Is the metric of n- sphere and we denote this metric as $d\psi^2$

The corresponding energy momentum tensor is given by

$$T_{\gamma\delta} = T_{\gamma\delta}^{(n)} + T_{\gamma\delta}^{(m)}$$

(3)

$$T_{\gamma\delta}^{(n)} = \mu\, l_\gamma l_\delta$$

(4)

And $T_{\gamma\delta} = (P+\rho)\left(l_\gamma n_\delta + l_\delta n_\gamma\right) + P g_{\gamma\delta}$

(5)

Here the above P and ρ represents the thermodynamic pressure and energy density, where as μ represents energy density of Vaidya null radiation.

l_γ, n_δ are linearly independent two Eigen vectors of energy momentum tensor.

These Eigen vectors are having the components

$$l_\gamma = \lambda_\gamma^0 ,$$

(6)

$$n_\gamma = \frac{1}{2}\left[1 - \frac{m(w,r)}{r^{n-1}}\right]\lambda_\gamma^0 - \lambda_\gamma^1 ,$$

(7)

$$l_\upsilon n^\upsilon = -1, \qquad l_\upsilon l^\upsilon = n_\upsilon n^\upsilon = 0$$

(8)

Especially for the above Eq. (5) when P = ρ = 0 then it reduces to Vaidya solution of higher dimensional space time with m= m (w) [5]

Now we consider EMT of Eq.(7) as the general case.

The energy conditions for the above will be as follows:

1. The dominant energy conditions are
   μ ≥ 0, ρ ≥ P ≥ 0
   (9)
2. The weak and strong energy conditions are
   μ ≥ 0, P ≥ 0 , ρ ≥ 0
   (10)

Einstein field equations is given by

$$G_{\gamma\delta} = KT_{\gamma\delta}$$

(11)

Where $G_{\gamma\delta}$ is Einstein tensor?

K is Gravitational constant

From Eqs (1), (3) and (4) which is having Stress Energy tensor is given by

$$\rho = \frac{nm'}{k(n-1)r^n}$$

(12)

$$P = \frac{-m''}{k(n-1)r^{n-1}}$$

(13)

$$\mu = \frac{n\dot{m}}{k(n-1)r^n}$$

(14)

Here dash and dot represents differentiation with respect to ‘r’ and ‘u’ respectively.

From the above equations

The limitations on ‘m’ should be

$(i)\ m' \geq 0 ,\ m'' \geq 0 \quad (ii)\ \dot{m} > 0$ to satisfy the energy conditions

(i)indicates the mass function either increases with ‘r’ or is constant

(ii)indicates the matter within radius ‘r’ increases with time.

III. (n+2) DIMENSION SPACE TIME IN MONOPOLE VAIDYA SOLUTION

The mass function in four dimensions in Wang theory is

$$m(w,r) = \beta r \qquad 0 < \beta < 1$$

where β is arbitrary constant [11]

The mass function in (n+2) dimensional Vaidya space time is taken as ([1], [6])

$$m(w,r) = \frac{3}{4}\beta r^{n-1} \qquad 0 < \beta < 1 \tag{15}$$

The mass function of Vaidya space time in (n+2) dimension is given by

$$m(w,r) = g(w) \tag{16}$$

In terms of mass function the energy momentum tensor is linear. So, a linear superposition of particular solution is also $m(w,r) = \frac{3}{4}\beta r^{n=1} + g(w)$

(17)

solution of Einstein's eq (1) by combining eqs (15) and (16)

The Physical situation is for w < 0
If g(w) = 0 then the space time is (n+2) dimensional Minkowskian monopole field.
At r = 0; w=0 of growing mass g(w) then the radiation is focused into central singularity.
For w > T the exterior space time settles into (n+2) dimensional Reisster- Nordstom Monopole field solution

Hence by using Eq (17), Eq (1) can be written as

$$ds^2 = -\left(1 - \left(\frac{3/4\,\beta r^{n-1}}{r^{n-1}} + \frac{g(w)}{r^{n-1}}\right)\right) dw^2 + 2\,dwdr + r^2 d\psi^2$$

$$ds^2 = -\left(1 - \frac{3}{4}\beta - \frac{g(w)}{r^{n-1}}\right) dw^2 + 2\,dw\,dr + r^2 d\psi^2$$

(18)

The above metric is also called as (n+2) dimensional generalized Vaidya Monopole metric.

IV. NATURE OF SINGULARITY

Here we are going to investigate the nature of singularity in (n+2) dimensional monopole Vaidya solution. By the use of reference [6] we investigate the nature of singularity ([3, 8, 13]).

If the radial null geodesics equation gives atleast one real and positive root then the central singularity is said to be naked singularity.

For the metric obtained in Eq (18) the outgoing radial null geodesics is given by

$$\frac{dr}{dw} = \frac{1}{2}\left[1 - \frac{3}{4}\beta - \frac{g(w)}{r^{n-1}}\right] \tag{19}$$

For the above equation generally there is no analytic solution to g (w)

However if we choose $g(w) \propto w^{n-1}$ then eq (17) becomes homogeneous and we can solve interms of elementary function.

So, we choose $g(w) = \frac{3}{4}\tau w^{n-1}$

(20)

Now, Vaidya space time (18) becomes

$$ds^2 = -\left[1 - \frac{3}{4}\beta - \frac{3/4\,\tau w^{n-1}}{r^{n-1}}\right] dw^2 + 2dwdr + r^2 d\psi^2$$

(21)

Here we consider radial null geodesics defined by $ds^2 = 0$ by taking $\dot\theta_1 = \dot\theta_2 = \dot\theta_3 = \dot\theta_4 = \ldots\ldots\ldots\ldots = \dot\theta_n = 0$

to investigate the structure of the collapse.

The radial null geodesics for the metric (21) should satisfy the null condition

$$\frac{dw}{dr} = \frac{2}{1 - \frac{3}{4}\beta - \frac{3/4\,\tau w^{n-1}}{r^{n-1}}} \tag{22}$$

The coordinate w is an advanced time coordinate. The above equation has a singularity at $r \to 0,\ w \to 0$
Here we are considering the limiting value as $X = \frac{w}{r}$ along the singular geodesic to approach a singularity
Hence, for the geodesic tangent to exist at this point uniquely we should have the following

$$X_0 = \lim_{\substack{w\to 0\\ r\to 0}} \frac{w}{r} = \lim_{\substack{w\to 0\\ r\to 0}} \frac{dw}{dr}$$

$$X_0 = \lim_{\substack{w\to 0\\ r\to 0}} \frac{2}{1 - \frac{3}{4}\beta - \frac{3/4\,\tau w^{n-1}}{r^{n-1}}} \tag{23}$$

$$X_0 = \frac{2}{1 - \frac{3}{4}\beta - \frac{3}{4}\tau X_0^{n-1}} \tag{24}$$

$$X_0 - \frac{3}{4}\beta X_0 - \frac{3}{4}\tau X_0^{n} = 2$$

$$\frac{3}{4}\tau X_0^{n} + \frac{3}{4}\beta X_0 - X_0 + 2 = 0$$

$$3\tau X_0^{n} + 3\beta X_0 - 4X_0 + 8 = 0$$

$$3\tau X_0^{n} - (4 - 3\beta) X_0 + 8 = 0 \tag{25}$$

From this Eq. we decide the nature of the singularity. The singularity will be naked if there exists at least one real and positive root. If there is no positive root then it ends into a black hole.

To investigate whether the naked singularities will arise or not, we take some different values of τ , β and n

Case (i)

If we substitute $n = 2$ then the Eq (21) reduces to 4D Monopole Vaidya space time. Here we select the values as $n = 2$, $\beta = 0.6$ then the Eq (25) becomes

$$3\tau X_0^{\,2} - 2.2\ X_0 + 8 = 0$$

Let us verify for $\tau = 0.01$ then we obtain a one of the real positive root as $X_0 = 3.837$ which clearly admits that, it is a naked singularity.

Case (ii)

Now we verify for $n = 3$ i.e we get the eq. as five dimensional Monopole Vaidya solution. For $n = 2$, $\beta = 0.6$ we get the equation

$$3\tau X_0^{\,3} - 2.2\ X_0 + 8 = 0$$

Here one of the values appears as 3.706 which is a real and positive value. So, here also we ensure that it is a naked singularity.

In the same way let us take for different values of n

Now we take $n = 4$ i.e for six dimensional then the equation reduces to

$$3\tau X_0^{\,4} - (4 - 3\beta)\ X_0 + 8 = 0$$

Now the roots which are going to obtained for different values of τ and β are given in the following table

TABLE I

VALUES of X_0 for DIFFERENT VALUES of β and τ in 6D

for n=4 (i.e 6D)				
τ	X_0			
	β=0.1	β=0.3	β=0.5	β=0.7
1*10^{-6}	106.51	100.224	93.012	84.425
1*10^{-7}	84.384	79.363	73.591	66.695
3*10^{-6}	73.621	69.216	64.144	58.068
4*10^{-6}	66.821	62.804	58.174	52.616
5*10^{-6}	61.977	58.237	53.921	48.731
6*10^{-6}	58.278	54.749	50.673	45.764
7*10^{-6}	55.321	51.96	48.077	43.391
8*10^{-6}	52.879	49.658	45.933	41.432
9*10^{-6}	50.814	47.71	44.119	39.774

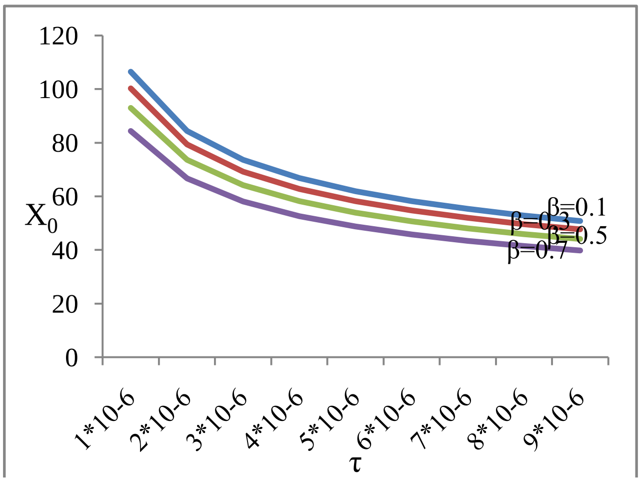

Fig.1 Graph of the Values of X_0 against the values of τ in 6D

From the graph we observe that at the initial state the value of X_0 is at peak and the values of X_0 decreases when increase the value of β.

It is also observed that the peak shifted towards lower values of X_0 for the different values of τ. As we increase the values of β the values of τ of X_0 decreases.

For $n = 5$ (i.e for 7D) the equation becomes

$$3\tau X_0^{\,5} - (4 - 3\beta)\ X_0 + 8 = 0$$

Then for this equation the roots of different values of β in seven dimensional Monopole Vaidya space time is shown below in the following table

TABLE 2

VALUES of X_0 for DIFFERENT VALUES of β and τ in 7D

for n=5 (i.e 7D)				
τ	X_0			
	β=0.1	β=0.3	β=0.5	β=0.7
1*10^{-7}	58.708	56.033	52.897	49.053
2*10^{-7}	49.277	47.008	44.342	41.059
3*10^{-7}	44.471	42.41	39.982	36.983
4*10^{-7}	41.345	39.418	37.146	34.331
5*10^{-7}	39.07	37.241	35.081	32.4
6*10^{-7}	37.303	35.55	33.478	30.901
7*10^{-7}	35.871	34.179	32.178	29.684
8*10^{-7}	34.674	33.034	31.091	28.667
9*10^{-7}	33.651	32.054	30.162	27.798

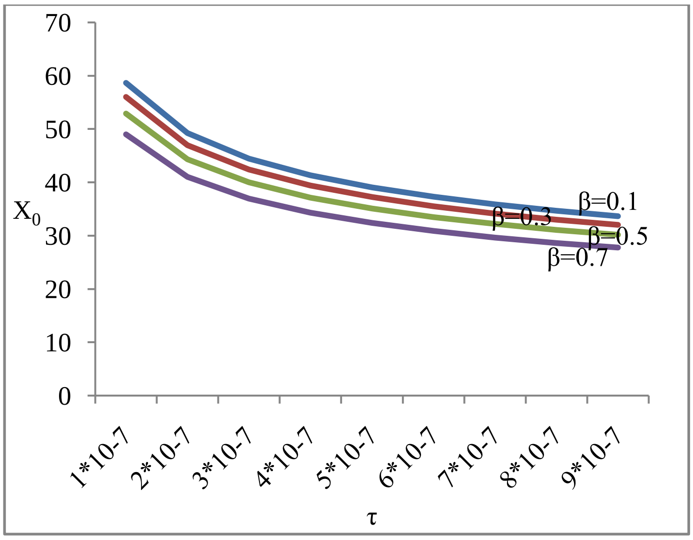

Fig.2 Graph of the Values of X_0 against the values of τ in 7D

We observe in 7D space time, the value of X_0 decrease as we increase the value of τ. Also for increasing β the value decreases rapidly.

Now for n = 6 (i.e for 8D) the equation becomes

$$3\tau X_0{}^6 - (4-3\beta)\, X_0 + 8 = 0$$

Then for this equation the roots of different values of β in 8D

Monopole Vaidya space time is shown below in the following table.

TABLE III

VALUES of X_0 for DIFFERENT VALUES of β and τ in 8D

for n=6 (i.e 8D)				
τ	X_0			
	β=0.1	β=0.3	β=0.5	β=0.7
$1*10^{-8}$	41.069	39.535	37.71	35.427
$2*10^{-8}$	35.693	34.345	32.736	30.712
$3*10^{-8}$	32.876	31.625	30.129	28.241
$4*10^{-8}$	31.012	29.825	28.403	26.604
$5*10^{-8}$	29.638	28.498	27.131	25.397
$6*10^{-8}$	28.559	27.456	26.133	24.45
$7*10^{-8}$	27.678	26.605	25.317	23.675
$8*10^{-8}$	26.936	25.889	24.63	23.023
$9*10^{-8}$	26.298	25.272	24.038	22.462

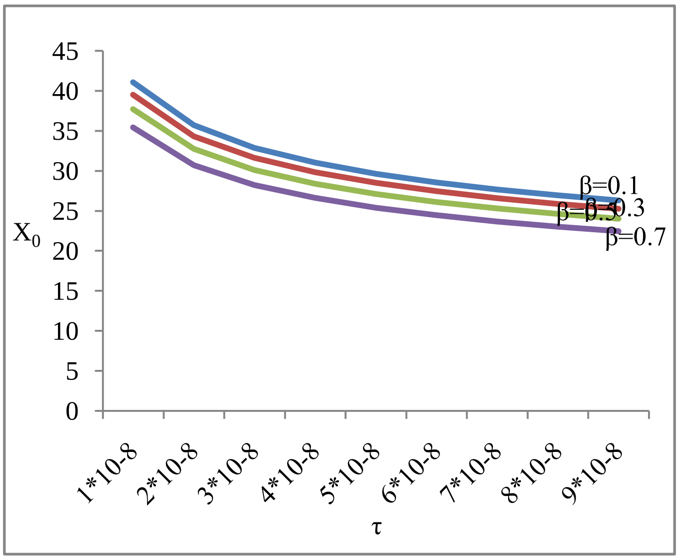

Fig.3. Graph of the Values of X_0 against the values of τ in 8D

We observe in 8D space time, the value of X_0 decrease as we increase the value of τ. Also for increasing β the value decreases rapidly.

Now for n = 7 (i.e for 9D) the equation becomes

$$3\tau X_0{}^7 - (4-3\beta)\, X_0 + 8 = 0$$

Then for this equation the roots of different values of β in 9D Monopole Vaidya space time is shown below in the following table.

TABLE IV

VALUES of X_0 for DIFFERENT VALUES of β and τ in 9D

for n=7 (i.e 9D)				
τ	X_0			
	β=0.1	β=0.3	β=0.5	β=0.7
$1*10^{-9}$	32.373	31.344	30.107	28.535
$2*10^{-9}$	28.798	27.872	26.755	25.328
$3*10^{-9}$	26.89	26.019	24.966	23.616
$4*10^{-9}$	25.163	24.778	23.768	22.469
$5*10^{-9}$	24.663	23.856	22.877	21.615
$6*10^{-9}$	23.913	23.127	22.173	20.941
$7*10^{-9}$	23.296	22.528	21.595	20.387
$8*10^{-9}$	22.774	22.021	21.105	19.917
$9*10^{-9}$	22.324	21.583	20.682	19.512

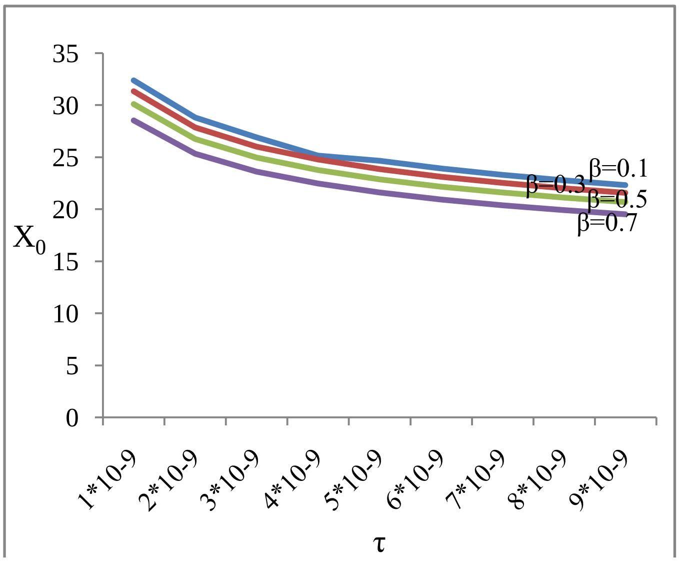

Fig.4. Graph of the Values of X_0 against the values of τ in 9D

We observe in 9D space time, the value of X_0 decrease as we increase the value of β. Also for increasing β the value of X_0 decreases rapidly.

Now for n = 8 (i.e for 10D) the equation becomes $3\tau X_0^{8} - (4-3\beta)\, X_0 + 8 = 0$

Then for this equation the roots of different values of β in10D Monopole Vaidya space time is shown below in the following table.

TABLE V

VALUES of X_0 for DIFFERENT VALUES of β and τ in 10D

for n=8 (i.e 10D)				
τ	X_0			
	β=0.1	β=0.3	β=0.5	β=0.7
$1*10^{-10}$	27.319	26.652	25.644	24.463
$2*10^{-10}$	24.711	24.019	23.176	22.085
$3*10^{-10}$	23.301	22.643	21.841	20.799
$4*10^{-10}$	22.349	21.714	20.939	19.93
$5*10^{-10}$	21.637	21.02	20.265	19.279
$6*10^{-10}$	21.072	20.468	19.729	18.763
$7*10^{-10}$	20.605	20.013	19.288	18.337
$8*10^{-10}$	20.209	19.627	18.912	17.975
$9*10^{-10}$	19.866	19.292	18.587	17.661

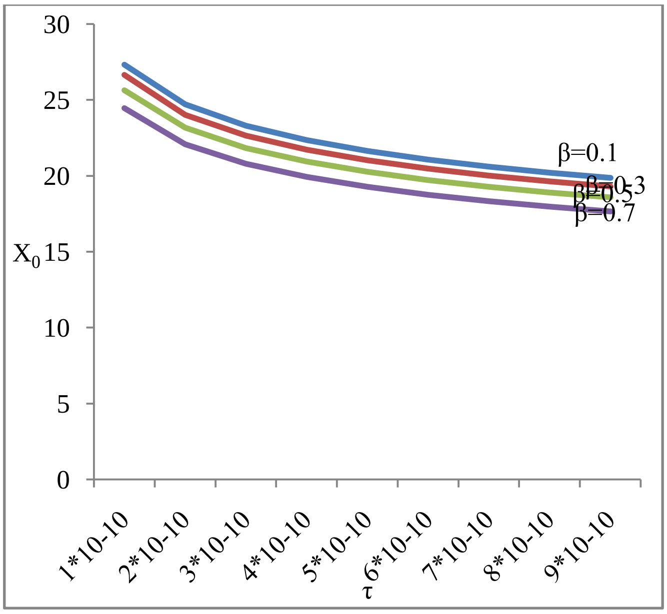

Fig.5. Graph of the Values of X_0 against the values of β and τ in10D

We observe in 10D space time, the value of X_0 decrease as we increase the value of β. Also for increasing β the value of X_0 decreases rapidly.

Case (iii)

For constant β

Let us choose β = 0.2. Then the equation becomes

$$3\tau X_0^{n} - 3.4\, X_0 + 8 = 0$$

Now for different values of τ we find the values of X_0 in the following table

TABLE VI

VALUES of X_0 for DIFFERENT VALUES of n and τ for β = 0.2

τ	X_0					
	n=4	n=5	n=6	n=7	n=8	n=9
$1*10^{-7}$	223.832	57.418	25.257	14.553	9.788	7.253
$2*10^{-7}$	177.491	48.183	21.918	12.912	8.821	6.611
$3*10^{-7}$	154.952	43.478	20.168	12.035	8.297	6.26
$4*10^{-7}$	140.71	40.417	19.01	11.448	7.943	6.02
$5*10^{-7}$	130.566	38.189	18.156	11.011	7.677	5.84
$6*10^{-7}$	122.82	36.459	17.486	10.666	7.466	5.896
$7*10^{-7}$	116.628	35.057	16.938	10.382	7.292	5.577
$8*10^{-7}$	111.516	33.885	16.477	10.142	7.144	5.475
$9*10^{-7}$	107.191	32.883	16.08	9.934	7.016	5.387

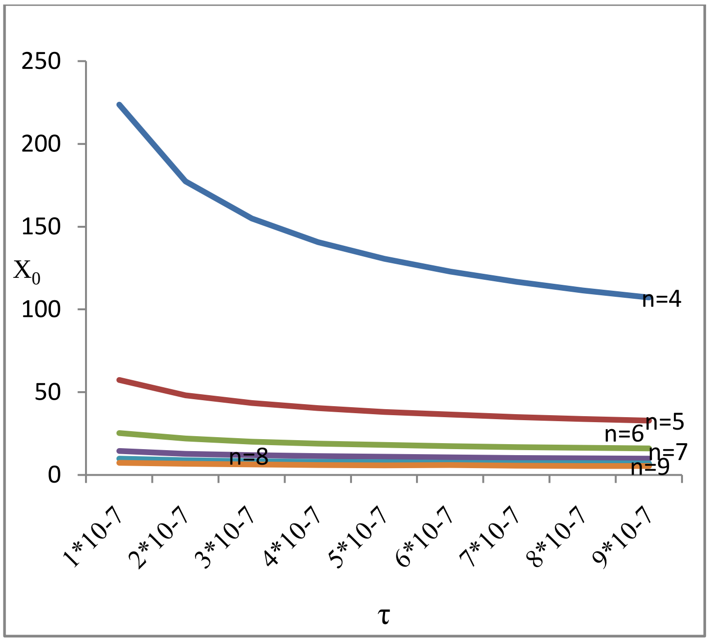

Fig.6. Graph of X_0 against the values of τ for β = 0.2

From the above graph we observed that for a fixed value β the values of X_0 decreases very quickly in higher dimensions. The value of X_0 is at the peak initially then it goes on decreasing with increasing value of τ.

If let us choose β = 0.4. Then the equation becomes $3\tau X_0{}^n - 2.8\ X_0 + 8 = 0$

Now for different values of τ we find the values of X_0 in the following table

TABLE VII

VALUES of X_0 for DIFFERENT VALUES of n and τ for β = 0.4

τ	X_0					
	n=4	n=5	n=6	n=7	n=8	n=9
$1*10^{-9}$	976.311	174.065	61.643	30.757	18.669	12.81
$2*10^{-9}$	774.701	146.255	53.585	27.343	16.862	11.707
$3*10^{-9}$	676.643	132.085	49.363	25.521	15.884	11.104
$4*10^{-9}$	614.683	122.868	46.569	24.3	15.224	10.694
$5*10^{-9}$	570.552	116.162	44.509	23.393	14.73	10.385
$6*10^{-9}$	536.854	110.953	42.893	22.676	14.338	10.14
$7*10^{-9}$	509.917	106.731	41.572	22.087	14.014	9.936
$8*10^{-9}$	487.676	103.202	40.46	21.588	13.739	9.763
$9*10^{-9}$	468.864	100.186	39.504	21.158	13.501	9.612

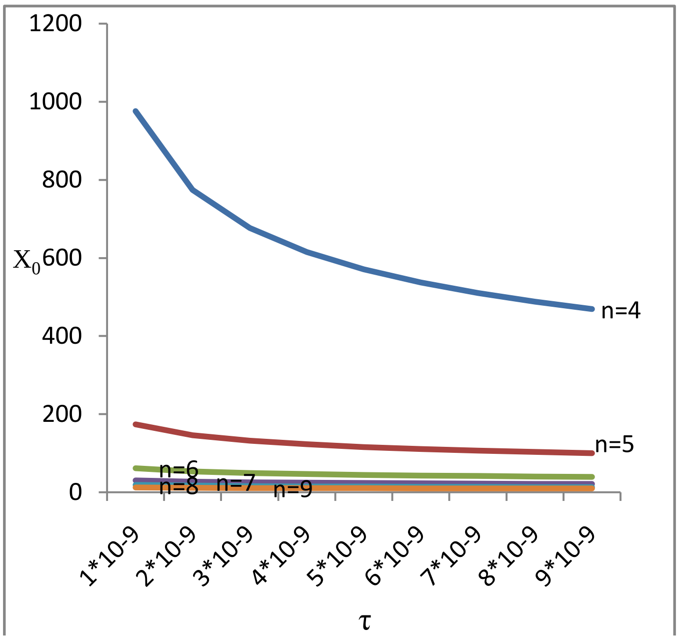

Fig.7. Graph of X_0 against the values of τ for β = 0.4

From the above graph we observed that for a fixed value β the values of X_0 decreases very quickly in higher dimensions.

The value of X_0 is at the peak initially then it goes on decreasing with increasing value of τ.

V. CONCLUDING REMARKS

Gravitational Collapse is one of the most interesting topics of the gravitational Physics and Astrophysics [9]. The end stage of the collapse was a big question to everyone. No one has identified exactly what is happening at the final stage. From the Cosmic Censorship Hypothesis initially we have an idea that the end state of singularity should be hidden by horizons. ([10], [12]).

From the past decades many examples has opposed this Cosmic Censorship Hypothesis. Even no one has given the exact proof. In this above example, it gives an idea that the end state of singularity is not hidden by a horizon, and is called as Naked Singularity.

In our present work, the final state of Monopole Vaidya Solution in (n+2) dimensional space time results in naked singularity. The results which are appeared in four dimensional space time will also valid in (n+2) dimensional space time.

For the different arbitrary dimensions we have shown that naked singularities occur in (n+2) dimensional space time of Monopole Vaidya solution. We also identified that by taking increasing values of β we got the decreasing values of X_0.

For the constant β the roots are going on increasing for the higher dimensions. This indicates that Black holes as well as naked singularities are obtained at the final state of the collapse. But we had point out that real and positive roots are obtained which indicates that the singularity is naked. This violates the Cosmic Censorship Hypothesis which says that only Black holes are observed. So, we conclude that singularity is naked in the final state of (n+2) dimensional space time of Monopole Vaidya solution.

REFERENCES

[1] Anzhong Wang and Yumei Wu. Generalized Vaidya solution, *Gen. Relativ. Gravit*., Vol.31, No.1, 107 , 1999.

[2] D. Christodoulou . *Commu. Math. Phys*. 93, 171, 1984.

[3] I.H.Dwivedi and P.S. Joshi , On the nature of naked singularities in Vaidya space-time, *Class. Quantum. Grav*., 6, 1599, 1989.

[4] D.M.Eardley and L.Smarr *Phys.Rev.D* 19, 2239, 1979.

[5] S.W.Hawking and G.F.R.Ellis The Large Scale Structure of Space time, *Cambridge University Press*, Cambridge, 1973.

[6] V.Husain, Exact solutions for null fluid collapse, *Phys. Rev*., D 53, R 1759, 1996.

[7] R.P.A.C. Newmann. *Class.Quantum grav*. 3, 527, 1986.

[8] K.D.Patil Structure of radial null geodesics in higher dimensional dust collapse , *Phys.Rev .D67*, 024017, 2003.

[9] K.D**.**Patil., Gravitational collapse in higher dimensional charged-Vaidya spacetime, *Pramana J. Phys.*, Vol. **60***(3)*, 423-431, 2003.

[10] K.D.Patil, The final fate of inhomogeneous dust collapse in higher dimensional space-time, *Indian J. Phys.*, Vol. 77, No.3, 2003.

[11] K.D.Patil., C.S.Khodre. and S.S.Zade, 'Final Fate of (n+2)-Dimensional Monopole Vaidya Solution', *International Journal of Mathematical Research & science,* Vol. 1, 2013.

[12] K.DPatil, R.V.Saraykar and S.H.Ghate *Pramana, J.Phys., Indian Academy of Science* Vol.52, No.2, 253, 1999.

[13] K.D.Patil and S.S.Zade, Nature of singularities in higher dimensiona Hussain Space time, *Int. J. Mod. Phys.D*15, 1359-1371, 2006.