Labeling Techniques of Holiday Star Graph

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Abstract - *This paper aims to focus on some labeling methods of Holiday Star Graph. We investigate Holiday Star Graph with six types of labeling; Cordial, H-cordial, Prime, Total prime, Vertex prime, Difference cordial.*

Keywords - *Graph Labeling, Holiday Star Graph, Cordial Labeling, Prime Labeling.*

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1. INTRODUCTION

We begin with simple, finite, undirected graph $G=(V(G),E(G))$ where $V(G)$ and $E(G)$ denotes the vertex set and the edge set respectively. For a finite set A, |A| denotes the number of elements of A. For all other terminology, we follow Gross [5]. We provide some useful definitions for the present work.

***Definition 1.1**:* The *graph labeling* is an assignment of numbers to the vertices or edges or both subject to certain condition(s).
A detailed survey of various graph labeling is explained in Gallian [4].

Definition 1.2: For a graph $G = (V(G), E(G))$, a mapping $f:V(G) \to \{0,1\}$ is called a *binary vertex labeling* of G and f(v) is called the *label* of the vertex v of G under f. For an edge $e = uv$, the induced edge labeling $f^*:E(G)\to\{0,1\}$ defined as $f^*(uv)=|f(u)-f(v)|$. Let $v_{f(0)}$, $v_{f(1)}$ be the number of vertices of G having labels 0 and 1 respectively under f and let $e_{f(0)}$, $e_{f(1)}$ be the number of edges having labels 0 and 1 respectively under f^*.

Definition 1.3: A binary vertex labeling f of a graph G is called a *cordial labeling* if $|v_{f(1)}-v_{f(0)}|\le 1$ and $|e_{f(1)}-e_{f(0)}|\le 1$. A graph G is said to be *cordial* if it admits *cordial labeling*.

The concept of cordial labeling was introduced by Cahit [1]. Lee and Liu [6] Lee proved that all complete bipartite graphs and all fans are cordial. Further, they proved that, the cycle C_n is cordial if and only if $n\neq 2(\text{mod}\,4)$, the wheel W_n is cordial if and only if $n\neq 3(\text{mod}\,4)$, $n\ge 3$. Prajapati and Gajjar [13] proved that complement of wheel graph and complement of cycle graph are cordial if $n\neq 4(\text{mod}\,8)$ or $n\neq 7(\text{mod}\,8)$. Prajapati and Gajjar [14] proved that cordial labeling in the context of duplication of cycle graph and path graph.

Definition 1.4: Let G be a graph with vertex set $V(G)$ and edge set $E(G)$ and let $f:E(G)\to\{0,1\}$. Define f^* on $V(G)$ by $f^*=\sum\{f(uv)/uv \in E(G)\}\ (mod\ 2)$. The function f is called an E- cordial labeling of G if $|v_{f(1)}-v_{f(0)}|\le 1$ and $|e_{f(1)}-e_{f(0)}|\le 1$. A graph is called *E-cordial* if it admits *E-cordial labeling*.

In 1997 Yilmaz and Cahit [19] introduced E-cordial labeling as a weaker version of edge-graceful labeling and with the blend of cordial labeling. They proved that the trees with n vertices, K_n, C_n are E-cordial if and only if $n\neq 2(\text{mod}\,4)$ while $K_{m,n}$ admits E-cordial labeling if and only if $m+n\neq 2(\text{mod}\,4)$.

Definition 1.5: A *prime labeling* of a graph G is an injective function $f:V(G)\to\{1,2,...,|V|\}$ such that for every pair of adjacent vertices u and v, $gcd(f(u),f(v)) = 1$. The graph which admits a *prime labeling* is called a *prime graph*.

The notion of a prime labeling was originated by Roger Entringer and was discussed in a paper by Tout et al. [16]. Many researchers have studied prime graphs. For e.g. Fu and Huang [3] have proved that P_n and $K_{1,n}$ are prime graphs. Lee et al. [7] have proved that W_n is a prime graph if and only if n is even. Vaidya and Prajapati [17] has proved that if $n_1 \geq 4$ is an even integer and n_2 is a natural number, then the graph obtained by identifying any of the rim vertices of a wheel W_{n_1} with an end vertex of a path graph P_{n_2} is a prime graph. Vaidya and Prajapati [18] have proved that switching the apex vertex in W_n is a prime graph and switching a rim vertex in W_n is a prime graph if $n+1$ is prime. In the same paper it has been proved that W_n is switching invariant if n is even.

Definition 1.6 G is called a *vertex prime graph* if there is a bijection $f: E(G) \to \{1,2,...,|E|\}$ such that for any vertex v, $\gcd_{uv \in E}\{f(uv)\} = 1$. The bijection f is called a *vertex prime labeling* of G.

Definition 1.7: Let $G=(V,E)$ be a graph with p vertices and q edges. A bijection $f: V(G) \to \{1,2,...,|V|+|E|\}$ is said to be a *total prime labeling* if for each edge $e = uv$, the labels assigned to u and v are relatively prime and for each vertex of degree at least 2, the greatest common divisor of the labels on the incident edges is 1. A graph which admits Total Prime Labeling is called *total prime graph*.

Prime labeling and vertex prime labeling are introduced in [16] and [2]. Combining these two, The notion of a total prime labeling was originated by Ramasubramanian and Kala [15] have proved that paths P_n, star $K_{1,n}$, bistar, comb, cycles C_n where n is even, helm H_n, $K_{2,n}$ and fan graph are total prime graph.

Definition 1.8: Let $G=(V,E)$ be a (p,q) graph, and f be a map from $V(G)$ to $\{1,2,...,p\}$. For each edge uv assign the label $|f(v)-f(u)| \leq 1$; f is called a difference cordial labeling if f is a one-to-one map and $|e_{f(1)} - e_{f(0)}| \leq 1$ where $e_{f(1)}$ denotes the number of edges labeled with 1 while $e_{f(0)}$ denotes the number of edges not labeled with 1. A graph with a difference cordial labeling is called a difference cordial graph.

Ponraj et al. [10] first introduced the concept of difference cordial labeling in 2013. After that, they introduced many concepts and studied some types of graphs that have this kind of labeling, such as path, cycle, complete graph, complete bipartite graph, bistar, wheel, web, sunfower graph, lotus inside a circle, pyramid, permutation graph, book with n pentagonal pages, t -fold wheel, and double fan, and some more standard graphs were investigated in [8, 9, 10, 11, 12] .

In this paper, for every natural number n the set $\{1,2,...,n\}$ will be denoted by $[n]$.

Origami is an ancient Japanese art of folding paper. The word origami comes from two Japanese words: ``ori", which means to fold, and ``kami", which means paper. Usually origami models are made strictly by folding paper. There is no cutting or gluing involved. Even if origami is mainly an artistic product, it has received a great deal of attention from mathematicians, because of its interesting algebraic and geometrical properties. We present a new graph inspired from a model of origami namely Holiday Star.

Definition 1.9: Let $v_1, v_2, ..., v_{4n-1}, v_{4n}$ be consecutive $4n$ vertices of cycle C_{4n}, $n \geq 3$. Let u_0 be central vertex and $u_1, u_2, ..., u_{2n-1}, u_{2n}$ be end vertices of star $K_{1,2n}$. join u_0 to v_{4i-3} by an edge; join u_{2i-1} to v_{4i-2} and u_{2i} by an edge. for each $i \in [n]$. The resulting graph is called *holiday star graph* HS_n. which is shown in figure 1.

Example:

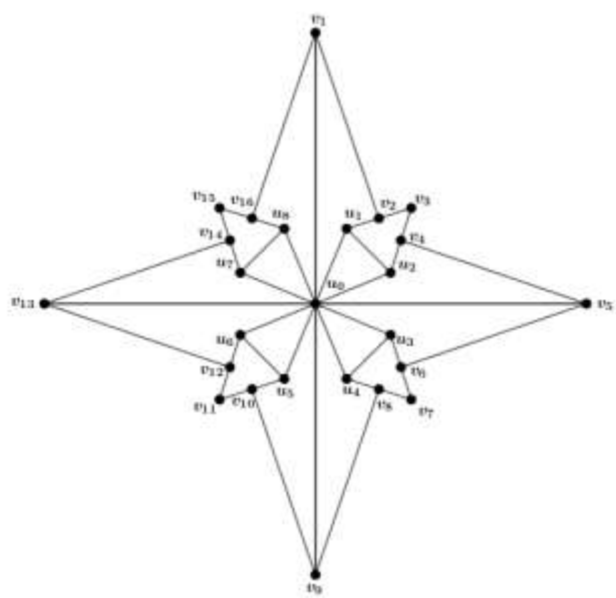

Figure 1: The Holiday Star Graph HS_4.

2. MAIN RESULTS

Theorem 2.1: HS_n is cordial.

Proof: For the graph HS_n,

$V(HS_n) = \{u_0, u_i / 1 \le i \le 2n\} \cup \{v_i / 1 \le i \le 4n\}$ and

$E(HS_n) = \{u_0u_{2i-1}, u_0u_{2i}, u_{2i-1}v_{4i-2}, u_{2i}v_{4i}, u_{2i-1}u_{2i}, u_0v_{4i-3} / 1 \le i \le n\} \cup \{v_iv_{i+1} / 1 \le i \le 4n-1\} \cup \{v_{4n}v_1\}$.

Therefor $|V(HS_n)| = 6n+1$ and $|E(HS_n)| = 10n$.

Define $f: V(HS_n) \to \{0,1\}$ as follows:

$$f(x) = \begin{cases} 0 & if\ \ x = u_0; \\ 1 & if\ \ x \in \{u_{2i-1}, v_{4i-3}, v_{4i-2}\}, i \in [n]; \\ 0 & if\ \ x \in \{u_{2i}, v_{4i-1}, v_{4i}\}, i \in [n]. \end{cases}$$

Thus $v_{f(1)} = 3n$ and $v_{f(0)} = 3n+1$. The induced edge Labeling $f^*: E(HS_n) \to \{0,1\}$ is $f^*(uv) = |f(u) - f(v)|$, for every edge $e = uv \in E$. Therefore

$$f^*(e) = \begin{cases} 1 & if\ e \in \{u_0u_{2i-1}, u_{2i-1}u_{2i}, u_0v_{4i-3}, v_{4i-2}v_{4i-1}\},\ i \in [n]; \\ 0 & if\ e \in \{u_0u_{2i}, u_{2i-1}v_{4i-2}, u_{2i}v_{4i}, v_{4i-1}v_{4i}, v_{4i-3}v_{4i-2}\},\ i \in [n]; \\ 1 & if\ e = v_{4i}v_{4i+1},\ i \in [n-1]; \\ 1 & if\ \ e = v_{4n}v_1. \end{cases}$$

Thus $e_{f(1)} = 5n$ and $e_{f(0)} = 5n$. Therefore f satisfies the conditions $|v_{f(1)} - v_{f(0)}| \le 1$ and $|e_{f(1)} - e_{f(0)}| \le 1$ for cordial labeing . So, f admits cordial labeling of HS_n. Hence HS_n is cordial.

Theorem 2.2: HS_n is E-cordial.

Proof: Proof: For the graph HS_n,

$V(HS_n) = \{u_0, u_i / 1 \le i \le 2n\} \cup \{v_i / 1 \le i \le 4n\}$ and

$E(HS_n) = \{u_0u_{2i-1}, u_0u_{2i}, u_{2i-1}v_{4i-2}, u_{2i}v_{4i}, u_{2i-1}u_{2i}, u_0v_{4i-3} / 1 \le i \le n\} \cup \{v_iv_{i+1} / 1 \le i \le 4n-1\} \cup \{v_{4n}v_1\}$.

Therefor $|V(HS_n)| = 6n+1$ and $|E(HS_n)| = 10n$.

Define $f: E(HS_n) \to \{0,1\}$ as follows:

$$f(e) = \begin{cases} 1 & if\ e \in \{u_0u_{2i-1}, u_{2i-1}u_{2i}, u_{2i-1}v_{4i-2}, v_{4i-3}v_{4i-2}\},\ i \in [n]; \\ 0 & if\ e \in \{u_0u_{2i}, u_{2i}v_{4i}, v_{4i-2}v_{4i-1}, v_{4i-1}v_{4i}, u_0v_{4i-3}\},\ i \in [n]; \\ 1 & if\ e = v_{4i}v_{4i+1},\ i \in [n-1]; \\ 1 & if\ \ e = v_{4n}v_1. \end{cases}$$

Thus $e_{f(1)} = 5n$ and $e_{f(0)} = 5n$.

The induced vertex labeling $f^*: V(HS_n) \to \{0,1\}$ is $f^*(V) = \sum\{f(uv) / uv \in E(HS_n)\}\ (mod\ 2)$. Therefore

$$f^*(x) = \begin{cases} \frac{1+(-1)^{n+1}}{2} & if\ \ x = u_0; \\ 1 & if\ \ x \in \{u_{2i-1}, u_{2i}, v_{4i}\}, i \in [n]; \\ 0 & if\ \ x \in \{v_{4i-3}, v_{4i-1}, v_{4i-2}\}, i \in [n]. \end{cases}$$

Thus $v_{f(1)} = 3n + \frac{1+(-1)^n}{2}$ and $v_{f(0)} = 3n + \frac{1+(-1)^{n+1}}{2}$.

Therefore f satisfies the conditions $|v_{f(1)} - v_{f(0)}| \le 1$ and $|e_{f(1)} - e_{f(0)}| \le 1$ for E-cordial labeing. So, f admits E-cordial labeling of HS_n. Hence HS_n is E-cordial.

Theorem 2.3: HS_n is prime graph.

Proof: For the graph HS_n,

$V(HS_n) = \{u_0, u_i / 1 \le i \le 2n\} \cup \{v_i / 1 \le i \le 4n\}$ and

$E(HS_n) = \{u_0u_{2i-1}, u_0u_{2i}, u_{2i-1}v_{4i-2}, u_{2i}v_{4i}, u_{2i-1}u_{2i}, u_0v_{4i-3} / 1 \le i \le n\} \cup \{v_iv_{i+1} / 1 \le i \le 4n-1\} \cup \{v_{4n}v_1\}$.

Therefor $|V(HS_n)| = 6n+1$ and $|E(HS_n)| = 10n$.

Define $f: V(HS_n) \to [6n+1]$ as follows:

$$f(x) = \begin{cases} 1 & if\ \ x = u_0; \\ 6i+1 & if\ \ x = u_{2i-1},\ i \in [n]; \\ 6i & if\ \ x = u_{2i},\ i \in [n]; \\ 6i-4 & if\ \ x = v_{4i-3},\ i \in [n]; \\ 6i-3 & if\ \ x = v_{4i-2},\ i \in [n]; \\ 6i-2 & if\ \ x = v_{4i-1},\ i \in [n]; \\ 6i-1 & if\ \ x = v_{4i},\ i \in [n]. \end{cases}$$

Clearly f is an injective function. Let e be an arbitrary edge of HS_n. To prove f is a prime labeling of HS_n we have the following cases:

$If\ \ e=u_0u_{2i-1},\ gcd(f(u_0),\ f(u_{2i-1}))=gcd(1,6i+1)=1,\ i\in[n].$
$If\ \ e=u_0u_{2i},\ gcd(f(u_0),\ f(u_{2i}))=gcd(1,6i)=1,\ i\in[n].$
$If\ \ e=u_{2i-1}v_{4i-2},\ gcd(f(u_{2i-1}),\ f(v_{4i-2}))=gcd(6i+1,6i-3)=1,\ i\in[n].$
$If\ \ e=u_{2i}v_{4i},\ gcd(f(u_{2i}),\ f(v_{4i}))=gcd(6i,6i-1)=1,\ i\in[n].$
$If\ \ e=u_{2i-1}u_{2i},\ gcd(f(u_{2i-1}),\ f(u_{2i}))=gcd(6i+1,6i)=1,\ i\in[n].$
$If\ \ e=u_0v_{4i-3},\ gcd(f(u_0),\ f(v_{4i-3}))=gcd(1,6i-4)=1,\ i\in[n].$
$If\ \ e=v_{4i-2}v_{4i-3},\ gcd(f(v_{4i-2}),\ f(v_{4i-3}))=gcd(6i-3,6i-4)=1,\ i\in[n].$
$If\ \ e=v_{4i-2}v_{4i-1},\ gcd(f(v_{4i-2}),\ f(v_{4i-1}))=gcd(6i-3,6i-2)=1,\ i\in[n].$
$If\ \ e=v_{4i-1}v_{4i},\ gcd(f(v_{4i-1}),\ f(v_{4i}))=gcd(6i-2,6i-1)=1,\ i\in[n].$
$If\ \ e=v_{4i}v_{4i+1},\ gcd(f(v_{4i}),\ f(v_{4i+1}))=gcd(6i-1,6i+2)=1,\ i\in[n-1].$
$If\ \ e=v_{4n}v_1,\ gcd(f(v_{4n}),\ f(v_1))=gcd(6n-1,2)=1.$

Thus, f is an injection and $gcd(f(u), f(v))=1$ for every pair of adjacent vertices u and v of HS_n. So f admits prime labeling of HS_n. Hence HS_n is a prime graph.

Theorem 2.4: HS_n is vertex prime graph.

Proof: For the graph HS_n,

$V(HS_n)=\{u_0,u_i\,/\,1\le i\le 2n\}\cup\{v_i\,/\,1\le i\le 4n\}$ and
$E(HS_n)=\{u_0u_{2i-1},u_0u_{2i},u_{2i-1}v_{4i-2},u_{2i}v_{4i},u_{2i-1}u_{2i},u_0v_{4i-3}\,/\,1\le i\le n\}$
$\cup\{v_iv_{i+1}\,/\,1\le i\le 4n-1\}\cup\{v_{4n}v_1\}$.

Therefor $|V(HS_n)|=6n+1$ and $|E(HS_n)|=10n$.

Define $f:E(HS_n)\to\{10n\}$ as follows:

$$f(x)=\begin{cases}10i-9 & if\ \ x=u_0v_{4i-3},\ i\in[n];\\ 10i-8 & if\ \ x=v_{4i-2}v_{4i-3},\ i\in[n];\\ 10i-7 & if\ \ x=u_0u_{2i-1},\ i\in[n];\\ 10i-6 & if\ \ x=u_{2i-1}v_{4i-2},\ i\in[n];\\ 10i-5 & if\ \ x=v_{4i-2}v_{4i-1},\ i\in[n];\\ 10i-4 & if\ \ x=u_{2i-1}u_{2i},\ i\in[n];\\ 10i-3 & if\ \ x=u_0u_{2i},\ i\in[n];\\ 10i-2 & if\ \ x=u_{2i}v_{4i},\ i\in[n];\\ 10i-1 & if\ \ x=v_{4i-1}v_{4i},\ i\in[n];\\ 10i & if\ \ x=v_{4i}v_{4i+1},\ i\in[n-1];\\ 10n & if\ \ x=v_{4n}v_1.\end{cases}$$

Clearly f is an bijection. Let v be an arbitrary vertex of HS_n. To prove f is a vertex prime labeling of HS_n we have the following cases:

$If\ \ v=u_0,\ gcd\left(f(u_0u_{2i-1}),\ f(u_0u_{2i}),\ f(u_0v_{4i-3})\right)$
$=gcd(10i-7,10i-3,10i-9)=1,\ i\in[n].$

$If\ \ v=u_{2i-1},\ gcd\left(f(u_0u_{2i-1}),\ f(v_{4i-2}u_{2i-1}),\ f(u_{2i}u_{2i-1})\right)$
$=\text{gcd}(10i-7,10i-6,10i-4)=1,\ i\in[n].$

$If\ \ v=u_{2i},\ gcd\left(f(u_0u_{2i}),\ f(v_{4i}u_{2i}),\ f(u_{2i}u_{2i-1})\right)$
$=gcd(10i-3,10i-2,10i-4)=1,\ i\in[n].$

$If\ \ v=v_{4i-2},gcd\left(f(v_{4i-2}v_{4i-1}),\ f(v_{4i-2}v_{4i-3}),\ f(v_{4i-2}u_{2i-1})\right)$
$=gcd(10i-5,10i-8,10i-6)=1,\ i\in[n].$

$If\ \ v=v_{4i-1},gcd\left(f(v_{4i-1}v_{4i}),\ f(v_{4i-2}v_{4i-1})\right)$
$=gcd(10i-1,10i-5)=1,\ i\in[n].$

$If\ \ v=v_{4i},\ gcd\left(f(v_{4i}v_{4i+1}),\ f(v_{4i}u_{2i}),\ f(v_{4i}v_{4i-1})\right)$
$=gcd(10i,10i-2,10i-1)=1,\ i\in[n-1].$

$If\ \ v=v_{4i-3},\ gcd\left(f(v_{4i-3}v_{4i-2}),\ f(v_{4i-3}u_{4i-4}),\ f(u_0u_{4i-3})\right)$
$=gcd(10i-8,10i-10,10i-9)=1,\ i\in[n]-\{1\}.$

$If\ \ v=v_{4n},\ gcd\left(f(v_{4n}v_1),\ f(v_{4n}u_{2n}),\ f(v_{4n}v_{4n-1})\right)$
$=gcd(10n,10n-2,10n-1)=1.$

$If\ \ v=v_1,gcd\left(f(v_{4n}v_1),\ f(v_1v_2),\ f(v_1u_0)\right)$
$=gcd(10n,2,1)=1.$

Thus, f is an bijection and $\gcd_{uv\in E}\{f(uv)\}=1$. The edges are labeled such that for any vertex v_i, the gcd of all the edges incident with v_i is 1. So f admits vertex prime labeling of HS_n. Hence HS_n is a vertex prime graph.

Theorem 2.5: HS_n is total prime graph.

Proof: For the graph HS_n,

$V(HS_n)=\{u_0,u_i\,/\,1\le i\le 2n\}\cup\{v_i\,/\,1\le i\le 4n\}$ and
$E(HS_n)=\{u_0u_{2i-1},u_0u_{2i},u_{2i-1}v_{4i-2},u_{2i}v_{4i},u_{2i-1}u_{2i},u_0v_{4i-3}\,/\,1\le i\le n\}$
$\cup\{v_iv_{i+1}\,/\,1\le i\le 4n-1\}\cup\{v_{4n}v_1\}$.

Therefor $|V(HS_n)|=6n+1$ and $|E(HS_n)|=10n$.

Define $f:(V(HS_n)\cup E(HS_n))\to\{16n+1\}$ as follows:

$$f(x)=\begin{cases}1 & \text{if } x=u_0;\\ 6i+1 & \text{if } x=u_{2i-1},\ i\in[n];\\ 6i & \text{if } x=u_{2i},\ i\in[n];\\ 6i-4 & \text{if } x=v_{4i-3},\ i\in[n];\\ 6i-3 & \text{if } x=v_{4i-2},\ i\in[n];\\ 6i-2 & \text{if } x=v_{4i-1},\ i\in[n];\\ 6i-1 & \text{if } x=v_{4i},\ i\in[n];\\ 6n+10i-8 & \text{if } x=u_0v_{4i-3},\ i\in[n];\\ 6n+10i-7 & \text{if } x=v_{4i-2}v_{4i-3},\ i\in[n];\\ 6n+10i-6 & \text{if } x=u_0u_{2i-1},\ i\in[n];\\ 6n+10i-5 & \text{if } x=u_{2i-1}v_{4i-2},\ i\in[n];\\ 6n+10i-4 & \text{if } x=v_{4i-2}v_{4i-1},\ i\in[n];\\ 6n+10i-3 & \text{if } x=u_{2i-1}u_{2i},\ i\in[n];\\ 6n+10i-2 & \text{if } x=u_0u_{2i},\ i\in[n];\\ 6n+10i-1 & \text{if } x=u_{2i}v_{4i},\ i\in[n];\\ 6n+10i & \text{if } x=v_{4i-1}v_{4i},\ i\in[n];\\ 6n+10i+1 & \text{if } x=v_{4i}v_{4i+1},\ i\in[n-1];\\ 16n+1 & \text{if } x=v_{4n}v_1.\end{cases}$$

Clearly f is a bijection. Let e and v be an arbitrary edge and vertex of HS_n. To prove f is a total prime labeling of HS_n. we have the following cases:

Then for any edge,

If $e=u_0u_{2i-1},\ gcd(f(u_0), f(u_{2i-1}))=gcd(1,6i+1)=1,\ i\in[n]$.

If $e=u_0u_{2i},\ gcd(f(u_0), f(u_{2i}))=gcd(1,6i)=1,\ i\in[n]$.

If $e=u_{2i-1}v_{4i-2},\ gcd(f(u_{2i-1}), f(v_{4i-2}))=gcd(6i+1,6i-3)=1,\ i\in[n]$.

If $e=u_{2i}v_{4i},\ gcd(f(u_{2i}), f(v_{4i}))=gcd(6i,6i-1)=1,\ i\in[n]$.

If $e=u_{2i-1}u_{2i},\ gcd(f(u_{2i-1}), f(u_{2i}))=gcd(6i+1,6i)=1,\ i\in[n]$.

If $e=u_0v_{4i-3},\ gcd(f(u_0), f(v_{4i-3}))=gcd(1,6i-4)=1,\ i\in[n]$.

If $e=v_{4i-2}v_{4i-3},\ gcd(f(v_{4i-2}), f(v_{4i-3}))=gcd(6i-3,6i-4)=1,\ i\in[n]$.

If $e=v_{4i-2}v_{4i-1},\ gcd(f(v_{4i-2}), f(v_{4i-1}))=gcd(6i-3,6i-2)=1,\ i\in[n]$.

If $e=v_{4i-1}v_{4i},\ gcd(f(v_{4i-1}), f(v_{4i}))=gcd(6i-2,6i-1)=1,\ i\in[n]$.

If $e=v_{4i}v_{4i+1},\ gcd(f(v_{4i}), f(v_{4i+1}))=gcd(6i-1,6i+2)=1,\ i\in[n-1]$.

If $e=v_{4n}v_1,\ gcd(f(v_{4n}), f(v_1))=gcd(6n-1,2)=1$.

Then for any vertex,

If $v=u_0,\ gcd\left(f(u_0u_{2i-1}), f(u_0u_{2i}), f(u_0v_{4i-3})\right)$
$=gcd(6n+10i-6, 6n+10i-2, 6n+10i-8)=1,\ i\in[n]$.

If $v=u_{2i-1},\ gcd\left(f(u_0u_{2i-1}), f(v_{4i-2}u_{2i-1}), f(u_{2i}u_{2i-1})\right)$
$=gcd(6n+10i-6, 6n+10i-5, 6n+10i-3)=1,\ i\in[n]$.

If $v=u_{2i},\ gcd\left(f(u_0u_{2i}), f(v_{4i}u_{2i}), f(u_{2i}u_{2i-1})\right)$
$=gcd(6n+10i-2, 6n+10i-1, 6n+10i-3)=1,\ i\in[n]$.

If $v=v_{4i-2}, gcd\left(f(v_{4i-2}v_{4i-1}), f(v_{4i-2}v_{4i-3}), f(v_{4i-2}u_{2i-1})\right)$
$=gcd(6n+10i-4, 6n+10i-7, 6n+10i-5)=1,\ i\in[n]$.

If $v=v_{4i-1}, gcd\left(f(v_{4i-1}v_{4i}), f(v_{4i-2}v_{4i-1})\right)$
$=gcd(6n+10i, 6n+10i-4)=1,\ i\in[n]$.

If $v=v_{4i},\ gcd\left(f(v_{4i}v_{4i+1}), f(v_{4i}u_{2i}), f(v_{4i}v_{4i-1})\right)$
$=gcd(6n+10i+1, 6n+10i-1, 6n+10i)=1,\ i\in[n-1]$.

If $v=v_{4i-3},\ gcd\left(f(v_{4i-3}v_{4i-2}), f(v_{4i-3}u_{4i-4}), f(u_0u_{4i-3})\right)$
$=gcd(6n+10i-7, 6n+10i-9, 6n+10i-8)=1,\ i\in[n]-\{1\}$.

If $v=v_{4n},\ gcd\left(f(v_{4n}v_1), f(v_{4n}u_{2n}), f(v_{4n}v_{4n-1})\right)$
$=gcd(16n+1, 16n-1, 16n)=1$.

If $v=v_1, gcd\left(f(v_{4n}v_1), f(v_1v_2), f(v_1u_0)\right)$
$=gcd(16n+1, 16n+3, 16n+2)=1$.

So, f is an bijection and According to this pattern, the vertices are labeled such that for any edge $e=uv\in HS_n,\ gcd(f(u), f(v))=1$. Also the edges are labeled such that for any vertex v_i, the gcd of all the edges incident with v_i is 1. So f admits total prime labeling of HS_n. Hence HS_n is a total prime graph.

Theorem 2.5: HS_n is difference cordial graph.

Proof: For the graph HS_n,

$V(HS_n)=\{u_0, u_i / 1\le i\le 2n\}\cup\{v_i / 1\le i\le 4n\}$ and

$E(HS_n)=\{u_0u_{2i-1}, u_0u_{2i}, u_{2i-1}v_{4i-2}, u_{2i}v_{4i}, u_{2i-1}u_{2i}, u_0v_{4i-3} / 1\le i\le n\}$
$\cup\{v_iv_{i+1} / 1\le i\le 4n-1\}\cup\{v_{4n}v_1\}$.

Therefor $|V(HS_n)|=6n+1$ and $|E(HS_n)|=10n$.

Define $f: V(HS_n)\to\{6n+1\}$ as follows:

$$f(x)=\begin{cases}1 & \text{if } x=u_0;\\ 5i-3 & \text{if } x=v_{4i-3},\ i\in[n];\\ 5i-2 & \text{if } x=v_{4i-2},\ i\in[n];\\ 5i-1 & \text{if } x=u_{2i-1},\ i\in[n];\\ 5i & \text{if } x=u_{2i},\ i\in[n];\\ 5i+1 & \text{if } x=v_{4i},\ i\in[n];\\ 5n+i+1 & \text{if } x=v_{4i-1},\ i\in[n].\end{cases}$$

The induced edge labeling $f^*: E(HS_n) \to \{0,1\}$ is $f^*(uv) = |f(u) - f(v)|$, for every edge $e = uv \in E$. Therefore

$$f^*(e) = \begin{cases} 1 & if\ e \in \{v_{4i-3}v_{4i-2}, u_{2i-1}v_{4i-2}, u_{2i-1}u_{2i}, u_{2i}v_{4i}\},\ i \in [n]; \\ 0 & if\ e \in \{u_0u_{2i-1}, u_0u_{2i}, v_{4i-2}v_{4i-1}, v_{4i-1}v_{4i}\},\ i \in [n]; \\ 0 & if\ e = u_0v_{4i+1},\ i \in [n] - \{1\}; \\ 1 & if\ e = v_{4i}v_{4i+1},\ i \in [n-1]; \\ 1 & if\ e = u_0v_1; \\ 0 & if\ e = v_{4n}v_1. \end{cases}$$

Since $e_{f(0)} = e_{f(1)} = 5n$. Therefore f satisfies the conditions $|e_{f(1)} - e_{f(0)}| \le 1$ for difference cordial labeling. So, f admits difference cordial labeling of HS_n. Hence HS_n is difference cordial.

3. CONCLUSION

We have derived six new results by investigating some labeling techniques of holiday star graph. More exploration is possible for other graph families and in the context of different graph labeling problems.

REFERENCE:

[1] I. Cahit, Cordial Graphs: *A weaker version of graceful and harmonious Graphs*, Ars Combinatoria, 23(1987), 201-207.

[2] T. Deretsky, S. M. Lee and J. Mitchem, *On Vertex Prime Labelings of Graphs*, In: J. Alvi, G. Chartrand, O. Oel-lerman, A. Schwenk, Eds., Graph Theory, Combinatorics and Applications: Proceedings of the 6th International Conference Theory and Applications of Graphs, Wiley, New York, 1(1991), 359-369.

[3] H. L. Fu and K. C. Huang, *on prime labelings, Discrete Mathematics*, 127(1994), 181-186.

[4] J. A. Gallian, *A Dynamic Survey of Graph Labeling*, The Electronic Journal of Combinatorics 19 (2016), \#DS6. Online: http://www.combinatorics.org

[5] J. Gross and J. Yellen, *Graph Theory and Its Applications*, CRC Press, Boca Raton, 1999.

[6] S. M. Lee, A. Liu, *A construction of cordial graphs from smaller cordial graphs*, Ars Combinatoria, 32(1991), 209-214.

[7] S. M. Lee, I. Wui and J. Yeh, *On the Amalgamation of Prime Graphs*, Bulletin of the Malaysian Mathematical Sciences Society (Second Series), 11(1988), 59-67.

[8] Ponraj R, Narayanan SS. *Difference cordiality of some graphs obtained from double alternate snake graphs.* Global Journal of Mathematical Sciences: Theory and Practical 5(2013),167-175.

[9] Ponraj R, Narayanan SS. *Further results on difference cordial labeling of corona graphs.* J Indian Acad Math 35(2013),217-235.

[10] Ponraj R, Narayanan SS, Kala R. *Difference cordial labeling of graphs.* Global Journal of Mathematical Sciences: Theory and Practical 3(2013),193-201.

[11] Ponraj R, Narayanan SS, Kala R. *Difference cordial labeling of graphs obtained from double snake*. International Journal of Mathematics Research 5(2013),317-322.

[12] Ponraj R, Narayanan SS, Kala R. *A note on difference cordial graphs.* Palestine Journal of Mathematics 4(2015), 189-197.

[13] U. M. Prajapati and R. M. Gajjar, *Cordial Labeling of Complement of Some Graph*. Mathematics Today, 30(2015), 99-118.

[14] U. M. Prajapati and R. M. Gajjar, *Cordial Labeling in the Context of Duplication of some Graph Elements*. International Journal of Mathematics and Soft Computing, 6(2016), 65-73.

[15] Ramasubramanian, R. Kala, *Total Prime Graph*, International Journal Of Computational Engineering Research (ijceronline.com) Vol. 2 Issue. 5.

[16] A.Tout, A.N. Dabbouey and K.Howalla, *Prime Labeling of graphs*, National Academy Science Letters, 11(1982), 365-368.

[17] S. K. Vaidya and U. M. Prajapati, *Some Results on Prime and K-Prime Labeling*, Journal of Mathematics Research, 3(2011), 66-75.

[18] S. K. Vaidya and U. M. Prajapati, *Some Switching Invariant Prime Graphs*, Open Journal of Discrete Mathematics, 2(2012), 17-20.

[19] R. Yilmaz and I. Cahit, *E-cordial graphs*, Ars Combin., 46(1997), 251-266.