Fuzzy Soft Compact Spaces

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Abstract-
In this paper to study some basic properties of fuzzy soft topological spaces, we define quasi fuzzy soft compactness. We also introduced some basic definitions of compact space and theorems of the concept.

Keyword-Fuzzy soft topology, soft set, fuzzy soft compactness, weakly and Strong fuzzy compact spaces.

I. Introduction

In many complicated problems arising in the fields of engineering, social science, Economics, Environment, medical science etc. have various uncertainties. Fuzzy soft sets are very useful structures arising in many problems encountered in real life. The theory soft sets was initiated by Molodtsov[4] in 1999 for modeling uncertainty present in real life. Roughly speaking a soft set is a parameterized classification of the objects of the universe P.K.Maji , R. Biswas ,A.R.Roy[8] also initiated in more generalized concept of fuzzy soft sets which is a combination of fuzzy set and soft set . In recent times, researchers have contributed a lot towards fuzzification of soft set. Neog, Sut ,Hazarika[10] introduced fuzzy soft topological spaces . Some others ([5], [6], [7] , [9]) studied on the compact fuzzy soft topological spaces. My main aim in this paper is to develop the basic properties of quasi fuzzy soft compact spaces and establish several equivalent forms of fuzzy soft compactness which are useful applications.

II. Preliminaries

Definition 2.1[11]A pair (F,A) is called a fuzzy soft set over U where $F:A \rightarrow \breve{P}(U)$ is a mapping from a into $\breve{P}(U)$.

Definition 2.2[11]

For two fuzzy soft sets (F, A) and (G,B) in a fuzzy soft class(U,E) , we say that (F,A) is a fuzzy soft subset of (G,B) if

i)$A \subseteq B$

ii)For all $\varepsilon \in A$, $F(\varepsilon)$ is a sub set $G(\varepsilon)$ and is written as $F(A) \subseteq (G,B)$

Definition 2.3[13]

The complement of a fuzzy soft set (F,A) is denoted by $(F,A)^C$ and is defined by

$(F, A)^C = (F^C, A)$. Where $F^C: A \rightarrow \breve{P}(U)$ is mapping given by

$F^C(\alpha) = U—F(\alpha)=[F(\alpha)]^C \forall \alpha \in A$

Definition 2.4[12]

Let $A \subseteq E$. Then the mapping $F_A: E \rightarrow \breve{P}(U)$, defined by $F_A(e)=\mu^e F_A$ (a fuzzy subset of U) is called fuzzy soft over (U,E), Where $\mu^e F_A=\bar{0}$ if $e \in E-A$ and $\mu^e F_A \neq \bar{0}$ if $e \in A$. The set of all fuzzy soft set over (U, E) is denoted by FS (U, E)

Definition 2.5[13]

The fuzzy soft set FS (U, E) is called null fuzzy soft set and is denoted by $\bar{\emptyset}$. Here $F\varphi(e)=\bar{0}$ foe every $e \in E$

Definition 2.6[12]A fuzzy soft topology T on (U,E) is a family of fuzzy soft sets over (U,E) satisfying the following properties

(i)$\bar{\emptyset}, \bar{E} \in T$

(ii) If F_A, $G_B \in$ T then If $F_A \cap G_B \in$ T

(iii) If $F^{\alpha}A_{\alpha} \in$ T for all $\alpha \in \Delta$,then $\cup F^{\alpha}A_{\alpha} \in$ T

Definition 2.7[12]

If T is a fuzzy soft topology on (U,E) , then (U,E,T) is said to be a fuzzy soft topological space . Also each member of T is called a fuzzy soft open set in (U,E,T)

Definition 2.8[5]

Let (U, E,T) be a soft topological space. A soft separation of $\bar{U}$ is a pair of (F,E) ,(G,E) of no- null soft open sets over U such that $\bar{U}$ =(F,E) $\bar{U}$ (G,E) , (F,E) $\bar{\cap}$ (G,E) =$\emptyset_E$

III. Quasi Fuzzy Compact Space

Definition 3.1

A fuzzy $\breve{A}$ of fuzzy soft set is a cover of a fuzzy soft set B if and only if $B \subseteq \cup\{A: A \in \breve{A}\}$

It is a fuzzy soft open cover if and only if each member of $\breve{A}$ is a fuzzy soft open set. A sub cover of $\breve{A}$ is a subfamily of $\breve{A}$ which is also a cover.

Definition 3.2

A fuzzy topological space (X, T) is quasi fuzzy soft compact if and only if each open cover has a finite sub-cover.

Theorem3.1 A fuzzy topological space (X,T) is quasi fuzzy compact if and only if for every net $\{K_n, n \in D\}$ of fuzzy closed sets in (X,T) such that F- $\limsup_D(K_n)=\bar{0}$, there exists $n_0 \in D$ for which $K_n=\bar{0}$ for every $n \in D$, $n \geq n_0$

Example 3.1 A fuzzy soft topological space (X, T) is quasi compact if X is finite.

Example3.2 **Let** (X,T) and (Y,T$^{/}$) be two fuzzy soft topological space and if $T \subset T^{/}$. Then fuzzy soft topological space (X,T) is quasi compact if (Y,T$^{/}$) is quasi compact.

Definition 3.3 Let α be an infinite cardinal and β be any cardinal such that $\alpha<\beta$. Then a fuzzy topological space (X,T) is called quasi fuzzy (α,β) compact if for every open cover ν of fuzzy open sets of (X,T) which $|\nu|<\beta$ then there exists a subfamily of $\nu$of cardinality less than α covering the fuzzy topological space (X,T). $|\nu|$ Denotes the cardinality of ν

Remarks 3.1 If is easy to see that the notation of quasi fuzzy (ω,α) compactness, Where $\omega$is the first infinite cardinal coincide with the notion of quasi fuzzy compactness.

Definition 3.4 A fuzzy topological space (X,T) is quasi fuzzy countable compact if and only if each open countable cover has a finite sub cover.

IV. Weakly Fuzzy Soft Compact Space

Definition 4.1 A fuzzy topological space (X,T) is called weakly fuzzy soft compact if and only if for every open cover $\nu = \{\nu_j: j \in J\}$ of fuzzy open sets X, that is $\cup\{\nu_j: j \in J\} =\bar{1}$for every $\varepsilon>0$, there exist a finite subfamily $\{\nu_{j1},\nu_{j2},\nu_{j3},\ldots..\nu_{jn}\}$ of ν such that $\cup\{\{\nu_j: i=1,2,3,\ldots..n\} \geq \overline{1-\varepsilon}$

Definition 4.2 Let Ω be a class of directed sets . A fuzzy soft space X is called weakly fuzzy Ω compact if for every Ω net $\{K_n, n \in D\}$ of fuzzy closed sets in X such that F- $\limsup_D(K_n)=\bar{0}$ there exists $n_0 \in D$ for which $K_n \leq \bar{\varepsilon}$ for every $n \in D$, $n \geq n_0$

Remarks 4.1 above theorem it follows that Ω is the class of all directed sets, then the notation of weakly fuzzy Ω compactness coincides with the notion of weakly fuzzy compactness.

Definition4.3 Let α be an infinite cardinal and β be any cardinal such that $\alpha<\beta$. Then a fuzzy topological space (X,T) is called weakly fuzzy (α,β) compact if for every open cover ν of fuzzy open sets of X which $|\nu|<\beta$ and for every $\varepsilon>0$, then there exists a subfamily of ν_1 of νcardinality less than α covering the fuzzy topological space (X,T). $|\nu|$ Denotes the cardinality of ν or $\cup\{\{\nu: \nu \in \nu_1\} \geq \overline{1-\varepsilon}$

Remarks4.2 If is easy to see that the notation of weakly fuzzy (ω,α) compactness coincide with the notion of weakly fuzzy compactness.

V. Strong Fuzzy soft Compact Spaces

Definition 5.1 A fuzzy soft topological space(X,T) is called strong fuzzy soft compact iff it is a compact for every $\alpha\in[0,1)$

Theorem 5.1 A fuzzy soft topological space(X,τ) is strong fuzzy soft compact iff it is a compact for every $\alpha\in[0,1)$ and for every net$\{K_n, n\in D\}$ of fuzzy closed sets in X such that F- $\limsup_D(K_n)\leq\overline{1-\varepsilon}$,$\forall n\in D$, $n\geq n_0$

Definition 5.2 Let Ω be a class of directed sets. A fuzzy soft space X is called strong fuzzy soft Ω compact if for every $\alpha\in[0,1)$ and Ωnet$\{K_n, n\in D\}$ of fuzzy closed sets in X such that F- $\limsup_D(K_n)\leq\overline{1-\varepsilon}$,$\forall n_0\in D$, for which $n\geq n_0$,$(K_n)\leq\overline{1-\varepsilon}$,$n\in D$,$n\geq n_0$

Definition5.2 Let α be infinite cardinal and β be any cardinal such that $\alpha<\beta$. A fuzzy soft space X is called strong fuzzy soft (α,β) compact if for every $\alpha\in[0,1)$ and for every a shading family ν of fuzzy open sets of X Where $|\nu|<\beta$ has a shading subfamily ν_1 cardinality less than α.

Remarks 5.1 We see that the notation of strong fuzzy (ω,α) compactness coincides with the notation of strong fuzzy compactness.

Definition 5.4 A fuzzy soft topological space(X,τ) is strong fuzzy soft compact if it is a compact for every $\alpha\in[0,1)$ and for every net$\{K_n, n\in D\}$ of fuzzy closed sets of topological space $(X, i_\alpha(\tau))$ such that $\limsup_D(K'_n)=\phi$, then there exists an element $\forall n_0\in D$, $n\geq n_0$

VI. Compact Fuzzy Soft Spaces

Definition 6.1 A fuzzy soft topological space(X,τ) is called fuzzy soft compact if and only if every family ν of fuzzy open sets of X and for every $\alpha\in I$ such that$\cup\{\nu:\nu\in V\}\geq\overline{\alpha}$ and for every $\varepsilon\in(0,\alpha]$ there exists a finite subfamily ν_1 of ν such that $\cup\{\nu:\nu\in V_1\}\geq\overline{\alpha-\varepsilon}$

Definition 6.2 Let Ω be a class of directed sets .A fuzzy space X is called fuzzy Ω compact if for every ä$\in I$ for every Ω net $\{k_n, n\in D\}$ of fuzzy closed sets in X such that $\lim\sup_D(K_n)\leq\overline{1-\varepsilon}$,$\forall n_0\in D$, for which $k_n\leq\overline{1-\alpha+\varepsilon}$ for every $n\in D$, $n\geq n_0$

Definition 6.3 Let α be an infinite cardinal and β be a cardinal such that $\alpha<\beta$. A fuzzy space X is called fuzzy (α,β) compact if for every ä$\in I$, for every family ν of fuzzy open sets of α where $|\upsilon|<\beta$ such that $\cup\{\nu:\nu\in V\}\geq\overline{\alpha}$ and for every $\varepsilon\in(0,\alpha]$ there exists a finite subfamily ν_i of ν cardinality less than α such that $\cup\{\nu:\nu\in V_1\}\geq\overline{\alpha-\varepsilon}$

Remarks 6.1 We see that the notion of fuzzy $(\omega,-\alpha)$ compactness coincides with the notion of fuzzy compactness.

Theorems 6.1 Suppose that α is regular and for every $\gamma<\beta$ cardinality of the set of all subsets of the set of γ of cardinality less than α , then there exists a class Ω of directed sets such that a fuzzy space X is fuzzy Ω compact if and only if it is fuzzy (α,β) compact.

Definition 6.4 The fuzzy topological space (X,τ) is fuzzy compact if and only if each constant fuzzy set in (X,τ) is compact.

Definition 6.5 The fuzzy topology space (X,τ) is weakly fuzzy compact if and only if 1_x is fuzzy compact we have if (X,τ) is quasi fuzzy compact , it is weakly fuzzy compact otherwise all spaces fuzzy topological space.

VII. Conclusion

In this work, we introduce quasi fuzzy soft compactness and gave basic definitions and theorems of this concept. Also we introduced weakly and strong fuzzy soft compact spaces and fuzzy compact spaces.

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