Higher Six Dimensional Plane Gravitational Waves in Bimetric Relativity

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Abstract **In this paper, I studied the** $Z = \left(\sqrt{x^2+y^2+z^2}-t\right)$, $Z = \left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$

type plane gravitation waves for higher six dimensions and it will observed that the result for vacuum space and for matter cosmic strings respectively.

Key Words: Plane gravitation, Cosmic strings, Bimetric Relativity.

I Introduction

Plane gravitational waves are usually discussed as a special case of the well established plane fronted gravitational waves with parallel rays, the so called pp-waves specialization is quit technical, e.g. the curvature tensor must be complex recurrent with a recurrence vector which is collinear with areal null vector.

both for weak field approximation and for exact solutions of Einstein field equations. Mohseni, Tucker and Wang (2001) [17] have studied the motion of motion of spinning test particles in plane gravitational waves.

Dnato Bini, et al(2003)[2], used the killing vectors to reduce the equation of motion into first order system of differential equations.

Kasseri, S. Singh D.et al (2002)[11], analyzed the motion of electrically neutral massive spinning test particle in the plane gravitational and electromagnetic waves background . Mohseni and Sepangi (2008)[18] have studied the motion of classical spinning test particle in the field of a weak plane gravitational

H.Taub(1961)[22] propounded a rigorous discussion of plane gravitational waves, defined various terms by formulating a meaningful mathematical version and obtained numerous results. A fairly general cases of Plane gravitational waves is represented by the metric $ds^2 = -Adx^2 - 2Ddxdy - Bdy^2 - dz^2 + dt^2$ (1.1)

waves.Takeno(1958a)[20] has found out the solutions of plane gravitational waves of the field equations in general relativity and those of the various field equations in non-symmetric unified theories for the space-time.

$$ds^2 = -Adx^2 - 2Ddxdy - Bdy^2 - \varphi^2(-C+E)dz^2 - 2\varphi Edzdt + (C+E)dt^2$$

(1.2)

where A,B,C,D,E are functions of $Z = (z-t)$ and φ is a function of Z.

However, Takeno (1958b)[21] first mooted the idea of metric (1.2). He proposed that all components of the metric tensor may be chosen to be an arbitrary function of z and t, but not those of $Z = (z - t)$ or $\frac{t}{z}$ -type waves. He stated that the solutions of various field equations may be found out, though the calculations involved maybe very complicated. This approach motivates that the results corresponding to plane gravitational waves can be deducted by suitable choice of the phase function Z. accordingly the space-time metric (1.2) will become a generalized Takeno's space-time if the function φ, has not necessarily taken as a function of Z.

Lal and Ali(1970a, 1970b)[12] [13] have found the wave solutions of the field equations of general relativity and non- symmetric unified field theories of Einstein, Bonnor and Schroding in the space-time (1.2) by assuming $\varphi = -1, E = (x, y, Z)$ and other components viz, A,B,C, and D as a functions of

$Z = (z - t)$. The theory of plane wave solutions or the generalized plane waves solutions of the field equations in general relativity and non-symmetric unified field theories have been studied by many investigators {4],[5],[6] and [8],[9],[10].

The plane gravitational waves g_{ij} are mathematically exposed by Takeno (1961)[22] in general relativity. He had studied $(z - t)$ and $\frac{t}{z}$ -type plane gravitational waves and obtained the line element for both waves as

$$ds^2 = -Adx^2 - 2Ddxdy - Bdy^2 - (C - E)dz^2 - 2Edzdt + (C + E)dt^2 \quad (1.3)$$

and

$$ds^2 = -Adx^2 - 2Ddxdy - Bdy^2 - Z(C - E)dz^2 - 2ZEdzdt + (C + E)dt^2 \quad (1.4)$$

where A, B, C, D and E are functions of Z. Also Takno has shown the co-existence of plane gravitational waves with electromagnetic waves in V_4.

The field equation of bimetric relativity derived from variation principles are

The theory of plane gravitational waves of the field equations in general relativity and those of the field equations in non-symmetric unified field theory have been studied by many investigators, for example: Takeno [20], [21] and [22]; Pandey [19] ; Lal and Shafiullah [14],etc. the theory of plane gravitational waves in general relativity has been studied by Lu Hui qing [15] ; Bondi,H. et al[1]; Torre, C.G. [23]; Hogan, P.A. [7] and they obtained the solutions. The field equations of these theories differ only in some region where there exists some other field besides gravitation. Therefore, Takeno confined himself to that region in which both gravitational and electromagnetic fields are present. The electromagnetic field is composed of plane waves which are transverse electromagnetic waves of electric and magnetic fields propagating in one direction with unit velocity.

In this paper,I study the Z $= \left[\sqrt{x^2 + y^2 + z^2} - t\right]$

$Z = \left[\frac{t}{\sqrt{x^2+y^2+z^2}}\right]$ type plane waves for higher six dimensions, I will observe the result for vacuum space in the context of bimetric theory of relativity.

I. Field Equations of Bimetric Relativity

To remove some of the unsatisfactory features of the general theory of relativity, Rosen has proposed the bimetric theory of relativity, in which there exist two metric tensors at each point of space-time $-g_{ij}$ which describes gravitation, and the background metric $-\gamma_{ij}$, which enter into the field equations and interacts with g_{ij} but does not interacts directly with matter.

Accordingly, at each space-time point, one has two line elements

$$ds^2 = g_{ij} dx^i dx^j$$

And $$d\sigma^2 = \gamma_{ij} dx^i dy^j$$

where ds is the interval between two neighboring events as measured by a clock and a measuring rod. The interval $d\sigma$ is an abstract quantity not directly measurable. One can regard matter were present

$$K_i^j = N_i^j - \frac{1}{2}{}_N g_i^j = -8\pi\kappa T_i^j$$

(2.1)

where $N_i^j = \frac{1}{2}\gamma^{\alpha\beta}\left(g^{hj}g_{hi}|\alpha\right)|\beta$

$N = N_\alpha^\alpha$, $\kappa = \left(\frac{g}{\gamma}\right)^{\frac{1}{2}}$

(2.3)

$$g = \det g_{ij}, \gamma = \det \gamma_{ij}$$

(2.4)

a vertical bar (|) denotes a covariant differentiation with respect to γ_{ij}.

T_i^j is the energy momentum tensor for the matter.

II. $Z = \left[\sqrt{x^2+y^2+z^2} - t\right]$-Type Higher Six Dimensional Plane Gravitational Wave Vacuum Solutions

For $Z = \left[\sqrt{x^2+y^2+z^2-t}\right]$-type higher six dimensional gravitational waves, the line element in V_6 in general relativity with the proper choice of coordinate system as

$$ds^2 = -\frac{3A}{(x^2+y^2+z^2)}\left[x^2dx^2 + y^2dy^2 + z^2dz^2\right] - Bdu^2 - Cdv^2 + Adt^2$$

(3.1)

where $A = A(Z)$, $B = B(Z)$, $C = C(Z)$

and $Z = \left[\sqrt{x^2+y^2+z^2} - t\right]$.

Corresponding to this equation (3.1), we consider the line element for background metric γ_{ij} as

$$d\sigma^2 = -(dx^2 + dy^2 + dz^2 + du^2 + dv^2) + dt^2$$

(3.2)

Since γ_{ij} is Lorentz metric (-1,-1,-1,-1, 1), γ- covariant derivative become the ordinary partial derivative. For empty space-time field equation in bimetric relativity, we assume the form

$$N_i^j = 0$$

(3.3)

(2.2)

where N_i^j is already defined in equation (2.2)

Using the equations (3.1) to (3.3), we have the results

$$N_1^1 = N_2^2 = N_3^3 = N_4^4 = N_5^5 = N_6^6 = 0$$

(3.4)

Thus, the field equations (3.4) are identically satisfied. It implies that

$Z = \sqrt{x^2+y^2+z^2} - t$-type plane gravitational wave for higher six dimensional V_6 exist in bimetric relativity.

With the introduction constant λ, the field equations of empty space-time in bimetric relativity assume the form

$$N_i^j = \lambda g_i^j$$

(3.5)

Using the equations (3.1) to (3.5), we have

$$\lambda = 0$$

(3.6)

It implies that the equation (3.6) becomes the equation (3.4) which is identically satisfied for $\left[\sqrt{x^2+y^2+z^2} - t\right]$ - type plane gravitational wave in V_6.it means that there is no contribution of cosmological constant term λ in the field equation in bimetric relativity for $\left[\sqrt{x^2+y^2+z^2} - t\right]$ - type plane wave in V_6

III. $Z = \left[\frac{t}{\sqrt{x^2+y^2+z^2}}\right]$-Type Higher Six Dimensional Gravitational Wave Vacuum Solutions

Consider the higher six dimensional $Z = \left[\frac{t}{\sqrt{x^2+y^2+z^2}}\right]$-type plane wave space-time as

$$ds^2 = -\frac{3At^2}{(x^2+y^2+x^2)}\left[\frac{t}{\sqrt{x^2dx^2+y^2dy^2+z^2dz^2}}\right] - Bdu^2 - Cdv^2 + Adt^2$$

(4.1)

where $A = A(Z)$, $B = B(Z)$, $C = C(Z)$ and $Z = \left[\frac{t}{\sqrt{x^2+y^2+z^2}}\right]$, and the background metric, corresponding to the metric (4.1) is taken as (3.2).

For empty space-time field equation in bimetric relativity, we assume the form (3.3), and we obtain the field equations as

$$N_1^1 = N_2^2 = N_3^3 = N_6^6 = \frac{1}{2}\left(\frac{\bar{A}^2}{A^2} - \frac{\bar{\bar{A}}}{A}\right)\left[\frac{t^2-(x^2+y^2+z^2)}{(x^2+y^2+z^2)^2}\right] = 0$$

(4.2)

$$N_4^4 = \frac{1}{2}\left(\frac{\bar{B}^2}{B^2} - \frac{\bar{\bar{B}}}{B}\right)\left[\frac{t^2-(x^2+y^2+z^2)}{(x^2+y^2+z^2)^2}\right] = 0$$

(4.3)

$$N_5^5 = \frac{1}{2}\left(\frac{\bar{C}^2}{C^2} - \frac{\bar{\bar{C}}}{C}\right)\left[\frac{t^2-(x^2+y^2+z^2)}{(x^2+y^2+z^2)^2}\right] = 0$$

(4.4)

From equation (4.2), we get-

$$\left(\frac{\bar{A}^2}{A^2} - \frac{\bar{\bar{A}}^2}{A}\right) = 0,$$

(4.5)

which gives us

$$A = n_1 exp(m_1 Z)$$

(4.6)

where m_1, n_1 are constants of integration.

And from equation (4.3), we get

$$\left(\frac{\bar{B}^2}{B^2} - \frac{\bar{\bar{B}}}{B}\right) = 0$$

(4.7)

$$B = n_2 exp(m_2 Z)$$

(4.8)

where m_2, n_2 are constants of integration.

Similarly by the equation(4.4), we

$$\left(\frac{\bar{C}^2}{C^2} - \frac{\bar{\bar{C}}}{C}\right) = 0$$

(4.9)

which gives us

$$C = n_3 \exp(m_3 Z)$$

(4.10)

Thus, the metric (4.1) takes the form

$$ds^2 = \frac{3t^2 n_1 exp(m_1 Z)}{(x^2+y^2+z^2)^2}[x^2+y^2+z^2] - n_2 exp(m_2 Z)du^2 - n_3 exp(m_3 Z) + n_1 exp(m_1 Z)dt^2$$

(4.11)

This study can further be extended with the introduction of cosmological constant λ in the field equations, which is defined in the equation (3.5).

Thus, we get

$$\frac{1}{2}\left(\frac{\bar{A}^2}{A^2} - \frac{\bar{\bar{A}}}{A}\right)\left[\frac{t^2-(x^2+y^2+z^2)}{(x^2+y^2+z^2)^2}\right] = \lambda,$$

(4.12)

$$\frac{1}{2}\left(\frac{\bar{B}^2}{B^2} - \frac{\bar{\bar{B}}}{B}\right)\left[\frac{t^2-(x^2+y^2+z^2)}{(x^2+y^2+z^2)^2}\right] = \lambda$$

(4.13)

$$\frac{1}{2}\left(\frac{\bar{C}^2}{C^2} - \frac{\bar{\bar{C}}}{C}\right)\left[\frac{t^2-(x^2+y^2+z^2)}{(x^2+y^2+z^2)^2}\right] = \lambda$$

(4.14)

Using the equation (4.12), we have

$$A = exp(D\lambda Z^2 + EZ + F)$$

(4.15)

where E,F are constants of integration

And

$$D = \left(\frac{[x^2+y^2+z^2]^2}{t^2-[x^2+y^2+z^2]}\right)$$

(4.16)

Similarly , by using the equation (4.13) and (4.14) , we have

$$B = exp(D\lambda Z^2 + GZ + H)$$

(4.17)

$$C = \exp(D\lambda Z^2 + IZ + J)$$

(4.18)

where G ,H, I and J are constants of integration.

Thus the metric (4.1) takes the form

$$ds^2 = -\frac{3t^2 e^{(D\lambda Z^2+EZ+F)}}{(x^2+y^2+z^2)^2}[x^2dx^2 + y^2dy^2 + z^2dz^2] - e^{(D\lambda Z^2+GZ+H)}du^2 - e^{(D\lambda Z^2+IZ+J)}dv^2 + e^{(D\lambda Z^2+EZ+F)}dt^2$$

(4.19)

Thus, $Z = \left[\frac{t}{\sqrt{x^2+y^2+z^2}}\right]$ - type higher six dimensional plane gravitational wave exists in biometric relativity with and without cosmological constant λ respectively.

V .Conclusion:

In the study of $Z = \left[\sqrt{x^2 + y^2 + z^2} - t\right]$- type plane gravitational wave in V_6, there is no contribution from cosmological constant λ. And further the study of $Z = \left[\frac{t}{\sqrt{x^2+y^2+z^2}}\right]$ - type higher six dimensional plane gravitational wave, I observed that the wave exists in bimetric relativity with and without cosmological constant λ respective.

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