Fuzzy Transportation Problem Using Improved Fuzzy Russells Method

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ABSTRACT: In this paper, we investigate the new idea of optimal solution of squared triangular and trapezoidal fuzzy number via fuzzy russal's method. This method is a modification of yager's ranking method. A new algorithm is investigated and a suitable optimal solution is obtained. A numerical example is given based on the algorithms.

KEY WORDS: Ranking function, Triangular, Trapezoidal Number, Transportation problem, Fuzzy Dominant Method.

I. INTRODUCTION: In Mathematics and Economics, transportation theory is a name given to the study of optimal transportation and allocation of resources. The problem was formalized by the French Mathematician Gaspard Monge in [1781]. Tolstoi was one of the first to study the transportation problem mathematically. The transportation problem (TP) refers to a special class of linear programming problem. When the theory of fuzzy sets was first introduced by Zadeh[20]. If shipping cost, are assumed to be proportional to the amount shipped from each origin to each destination so as to minimize total shipping cost turns out to be a linear programming problem. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities. Most of the existing techniques provide only crisp solutions for the fuzzy transportation problem. Shiang-Tai Liu and Chiang Kao [16],Chanas et al. [3], Chanas and Kuchta [2], proposed a method for solving fuzzy transportation problem. Nagoor Gani and Abdul Rezak [13] obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. Pandian et al. [15], proposed a method namely, zero point method, for finding a fuzzy optimal solution for a fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers. Amarpreet kaur[1] proposed a new method for solving fuzzy transportation problem by assuming that a decision maker is uncertain about the precise values of the transportation cost, availability and demand of the product. In many fuzzy decision problems, the data are represented in terms of fuzzy numbers. In a fuzzy transportation problem, all parameters are fuzzy numbers. Fuzzy numbers may be normal or abnormal, triangular or trapezoidal. Ranking method is used to change the fuzzy number into crisp form. The method for ranking was first proposed by Jain [8]. Yager [19] proposed four indices which may be employed for the purpose of ordering fuzzy quantities in [0, 1 . In this paper, we investigate the new idea of optimal solution of squared triangular and trapezoidal fuzzy number via fuzzy russal's method . This method is a modification of yager's ranking method. New algorithms is investigated and suitable optimal solutions is obtained. A numerical example is given based on the algorithms.

II. PRELIMINARIS

2.1 DEFINITION (FUZZY NUMBER):

A real fuzzy number a is a fuzzy subset of the real number R with membership function a

$\mu_{\tilde{a}}$ satisfying the following conditions,

$\mu_{\tilde{a}}$ is continuous from R to the closed interval [0,1]

$\mu_{\tilde{a}}$ is strictly increasing and continuous on $[a_1,a_2]$

$\mu_{\tilde{a}}$ is strictly decreasing and continuous on $[a_3,a_4]$

2.2 DEFINITION: A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0, 1]. (i,e) A = $\{(x, \mu_A(x) ; x \in X\}$, Here $\mu_A: X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of x ε X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

2.3 DEFINITION:

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist at least one x ε X such that $\mu_A(x) = 1$.

2.4 DEFINITION:

The fuzzy set A is convex if and only if, for any x_1, $x_2 \in X$, the membership function of A satisfies the inequality $\mu_A\{\lambda x_1+(1-\lambda)x_2\} \geq \min \{\mu_A(x_1), \mu_A(x_2)\}$. $0\leq\lambda\leq1$.

2.5 (TRIANGULAR FUZZY NUMBER) :

For a triangular fuzzy number A(x), it can be represented by A(a,b,c ; 1) with membership function μ(x) given by

$$\mu_A(x) = \begin{cases} (x-a)/(b-a) & , \quad a \le x \le b \\ 1 & , \quad x=b \\ (c-x)/(c-b) & , \quad c \le x \le d \\ 0 & , \quad \text{otherwise} \end{cases}$$

2.6 TRAPEZOIDAL FUZZY NUMBER:

A fuzzy number Ã(a,b,c,d) is said to be a trapezoidal fuzzy number if its membership function is given by,

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , \quad x < m \\ (x-a)/(b-a) & , \quad a \le x \le b \\ (d-x)/(d-c) & , \quad c \le x \le d \\ 0 & , \quad x > d \end{cases}$$

2.7 PROPERTIES OF TRAPEZOIDAL FUZZY NUMBER:

- Trapezoidal fuzzy number Ã= (a, b, c, d) is said to be non negative trapezoidal fuzzy number if and only if a-c ≥ 0.
- A trapezoidal fuzzy number Ã= (a, b, c, d) is said to be zero trapezoidal fuzzy number if and only if a = 0, b = 0, c = 0, d = 0.
- Two trapezoidal fuzzy numbers $\tilde{A}_1=(a_1,b_1,c_1,d_1)$ and $\tilde{A}_2 = (a_2,b_2,c_2,d_2)$ are said be equal i.e. $\tilde{A}_1 = \tilde{A}_2$, if and only if $a_1= a_2$, $b_1= b_2$, $c_1= c_2$, $d_1 = d_2$.

2.8 FUZZY SET:

A Fuzzy set A is defined as the set of ordered pairs (X, μ_A(x)) ,where x is an element of the universe of discourse U and $\mu_A(x)$ is the membership function, that attributes to each X Є U a real number Є[0,1]□, describing the degree to which X belongs to the set.

EXAMPLE:

Let X = {a, b, c, d }, Define μA : X →□[0,1] as follows:

μ_A (a) = 0, μ_A (b) = 0.4, μ_A (c) = 0.6, μ_A (d) = 1

Then the class, A = {(a, 0), (b, 0.4), (c, 0.6), (d, 1)} is a fuzzy set on X.

2.9 CRISP SET:

A crisp set is a special case of a Fuzzy set, in which the membership function only takes two values, commonly defined as 0 and 1.

3. ARITHMETIC OPERATION :

Let A_1 and A_2 be two trapezoidal fuzzy numbers parameterized by the quadruple and (a_1,a_2,a_3,a_4) (b_1,b_2,b_3,b_4) respectively. The simplified fuzzy number arithmetic operations between the trapezoidal fuzzy numbers A_1 and A_2 are as follows, □ □

Fuzzy numbers addition $(a_1,a_2,a_3,a_4) + (b_1,b_2,b_3,b_4) = (a_1 +b_1 ,a_2 +b_2 ,a_3+b_3 ,a_4+b_4)$

$$\text{Fuzzy numbers subtraction } \rho = \frac{\tilde{E}}{J_c\,(T=\text{constant.})\ (P.(\tilde{E}/E_c)^p + (1-P))}$$

Fuzzy numbers of subtraction $(a_1,a_2,a_3,a_4) - (b_1,b_2,b_3,b_4) = (a_1 -b_4 ,a_2 -b_3 ,a_3-b_2 ,a_4-b_1)$

Multiplication used by Stephen Dinegar.D &Palanivel.K [17]:

$\tilde{a}.\tilde{b}= [\ a_1/4(b_1+b_2 +b_3+b_4), a_2/4(b_1 +b_2 +b_3 +b_4), a_3/4 (b_1+b_2 +b_3 +b_4), a_4/4(b_1 +b_2+b_3 +b_4)]$, $R(a)>0$

$\tilde{a}.\tilde{b}= [\ a_4/4(b_1+b_2 +b_3+b_4), a_3/4(b_1 +b_2 +b_3 +b_4), a_2/4 (b_1+b_2 +b_3 +b_4), a_1/4(b_1 +b_2+b_3 +b_4)]$, $R(a)<0$

EXAMPLE:

Let A_1 and A_2 be two trapezoidal fuzzy numbers, where

$A_1= (1, 2, 3, 4)$ and $A_2= (5, 6, 7, 8)$.

Then, $A_1 + A_2 = (1,2,3,4) + (5,6,7,8) =(6,8,10,12)$

$A_1= (1,2,3,4)$ and $A_2 = (5,6,7,8)$ then $A_1-A_2=(-7,-5,-3,-1)$

3.1 MATHEMATICAL FORMULATION OF FUZZY TRANSPORTATION PROBLEM :

Minimize: $Z =\sum \sum \tilde{C}_{ij}\tilde{X}_{ij}$

Subject to, $\sum_{j=1}^{n} \tilde{X}_{ij} \leq \tilde{a}_i$ for $i=1,2,\ldots\ldots,m$

$$\sum_{i=1}^{m} \tilde{X}_{ij} \geq \tilde{b}_j \qquad \text{for} \quad j=1,2,\ldots\ldots,n$$

$$\tilde{x}_{ij} \geq 0 \qquad \text{for} \quad i=1,2,\ldots\ldots,m \quad \text{for} \quad j=1,2,\ldots\ldots,n$$

where $\tilde{a}_i=(a_1,a_2,a_3,a_4)$, $\tilde{b}_j = (b_1,b_2,b_3,b_4)$ and $\tilde{c}_{ij}$ $(c_{ij}, c_{ij}, c_{ij}, c_{ij})$ representing the uncertain supply and demand for the transportation problem

3.2 RANKING FUNCTION

Many different approaches for the ranking of fuzzy numbers have been proposed in the literature. In this Fuzzy Russell's method we use Yager's[19] ranking method. A ranking function

R: F(R) R which maps each fuzzy number into the real line, F(R) represents the set of all trapezoidal fuzzy number. If R be any ranking function,

$$R(\tilde{a}) = \frac{a_1+a_2+a_3+a_4}{4}$$

For any two trapezoidal Fuzzy number $\tilde{A}=(a_1,a_2,a_3,a_4)$, $\tilde{B}= (b_1,b_2,b_3,b_4)$ then we have,

(i)$\tilde{A}\leq\tilde{B} \Leftrightarrow R(\tilde{A})\leq R(\tilde{B})$ (II) $\tilde{A}\geq\tilde{B} \Leftrightarrow R(\tilde{A})\geq R(\tilde{B})$ (III) $\tilde{A}=\tilde{B} \Leftrightarrow R(\tilde{A})=R(\tilde{B})$

III. THE COMPUTATIONAL PROCEDURE FOR FUZZY RUSSELL'S METHOD:

In this section we proposes modified method called as Fuzzy Russell's method is used for finding initial basic feasible solution for Fuzzy transportation problem. The solution procedure as follows,

3.4 ALGORITHM FOR FUZZY METHOD:

Step 1: Calculate the quantities, and using $\tilde{U}_i$ and $\tilde{V}_j$ and Δ_{ij} using

$$\tilde{U}_i = \max_{1\leq j\leq n} \{C_{ij}\} \quad \text{for } i=1,2,\ldots\ldots,m$$

$$\tilde{V}_i = \max_{1\leq i\leq m} \{C_{ij}\} \quad \text{for } j=1,2,\ldots\ldots,n$$

And $\Delta_{ij} = C_{ij} - \tilde{U}_i - \tilde{V}_i$ for all I,j.

STEP 2:

Select the variables x_{ij} ($xij^{(1),}$ $xij^{(2),}$ $xij^{(3),}$ $xij^{(4)}$) having the most negative value of Δij .If there are ties in the value of Uij ,Select Vij x ($xij^{(1),}$ $xij^{(2),}$ $xij^{(3),}$ $xij^{(4)}$) with the smallest unit cost c_{ij} ($cij^{(1),}$ $cij^{(2)}$, $cij^{(3)}$,$cij^{(4)}$).If there are ties again in the value of c_{ij} ($cij^{(1),}cij^{(2),}cij^{(3),}cij^{(4)),}$select x_{ij} ($xij^{(1),}$ $xij^{(2)}$, $xij^{(3),}xij^{(4)}$) with the largest amount of remaining source supply or destination demand.

STEP3:

Set the activity level of x_{ij} ($xij^{(1),}$ $xij^{(2),}$ $xij^{(3),}xij^{(4))}$ equal to the smaller value between the source supply a_i and the destination demand b_j .

STEP4:

Subtract x_{ij} ($xij^{(1),}$ $xij^{(2),}$ $xij^{(3),}$ $xij^{(4)}$) from a_i and b_j found in step3. Eliminate from the transportation table the row or column that results in a zero supply or destination demand after this subtraction. Stop if all (i = 1, 2,…, m) and (j = 1, 2,…, n) are zero, otherwise go to step1 a_i b_j

IV. NUMERICAL EXAMPLE:

(I) Consider the Fuzzy transportation problem. Here cost value, supplies and demands are trangular fuzzy number. Here and are Fuzzy Supply and Fuzzy Demand. Fuzzy Russell's method is used to finding the initial basic feasible a_i b_jThe fuzzy transportation cost for unit quantity of the product from i^{th} source j^{th} destination if c_{ij} where

$$(C_{ij})_{3x3} = \begin{pmatrix} (1,\ 4,\ 9) & (16,25,36) & (9,36,49) \\ (16,25,64) & (36,64,81) & (4,49,64) \\ (4,25,81) & (25,36,64) & (49,64,81) \end{pmatrix}$$

Fuzzy availability of the product at the source are (4 ,25, 36) (16, 36 ,49) (25, 49 ,81) and the fuzzy demand of the product at destination are (16, 25 ,36) (4 ,49, 81) (25 ,36 ,49).

SOLUTION: Therefore

$$\sum_{i=1}^{n} \tilde{a}_i = \sum_{j=1}^{m} \tilde{b}_j$$

the problem is balanced fuzzy transportation problem. There exists a fuzzy initial basic feasible Solution Now we applying the Fuzzy Russell's method for fuzzy transportation problem we have

The fuzzy Transportation problems are given in **Table-1**

Sources	D_1	D_2	D_3	Supply
S_1	(1 , 4 , 9)	(16 ,25, 36)	(9, 36 ,49)	(4 ,25, 36)
S_2	(16, 25, 64)	(36, 64 ,81)	(4, 49 ,64)	(16, 36 ,49)
S_3	(4 ,25 ,81)	(25, 36, 64)	(49,64 ,81)	(25, 49 ,81)
Demand	(16, 25 ,36)	(4 ,49, 81)	(25 ,36 ,49)	

The fuzzy Transportation problems are given in **Table-2**

Sources	D_1	D_2	D_3	Supply
S_1	(1 , 4 , 9) **(4 ,25, 36)**	(16 ,25, 36)	(9, 36 ,49)	(4 ,25, 36)
S_2	(16, 25, 64) **(12,0,0)**	(36, 64 ,81)	(4, 49 ,64) **(4 ,36,49)**	(16, 36 ,49)
S_3	(4 ,25 ,81)	(25, 36, 64) **(4 ,49,81)**	(49,64 ,81) **(21,0,0)**	(25, 49 ,81)
Demand	(16, 25 ,36)	(4 ,49, 81)	(25 ,36 ,49)	

Therefore the initial basic feasible solution is, Minimum Z = ($Z^{(1),}$ $Z^{(2),}$ $Z^{(3)}$)

=(1 , 4 , 9)**(4 ,25, 36)**+(4, 49 ,64) **(4 ,36,49)**+(16, 25, 64)**(12,0,0)**+(25, 36, 64) **(4 ,49,81)**+(49,64 ,81)**(21,0,0)**

Minimum Z ($Z^{(1),}$ $Z^{(2),}$ $Z^{(3)}$)**=(**1341,3628,8644)

The crisp value of the Fuzzy Transportation problem is 4310.25.

II NUMERICAL EXAMPLE: (TRAPEZOIDAL SQUARED FUZZY NUMBERS):

A company has four sources S_1, S_2, S_3, S_4 and destinations D_1, D_2, D_3, D_4. The fuzzy transportation cost for unit quantity of product from i^{th} sources j^{th} destinations is C_{ij}

$$\text{where } C_{ij} = \begin{pmatrix} (1,4,9,16) & (4,9,16,25) & (9,16,25,36) & (16,25,36,49 \\ (4,9,16,25) & (9,16,25,36) & (16,25,36,49) & (25,36,49,64) \\ (9,16,25,36) & (16,25,36,49) & (25,36,49,64) & (36,49,64,81) \\ (16,25,36,49) & (25,36,49,64) & (36,49,64,81) & (25,36,49,81) \end{pmatrix}$$

Fuzzy availability of the product at source are (16 25 36 49) (36 49 64 81) (25 36 49 64) (4 16 25 36) and there Fuzzy demand of the product and destination are (36 49 64 81) (25 36 49 64) (4 16 25 36) (16 25 36 49

SOLUTION:

The fuzzy Transportation problems are given in **Table-1**

Sources	D_1	D_2	D_3	D_4	Supply
S_1	(1, 4 , 9, 16)	(4, 9,16,25)	(9,16,25,36)	(16,25,36,49)	(16,25,36,49)
S_2	(4, 9,16, 25)	(9,16,25,36)	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)
S_3	(9,16,25,36)	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)	(25,36,49,64)
S_4	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)	(25,36,49,81)	(4,16,25,36)
Demand	(36,49,64,81)	(25,36,49,64)	(4,16,25,36)	(16,25,36,49)	

Table-2

Sources	D_1	D_2	D_3	D_4	Supply
S_1	(1,4,9,16) **(16,25,36,49)**	(4, 9,16,25)	(9,16,25,36)	(16,25,36,49)	(16,25,36,49)
S_2	(4, 9,16, 25) **(-13,13,39,65)**	(9,16,25,36) **(-11,11,33,60)**	(16,25,36,49) **(-89,23,40,105)**	(25,36,49,64)	(36,49,64,81)
S_3	(9,16,25,36)	(16,25,36,49)	(25,36,49,64) **(-24,0,24,48)**	(36,49,64,81) **(16,25,36,49)**	(25,36,49,64)
S_4	(16,25,36,49)	(25,36,49,64) **(4,16,25,36)**	(36,49,64,81)	(25,36,49,81)	(4,16,25,36)
Demand	(36,49,64,81)	(25,36,49,64)	(4,16,25,36)	(16,25,36,49)	

Therefore the initial basic feasible solution is, Minimum Z = $(Z^{(1),} Z^{(2),} Z^{(3),} Z^{(4)})$
=(1, 4 , 9, 16) (16,25,36,49)+(4, 9,16, 25)(-13,13,39,65)+(9,16,25,36) (-11,11,33,60)+(16,25,36,49)(-89,-23,40,105)+(25,36,49,64) (-24,0,24,48) + (36,49,64,81) (16,25,36,49) + (25,36,49,64)(4,16,25,36)

Minimum Z = $(Z^{(1),} Z^{(2),} Z^{(3),} Z^{(4)})$
= (2505.5,3,748.25,5229.25,6973.25)

The crisp value of the Fuzzy Transportation problem is 4,614

V. CONCLUSION:

We proposed Fuzzy Dominant method to find the initial basic feasible solution using Yager's ranking method with trapezoidal fuzzy numbers. This method can be used for all kinds of fuzzy numbers. This method is very easy to apply and can be utilized for the fuzzy transportation problem. This technique can also be tried in solving other types of problem like, project schedules, assignment problem and network flow problem.

REFERENCES:

[1] Amarpreet kaur,Amit Kumar. 'A new method for solving fuzzy transportation problem using ranking function', Applied Mathematical Modeling, (2011), 35, pp: 5652-5661.

[2] Chanas, S and Kuchta, D.'A concept of the optimal solution of the transportation problem with fuzzy cost coefficients',Fuzzy sets and Systems,(1996),82, pp:299-305.

[3] Chanas.S, Kolodziejczyk.W and Machaj, A. 'A fuzzy approach to the transportation problem', Fuzzy Sets and Systems, (1984), 13, pp: 211–221.

[4] Campos, L. and Gonzalez Munoz, A. 'A subjective approach for ranking fuzzy number', Fuzzy Sets and Systems, (1989), 29, pp: 145-153.

[5] Campos, L. and Verdegay, J. L. 'Linear programming problem and ranking of fuzzy numbers', Fuzzy Sets and Systems, (1989),32,pp 1-11.

[6] F.L.Hitchcock. 'The distribution of a product from several source to numerous localities', J.Math.phys, (1941), 20,pp:224-230.

[7] Ismail Mohideen, S. and Senthil Kumar, P. 'A Comparative Study on Transportation Problem in Fuzzy Environment', International Journal of Mathematics Research, ISSN 0976-5840, (2010), Vol.2, Number 1, pp: 151-158.

[8] Jain, R. 'Decision-making in the presence of fuzzy variables', IEEE Transactions on Systems, Man and Cybernetics, (1976), 6, pp: 698-703.

[9]. R.Jahirhussain , P.Jayaraman fuzzy optimal transportation problem by improved zero suffix method via robust rank techniques International Journal of Fuzzy Mathematics and Systems (IJFMS). Volume 3 (2013) pp 303-311

[10] Kim, K. and Park, K. S. 'Ranking fuzzy number with index of optimism', Fuzzy Sets and Systems, (1990), 35, pp: 143-150.

[11] Liou, T. S. and Wang, M. J. 'Ranking fuzzy numbers with integral value', Fuzzy Sets and Systems, (1992), 50, pp: 247-255.

[12] S.Narayanamoorthy S.Saranya & S.Maheswari A Method for Solving Fuzzy Transportation Problem (FTP) using Fuzzy Russell's Method I.J. Intelligent Systems and Applications, 2013, 02, 71-75

[13] Nagoor Gani, A. and Abdul Razak, K. 'Two Stage Fuzzy Transportation Problem', Journal of Physical Sciences, (2006), Vol.10,pp:63-69.

[14] Narayanamoorthy, S., Anukokila P. 'Robust fuzzy transportation problems based on extension principle under uncertain Demands', IJMA, (2012), Vol.5.No.1. pp: 01-19.

[15] Pandian, P and Natarajan, G. 'A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problem',Applied Mathematical Sciences,(2010),4, pp: 79-90.

[16] Shiang-Tai Liu, and Chiang Kao. 'Solving fuzzy transportation problem based on extension principle', Journal of Physical Science, (2006), 10, pp: 63-69.

[17] Stephen Dinagar, D and Palanivel, K., 'The transportation problem in Fuzzy Environment'. International Journal of Algorithms, Computing and Mathematics, August 2009, Vol 2, Number 3.

[18] Tze-San lee. 'A complete Russell's method for the Transportation Problem', (1986), Vol.28, No.4.

[19] Yager, R.R. 'A procedure for ordering fuzzy subsets of the unit interval', Information Sciences, (1981), 24, pp: 143-161.

[20] Zadeh, L. A. Fuzzy sets, Information and Control, (1965), 8, pp: 338-353.