Normalized Hamming Similarity Measure for Intuitionistic Fuzzy Multi Sets and Its Application in Medical diagnosis

***P. Rajarajeswari,   **N. Uma**

** Department of Mathematics, Chikkanna Arts College, Tirupur, Tamil Nadu. (INDIA).*

*** Department of Mathematics, SNR Sons College, Coimbatore, Tamil Nadu. (INDIA).*

ABSTRACT

As Similarity measure for fuzzy sets is one of the important research topics of fuzzy set theory, there are several methods to measure similarity between two fuzzy sets (FS), Intuitionistic fuzzy sets (IFS) and Intuitionistic fuzzy multi sets (IFMS). In this paper, the Normalized Hamming Similarity measure of Intuitionistic Fuzzy Multi sets (IFMS) is introduced. This new measure for IFMS is based on the geometrical interpretation of IFS which involves both similarity and dissimilarity. Using this measure, the application of medical diagnosis and pattern recognition are shown.

KEYWORDS: Intuitionistic fuzzy set, Intuitionistic Fuzzy Multi sets, Normalized Hamming distance, Pattern Recognition, Medical Diagnosis.

I. INTRODUCTION

The Fuzzy set (*FS*) proposed by **Lofti A. Zadeh [1]** allows the uncertainty of a set with a membership degree (μ) between 0 and 1. That is, the membership function ($\mu \in [0,1]$) and the non membership function equals one minus the membership degree ($1 - \mu \in [0,1]$). The Intuitionistic Fuzzy sets (*IFS*), a generalisation of the Fuzzy set (*FS*) was introduced by **Krasssimir T. Atanassov [2, 3].** The *IFS* represent the uncertainty with respect to both membership ($\mu \in [0,1]$) and non membership ($\vartheta \in [0,1]$) such that $\mu + \vartheta \leq 1$. The number $\pi = 1 - \mu - \vartheta$ is called the hesitiation degree or intuitionistic index. The study of distance and similarity measure of *IFS*s by several authors like **Y. H. Li, D. L. Olson, Q. Zheng [4], Li and Cheng [5], Liang and Shi [6]** gives lots of measures, each representing specific properties and behaviour in real-life decision making and pattern recognition works. Based on Hamming distance, **Szmidt and Kacprzyk [7, 8, 9]** introduced the distance and similarity measure between IFSs and its application is widely used in various fields like medical diagnosis, logic programming, decision making. The Similarity based on Normalized Hamming distance and the Complement of *IFS* proposed by **Szmidt and Kacprzyk [10, 11, 12]** concludes that one can avoid conclusions about the strong similarity between intuitionistic fuzzy sets on the basis of the small distances between the referred sets.

The Fuzzy Multi set (*FMS*) introduced by **R. R. Yager [13]** can occur more than once with the possibly of the same or the different membership values, which is based on the Multi set **[14]** repeats the occurrences of any element. And recently, the new concept Intuitionistic Fuzzy Multi sets (*IFMS*) was proposed by **T.K Shinoj and Sunil Jacob John [15].**

As various distance and similarity methods of *IFS* are extended for *IFMS* distance and similarity measures **[16], [17], [18] and [19],** this paper is an extension of the new similarity measure based on Normalized Hamming distance and Complement of *IFS* to *IFMS.*

II. PRELIMINARIES

Definition: 2.1

Let X be a nonempty set. A **fuzzy *set*** *A* in *X* is given by $A = \{\langle x, \mu_A(x)\rangle / \ x \in X\}$ -- (2.1) where $\mu_A : X \to [0, 1]$ is the membership function of the fuzzy set A (i.e.) $\mu_A(x) \in [0,1]$ is the membership of $x \in X$ in A. The generalizations of fuzzy sets are the Intuitionistic fuzzy *(IFS)* set proposed by **Atanassov [1, 2]** is with independent memberships and non memberships.

Definition: 2.2

An ***Intuitionistic fuzzy set*** *(IFS), A* in *X* is given by $A = \{\langle x,\ \mu_A(x), \vartheta_A(x)\ \rangle / x \in X\}$ -- (2.2)

where $\mu_A : X \to [0,1]$ and $\vartheta_A : X \to [0,1]$ with the condition $0 \le \mu_A(x) + \vartheta_A(x) \le 1$, $\forall x \in X$
Here $\mu_A(x)$ *and* $\vartheta_A(x) \in [0,1]$ denote the membership and the non membership functions of the fuzzy set *A*;

For each Intuitionistic fuzzy set in *X*, $\pi_A(x) = 1 - \mu_A(x) - [1 - \mu_A(x)] = 0$ for all $x \in X$ that is

$\pi_A(x) = 1 - \mu_A(x) - \vartheta_A(x)$ is the hesitancy degree of $x \in X$ in *A*. Always $0 \le \pi_A(x) \le 1$, $\forall x \in X$.

The ***complementary set*** A^c of *A* is defined as $A^c = \{\langle x,\ \vartheta_A(x),\ \mu_A(x)\ \rangle / x \in X\}$ -- (2.3)

Definition: 2.3

Let X be a nonempty set. A ***Fuzzy Multi set (FMS)*** *A* in X is characterized by the count membership function Mc such that Mc : X → Q where Q is the set of all crisp multi sets in [0,1]. Hence, for any $x \in X$, Mc(x) is the crisp multi set from [0, 1]. The membership sequence is defined as

$(\mu_A^1(x)\ , \mu_A^2(x), \dots \dots \dots \mu_A^p(x))$ where $\mu_A^1(x) \ge \mu_A^2(x) \ge \cdots \ge \mu_A^p(x)$.

Therefore, A FMS *A* is given by $A = \{\langle\, x,\ (\mu_A^1(x)\ , \mu_A^2(x), \dots \dots \dots \mu_A^p(x))\ \rangle / x \in X\}$ -- (2.4)

Definition: 2.4

Let X be a nonempty set. A ***Intuitionistic Fuzzy Multi set (IFMS)*** *A* in X is characterized by two functions namely count membership function Mc and count non membership function NMc such that Mc : X → Q and NMc : X → Q where Q is the set of all crisp multi sets in [0,1]. Hence, for any $x \in X$, Mc(x) is the crisp multi set from [0, 1] whose membership sequence is defined as

$(\mu_A^1(x)\ , \mu_A^2(x), \dots \dots \dots \mu_A^p(x))$ where $\mu_A^1(x) \ge \mu_A^2(x) \ge \cdots \ge \mu_A^p(x)$ and the corresponding

non membership sequence NMc (x) is defined as $(\vartheta_A^1(x)\ , \vartheta_A^2(x), \dots \dots \dots \vartheta_A^p(x))$ where the non membership can be either decreasing or increasing function. such that $0 \le \mu_A^i(x) + \vartheta_A^i(x) \le 1, \forall x \in X \text{ and } i = 1, 2, \dots p.$

Therefore, An ***IFMS*** *A* is given by

$A = \{\langle\, x,\ (\mu_A^1(x)\ , \mu_A^2(x), \dots \dots \dots \mu_A^p(x)),\ (\vartheta_A^1(x)\ , \vartheta_A^2(x), \dots \dots \dots \vartheta_A^p(x))\ \rangle / x \in X\}$ -- (2.5)

where $\mu_A^1(x) \ge \mu_A^2(x) \ge \cdots \ge \mu_A^p(x)$

The ***complementary set*** A^c of *A* is defined as

$A^c = \{\langle x,\ (\vartheta_A^1(x)\ , \vartheta_A^2(x), \dots \dots \dots \vartheta_A^p(x)),\ (\mu_A^1(x)\ , \mu_A^2(x), \dots \dots \dots \mu_A^p(x)),\rangle / x \in X\}$ – (2.6)

$\textit{where } \vartheta_A^1(x) \ge \vartheta_A^2(x) \ge \cdots \ge \vartheta_A^p(x)$

Definition: 2.5

The **Cardinality** of the membership function Mc(x) and the non membership function NMc (x) is the length of an element x in an *IFMS* *A* denoted as η, defined as η = |Mc(x)| = |NMc(x)|

If A, B, C are the *IFMS* defined on X , then their cardinality η = Max { η(A), η(B), η(C) }.

Definition: 2.6

$S(\boldsymbol{A}, \boldsymbol{B})$ is said to be the **similarity measure** between A and B , where A, B ∈ X and X is an *IFMS*, as $S(\boldsymbol{A}, \boldsymbol{B})$ satisfies the following properties

1. $S(\boldsymbol{A}, \boldsymbol{B}) \in [0,1]$
2. $S(\boldsymbol{A}, \boldsymbol{B}) = 1$ if and only if A = B
3. $S(\boldsymbol{A}, \boldsymbol{B}) = S(\boldsymbol{B}, \boldsymbol{A})$

Definition: 2.7

The geometrical interpretation of Intuitionistic Fuzzy sets given in **Szmidt, Baldwin and Kacprzyk [10, 11, 12]** implies that any combination of the parameters characteristic for elements belonging to an Intuitionistic fuzzy set can be represented inside triangle. A similarity of any two elements X and F belonging to an Intuitionistic fuzzy sets is $Sim\,(X, F) = \frac{I_{IFS}(X\,,\;F\,)}{I_{IFS}(X\,,\;F^c\,)} = \frac{a}{b}$ -- (2.7)

Where F^c is the complement set of F, 'a' is the distance from X to F and 'b' is the distance from X to F^c .

III NORMALIZED HAMMING SIMILARITY MEASURE FOR *IFMS*

In the *IFMS*, the Normalized Hamming distance between two elements A and B is

$$N_D^*(A\,,B\,) = \frac{1}{\eta}\sum_{j=1}^{\eta}\left\{\frac{1}{2n}\sum_{i=1}^{n}\left(\left|\mu_A^j(x_i) - \mu_B^j(x_i)\right| + \left|\vartheta_A^j(x_i) - \vartheta_B^j(x_i)\right|\right)\right\} \quad \text{-- (3.3.1)}$$

with multi membership and multi non membership function

$$N_D^*(A\,,B\,) = \frac{1}{\eta}\sum_{j=1}^{\eta}\left\{\frac{1}{2n}\sum_{i=1}^{n}\left(\left|\mu_A^j(x_i) - \mu_B^j(x_i)\right| + \left|\vartheta_A^j(x_i) - \vartheta_B^j(x_i)\right|\right) + \left|\pi_A^j(x_i) - \pi_B^j(x_i)\right|\right\} \quad \text{--(3.3.2)}$$

with the multi membership, multi non membership and multi hesitation functions.

Then using the concept of Normalized Hamming distance, the Similarity measure in *IFMS* is

$$\boldsymbol{Sim}\,(\boldsymbol{A}, \boldsymbol{B}) = \frac{N_D^*(A\,,\;B\,)}{N_D^*(A\,,\;B^c\,)} \quad \text{-- (3.3.3)}$$

Where B^c is the complement set of B such that
$B^c = \{\langle x,\ (\vartheta_B^1(x)\ ,\vartheta_B^2(x), \dots \dots \dots \vartheta_B^p(x)\ ),\ (\ \mu_B^1(x)\ ,\mu_B^2(x), \dots \dots \dots \mu_B^p(x)\ )\rangle / \ x \in X\}$

$N_D^*(A\,,\ B\,)$ is the distance from A to B and $N_D^*(A\,,\ B^c\,)$ is the distance from A to B^c. And this similarity measure lies between 0 and ∞. (i.e.) $0 \le \boldsymbol{Sim}\,(\boldsymbol{A}, \boldsymbol{B}) \le \infty$ **.**

Let A and B are two *IFMS* defined on the same set of universe of discourse, then there are four possibilities of the similarity measure of A and B.

1. A and B are two exactly similar sets
2. A and B^c are two exactly similar sets
3. A is more similar to B than to B^c
4. A is more similar to B^c than to B

We clearly know that A can never be similar to B and B^c together. (i.e.) A = B = B^c is never possible.

Hence we have the four possible cases

Case (i) : $Sim\,(A,B)$ = 0 when A = B

Let the two *IFMS* A and B be equal (i.e.) **A = B**. This implies for any $\mu_A^j(x_i) = \mu_B^j(x_i)$ and $\vartheta_A^j(x_i) = \vartheta_B^j(x_i)$ which states that $|\mu_A^j(x_i) - \mu_B^j(x_i)|$ *and* $|\vartheta_A^j(x_i) - \vartheta_B^j(x_i)| = 0$. Hence $N_D^*(A\,,\,B\,) = 0$

$$\text{Therefore} \quad Sim\,(A,B) = \frac{N_D^*(A\,,\,B\,)}{N_D^*(A\,,\,B^c\,)} = \frac{0}{N_D^*(A\,,\,B^c\,)} = 0$$

Case (ii) : $Sim\,(A,B) = \infty$ when A = B^c

Let the two *IFMS* A and B^c be equal (i.e.) **A = B^c** implies $N_D^*(A\,,\,B^c\,) = 0$

$$\text{Therefore} \quad Sim\,(A,B) = \frac{N_D^*(A\,,\,B\,)}{N_D^*(A\,,\,B^c\,)} = \frac{N_D^*(A\,,\,B\,)}{0} = \infty$$

Case (iii) : $Sim\,(A,\,B) > 1$ when A is more similar to B than to B^c

A is more similar to B than to B^c refers $N_D^*(A\,,B\,) > N_D^*(A\,,\,B^c\,)$. Hence $Sim\,(A,B) = \frac{N_D^*(A\,,\,B\,)}{N_D^*(A\,,\,B^c\,)} > 1$

Case (iv) : $Sim\,(A,\,B) < 1$ when A is more similar to B^c than to B

A is more similar to B^c than to B refers $N_D^*(A\,,\,B^c\,) > N_D^*(A\,,B\,)$. Hence $Sim\,(A,B) = \frac{N_D^*(A\,,\,B\,)}{N_D^*(A\,,\,B^c\,)} < 1$

From the four cases it is clearly known that to measure the similarity between two *IFMS* A and B, one should be interested in the values $\mathbf{0 \leq Sim\,(A,\,B) < 1}$

Therefore the new similarity measure of similarity between two IFMS A and B is constructed for selecting the objects which are more similar than dissimilar.

IV MEDICAL DIAGNOSIS USING IFMS - NORMALIZED HAMMING DISTANCE

As Medical diagnosis contains lots of uncertainties, they are the most interesting and fruitful areas of application in Intuitionistic fuzzy set theory consisting of both the terms like membership and non membership function are considered to be the better one. Due to the increased volume of information available to physicians from new medical technologies, the process of classifying different set of symptoms under a single name of disease becomes difficult. Recently, there are various models of medical diagnosis under the general framework of fuzzy sets are proposed. In some practical situations, there is the possibility of each element having different membership and non membership functions. The proposed distance and similarity measure among the Patients Vs Symptoms and Symptoms Vs diseases gives the proper medical diagnosis. Here, the lowest similarity points out a proper diagnosis.

The unique feature of this proposed method is that it considers multi membership and non membership. By taking one time inspection, there may be error in diagnosis. Hence, this multi time inspection, by taking the samples of the same patient at different times gives best diagnosis.

Let P = { P_1, P_2, P_3, P_4 } be a set of Patients.

D = { Fever, Tuberculosis, Typhoid, Throat disease } be the set of diseases

and S = { Temperature, Cough, Throat pain, Headache, Body pain } be the set of symptoms.

Our solution is to examine the patient at different time intervals (three times a day), which in turn give arise to different membership and non membership function for each patient.

TABLE : 4.1 – IFMs Q : The Relation between Patient and Symptoms

Q	Temperature	Cough	Throat Pain	Head Ache	Body Pain
P_1	(0.6, 0.2) (0.7, 0.1) (0.5, 0.4)	(0.4, 0.3) (0.3, 0.6) (0.4, 0.4)	(0.1, 0.7) (0.2, 0.7) (0, 0.8)	(0.5, 0.4) (0.6, 0.3) (0.7, 0.2)	(0.2, 0.6) (0.3, 0.4) (0.4, 0.4)
P_2	(0.4, 0.5) (0.3, 0.4) (0.5, 0.4)	(0.7, 0.2) (0.6, 0.2) (0.8, 0.1)	(0.6, 0.3) (0.5, 0.3) (0.4, 0.4)	(0.3, 0.7) (0.6, 0.3) (0.2, 0.7)	(0.8, 0.1) (0.7, 0.2) (0.5, 0.3)
P_3	(0.1, 0.7) (0.2, 0.6) (0.1, 0.9)	(0.3, 0.6) (0.2, 0) (0.1, 0.7)	(0.8, 0) (0.7, 0.1) (0.8, 0.1)	(0.3, 0.6) (0.2, 0.7) (0.2, 0.6)	(0.4, 0.4) (0.3, 0.7) (0.2, 0.7)
P_4	(0.5, 0.4) (0.4, 0.4) (0.5, 0.3)	(0.4, 0.5) (0.3, 0.3) (0.1, 0.7)	(0.2, 0.7) (0.1, 0.6) (0, 0.7)	(0.5, 0.4) (0.6, 0.3) (0.3, 0.6)	(0.4, 0.6) (0.5, 0.4) (0.4, 0.3)

Let the samples be taken at three different timings in a day (morning, noon and night)

TABLE : 4.2 – IFMs R : The Relation among Symptoms and Diseases

R	Viral Fever	Tuberculosis	Typhoid	Throat disease
Temperature	(0.8, 0.1)	(0.2, 0.7)	(0.5, 0.3)	(0.1, 0.7)
Cough	(0.2, 0,7)	(0.9, 0)	(0.3, 0,5)	(0.3, 0,6)
Throat Pain	(0.3, 0.5)	(0.7, 0.2)	(0.2, 0.7)	(0.8, 0.1)
Head ache	(0.5, 0.3)	(0.6, 0.3)	(0.2, 0.6)	(0.1, 0.8)
Body ache	(0.5, 0.4)	(0.7, 0.2)	(0.4, 0.4)	(0.1, 0.8)

TABLE : 4.3 – The Normalized Hamming distance $N_D^*(Q, R)$ between IFMs Q and R :

$N_D^*(Q, R)$	Viral Fever	Tuberculosis	Typhoid	Throat disease
P_1	0.1633	0.3067	**0.1430**	0.4067
P_2	0.2607	**0.1833**	0.2533	0.3600
P_3	0.3533	0.3000	0.2600	**0.1200**
P_4	0.1767	0.3333	**0.1033**	0.3667

The lowest distance from the table 4.3 gives the proper medical diagnosis. **Patient P_1** suffers from **Typhoid, Patient P_2** suffers from **Tuberculosis, Patient P_3** suffers from **Throat disease** and **Patient P_4** suffers from **Typhoid.**

The table 4.3 refers that the distance measure of Patient P1 is for viral fever and typhoid are nearer. To avoid the conclusion based on the small distances between them, the complement distance measure is determined and hence the Normalized Hamming similarity measure is calculated.

TABLE : 4.4 – The Normalized Hamming distance $N_D^*(A, B^c)$ between IFMs Q and R :

$N_D^*(A, B^c)$	Viral Fever	Tuberculosis	Typhoid	Throat disease
P_1	0.3033	0.4000	0.2300	0.2000
P_2	0.1667	0.3767	0.1900	02867
P_3	0.1967	0.3167	0.2267	0.5433
P_4	0.2767	0.2033	0.2300	0.2600

TABLE : 4.5 – Similarity measure $Sim\,(A,B)$ based on Complement measure between IFMs Q and R

$Sim\,(A,B)$	Viral Fever	Tuberculosis	Typhoid	Throat disease
P_1	**0.5384**	0.7668	0.6277	2.0340
P_2	1.5639	**0.4866**	1.3330	1.2557
P_3	1.7961	0.9473	1.1469	**0.2209**
P_4	0.6386	1.6395	**0.4491**	1.4104

As we are interested in the values of $0 \leq Sim\,(A,\ B) < 1$, other values are highlighted and omitted. (i.e.) **the new similarity measure of medical diagnosis refers selecting the objects which are more similar than dissimilar.** The lowest value from the table 4.5 other than highlighted values gives **Patient P_1** suffers from Viral **Fever** instead of Typhoid**,** Patient **P_2** suffers from **Tuberculosis, Patient P_3** suffers from **Throat disease** and **Patient P_4** suffers from **Typhoid.**

V PATTERN RECOGNISION OF *IFMS* CORRELATION SIMILARITY MEASURE

EXAMPLE: 5.2

Let X = {A_1, A_2, A_3, A_4........ A_n } with A = { A_1, A_2 }; B ={ A_4, A_6}; C = { A_1, A_{10}} ; D = { A_4, A_6} ;
E = { A_4, A_6}are the *IFMS* defined as
A = { $\langle A_1 : (0.2,0.2)\rangle, \langle A_2 : (0.3,0.2)\rangle$ } ; B = { $\langle A_4 : (0.1,0.2)\rangle, \langle A_6 : (02,0.3)\rangle$ } ;
C = { $\langle A_1 : (0.1,0.1)\rangle, \langle A_{10} : (0.2,0.2)\rangle$ } ; D = { $\langle A_3 : (0.2,0.1)\rangle, \langle A_4 : (0.3,0.2)\rangle$ } ;
E = { $\langle A_1 : (0.5,0.4)\rangle, \langle A_4 : (0.8,0.1)\rangle$ }

The *IFMS* Pattern Y = { $\langle A_1 : (0.1,0.2)\rangle, \langle A_{10} : (0.2,0.3)\rangle$ }

Here, the cardinality η = 2 as $|Mc(A)| = |NMc(A)| = 2$ and $|Mc(B)| = |NMc(B)| = 2$,
then the Complement measure between the Patten (A, Y) = 3.0, Patten (B, Y) = **0,** Patten (C, Y) = 1**,** Patten (D, Y) = ∝, Patten (E, Y) = 1.167 and the **testing Pattern Y belongs to Pattern B type**
As the values larger than 1 are omitted, the Pattern B is closer.

EXAMPLE: 5.3

Let X = {A_1, A_2, A_3, A_4........ A_n } with X1 = { A_1, A_2 }; X2 ={ A_3, A_4 }; X3 = { A_1, A_4 } are the *IFMS* defined as
A = { $\langle A_1 : (0.4,0.2,0.1), (0.3,0.1,0.2), (0.2,0.1,0.2), (0.1,0.4,0.3)\rangle$,
$\langle A_2 : (0.6,0.3,0), (0.4,0.5,0.1), (0.4,0.3,0.2), (0.2,0.6,0.2)\rangle$ }
B = { $\langle A_3 : (0.5,0.2,0.3), (0.4,0.2,0.3), (0.4,0.1,0.2), (0.1,0.1,0.6)\rangle$
$\langle A_4 : (0.4,0.6,0.2), (0.4,0.5,0), (0.3,0.4,0.2), (0.2,0.4,0.1)\rangle$ }
C = { $\langle A_1 : (0.4,0.2,0.1), (0.3,0.1,0.2), (0.2,0.1,0.2), (0.1,0.4,0.3)\rangle$,
$\langle A_4 : (0.4,0.6,0.2), (0.4,0.5,0), (0.3,0.4,0.2), (0.2,0.4,0.1)\rangle$ }

then the Pattern D of *IFMS* referred as { $\langle A_5 : (0.4,0.6,0.2), (0.4,0.5,0), (0.3,0.4,0.2), (0.2,0.4,0.1)\rangle$,
$\langle A_6 : (0.4,0.2,0.2), (0.5,0.5,0), (0.2,0.4,0.2), (0.2,0.5,0.1)\rangle$ }

The cardinality η = 2 as $|Mc(A)| = |NMc(A)| = |Hc(A)| = 2$ and $|Mc(B)| = |NMc(B)| = |Hc(B)| = 2$,
then the Proposed measure between the Pattern (A, D) is 0.7561 ; the Pattern (B, D) is 0.9412 and the Pattern (C, D) is **0.7222.**
Hence, the testing Pattern D belongs to Pattern C type because of the lowest value.

VI. CONCLUSION

In this paper, the Normalized similarity measure of *IFMS* from *IFS* theory is introduced. The prominent characteristic of this proposed measure of any two *IFMS*s is constructed for selecting the objects which are more similar than dissimilar. The **pattern recognition**, **example 5.1,** shows that the new measure perform well in the case of membership and non membership function and **example 5.2** depicts that the proposed measure is effective with three representatives of *IFMS* – membership, non membership and hesitation functions.

Also the measure equals to zero if and only if the two *IFMS*s are the same, referred in the **example 5:1 of pattern recognition**. Finally, the medical diagnosis using the proposed measure has been given to show the efficiency of the developed IFMS. This measure makes it possible to avoid the diagnosis about the IFMS on the basis of the small distances between them.

REFERENCES

[1] Zadeh L. A., *Fuzzy sets, Information and Control 8* (1965) 338-353.

[2] Atanassov K., Intuitionistic fuzzy sets, *Fuzzy Sets and System 20* (1986) 87-96.

[3] Atanassov K., More on Intuitionistic fuzzy sets, Fuzzy *Sets and Systems 33* (1989) 37-46.

[4] Yanhong L., Olson D., Qin Z., Similarity between Intuitionistic Fuzzy sets : Comparative Analysis, *Pattern Recognition Letters, 28- 2* (2007) 278-285.

[5] Li D., Cheng C., New Similarity measures of Intuitionistic fuzzy sets and application to pattern recognition, *Pattern Recognition Letters , 23* (2002) 221-225.

[6] Liang Z., Shi P., Similarity measures on Intuitionistic fuzzy sets, *Pattern Recognition Letters, 24* (2003) 2687-2693.

[7] Szmidt E., Kacprzyk J., Remarks on some applications of Intuitionistic fuzzy sets in decision making, *Notes on IFS, 2,(1996)* 22-31.

[8] Szmidt E., Kacprzyk J., On measuring distances between Intuitionistic fuzzy sets, *Notes on IFS, Vol. 3* (1997) 1- 13.

[9] Szmidt E., Kacprzyk J., Distances between Intuitionistic fuzzy sets. *Fuzzy Sets System, 114* (2000) 505-518.

[10] Szmidt E., Kacprzyk J., Entropy for Intuitionistic Fuzzy sets, *Fuzzy Sets and System, 118, (2001)* 467-477.

[11] Szmidt E., Baldwin J., New similarity measure for Intuitionistic fuzzy set theory and mass assignment theory, *Notes on IFS, 9, (2003)* 60-79.

[12] Szmidt E., Kacprzyk J., A similarity measure for Intuitionistic fuzzy sets and its application in supporting medical diagnostic reasoning. *ICAISC 2004, Vol. LNAI 3070* (2004) 388-393.

[13] Yager R. R., On the theory of bags,(Multi sets), *Int. Jou. Of General System, 13* (1986) 23-37.

[14] Blizard W. D., Multi set Theory, *Notre Dame Journal of Formal Logic, Vol. 30, No. 1* (1989) 36-66.

[15] Shinoj T.K., Sunil Jacob John , Intuitionistic Fuzzy Multi sets and its Application in Medical Diagnosis, *World Academy of Science, Engineering and Technology, Vol. 61* (2012).

[16] P. Rajarajeswari., N. Uma., On Distance and Similarity Measures of Intuitionistic Fuzzy Multi Set, *IOSR Journal of Mathematics (IOSR-JM) Vol. 5, Issue 4* (Jan. - Feb. 2013) *19-23.*

[17] P. Rajarajeswari., N. Uma., Hausdroff Similarity measures for Intuitionistic Fuzzy Multi Sets and Its Application in Medical diagnosis, *International Journal of Mathematical Archive-4(9)*,(2013) *106-111.*

[18] P. Rajarajeswari., N. Uma., A Study of Normalized Geometric and Normalized Hamming Distance Measures in Intuitionistic Fuzzy Multi Sets, *International Journal of Science and Research (IJSR), Vol. 2, Issue 11,* November 2013, 76-80.

[19] P. Rajarajeswari., N. Uma., Intuitionistic Fuzzy Multi Similarity Measure Based on Cotangent Function, *International Journal of Engineering Research & Technology (IJERT) Vol. 2 Issue 11,* (Nov- 2013) 1323–1329.