

Intuitionistic Fuzzy β Generalized Homeomorphisms

M. Saranya

Assistant Professor, Department of Mathematics,
AJK College of Arts and Science, Coimbatore, Tamil Nadu, India

Abstract: In this paper we introduce the new class of homeomorphisms called intuitionistic fuzzy β generalized homeomorphisms. We also introduce $M - \beta$ generalized homeomorphisms in intuitionistic fuzzy topological spaces and investigate some of the properties. We provide the relation between intuitionistic fuzzy β generalized homeomorphisms and intuitionistic fuzzy $M - \beta$ generalized homeomorphisms. Also we prove that the set of all M -generalized semi-pre homeomorphisms forms a group under the operation of composition of maps.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy $\beta_g T_{1/2}$ space, intuitionistic fuzzy β generalized homeomorphisms and intuitionistic fuzzy $M - \beta$ generalized homeomorphisms.

AMS Classification code: 54A40, 03E72

I. INTRODUCTION

Atanassov [1] introduced the idea of intuitionistic fuzzy sets. Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. The notion of homeomorphisms plays a vital role in intuitionistic fuzzy topology as well as in topology. Here we introduce the new class of homeomorphisms called β generalized homeomorphisms in intuitionistic fuzzy topological spaces. We also introduce the $M - \beta$ generalized homeomorphisms in intuitionistic fuzzy topological spaces and investigate some of their properties. We provide the relation between intuitionistic fuzzy β generalized homeomorphisms and intuitionistic fuzzy $M - \beta$ generalized homeomorphisms. Also we prove that the set of all $M - \beta$ generalized homeomorphisms forms a group under the operation of composition of maps.

II. PRELIMINARIES

Definition 2.1 [1]: An *intuitionistic fuzzy set* (IFS for short) A is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$$

where the functions $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each

$x \in X$. Denote by $\text{IFS}(X)$, the set of all intuitionistic fuzzy sets in X .

An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A \rangle$ instead of denoting $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$.

Definition 2.2 [1]: Let A and B be two IFSs of the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$$

and

$$B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in X \}.$$

Then,

- (a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
- (b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,
- (c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle : x \in X \}$,
- (d) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X \}$,
- (e) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X \}$.

The intuitionistic fuzzy sets $0_{\sim} = \langle x, 0, 1 \rangle$ and $1_{\sim} = \langle x, 1, 0 \rangle$ are respectively the empty set and the whole set of X .

Definition 2.3 [2]: An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

- (i) $0_{\sim}, 1_{\sim} \in \tau$,
- (ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
- (iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS in short) in X . The complement A^c of an IFOS A in an IFTS (X, τ) is

called an *intuitionistic fuzzy closed set* (IFCS in short) in X .

Definition 2.4 [3]: An IFS A in an IFTS (X, τ) is said to be an *intuitionistic fuzzy β generalized closed set* (IF β GCS for short) if $\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IF β OS in (X, τ) .

The complement A^c of an IF β GCS A in an IFTS (X, τ) is called an *intuitionistic fuzzy β generalized open set* (IF β GOS for short) [4] in X .

Definition 2.5 [5]: If every IF β GCS in (X, τ) is an IFBCS in (X, τ) , then the space can be called as an *intuitionistic fuzzy β generalized $T_{1/2}$* (IF $\beta_g T_{1/2}$ in short) *space*.

Definition 2.6 [5]: An IFTS (X, τ) is an *intuitionistic fuzzy β generalized a $T_{1/2}$* (IF $\beta_{ga} T_{1/2}$ in short) *space* if every IF β GCS is an IFCS in X .

Definition 2.7 [6]: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an *intuitionistic fuzzy β generalized continuous* (IF β G continuous for short) **mapping** if $f^{-1}(V)$ is an IF β GCS in (X, τ) for every IFCS V of (Y, σ) .

Definition 2.8 [7]: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an *intuitionistic fuzzy β generalized irresolute* (IF β G irresolute) **mapping** if $f^{-1}(V)$ is an IF β GCS in (X, τ) for every IF β GCS V of (Y, σ) .

Definition 2.9 [8]: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an *intuitionistic fuzzy β generalized closed mapping* (IF β GCM for short) if $f(V)$ is an IF β GCS in Y for every IFCS V of X .

Definition 2.10 [8]: A mapping $f: X \rightarrow Y$ is said to be an *intuitionistic fuzzy β generalized open mapping* (IF β GOM for short) if $f(A)$ is an IF β GOS in Y for each IFOS A in X .

Definition 2.11 [8] : A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be an *intuitionistic fuzzy $M - \beta$ generalized closed mapping* (IFM β GCM for short) if $f(A)$ is an IF β GCS in Y for every IF β GCS A in X .

Definition 2.12 [2]: Let X and Y be two nonempty IFSs and $f: X \rightarrow Y$ be a function. If $A = \{ \langle x, (\mu_A(x), \nu_A(x)) / x \in X \rangle \}$ is an IFS in X , then the *image* of A under f , denoted by $f(A)$, is the IFS in Y defined by

$$f(A) = \{ \langle y, f(\mu_A)(y), f(\nu_A)(y) / y \in Y \rangle \}$$

where $f(\nu_A) = 1 - f(1 - \nu_A)$.

Definition 2.13 [2]: Let X and Y be two nonempty sets and $f: X \rightarrow Y$ be a function. If $B = \{ \langle y, (\mu_B(y), \nu_B(y)) / y \in Y \rangle \}$ is an IFS in Y ,

then the *preimage* of B under f is denoted and defined by

$$f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) / x \in X \rangle \}$$

where $f^{-1}(\mu_B)(x) = \mu_B(f(x))$ for every $x \in X$.

III. Intuitionistic Fuzzy β Generalized Homeomorphisms

In this section we introduce intuitionistic fuzzy β generalized homeomorphisms and investigate some properties.

Definition 3.1: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping. Then f is said to be an *intuitionistic fuzzy β generalized homeomorphism* (IF β GHM for short) if f is both an IF β G continuous mapping and an IF β GOM.

Example 3.2: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v) \rangle$. Then $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$.

Then, $IF\beta C(X) = \{0_{\sim}, 1_{\sim}, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a + \nu_a \leq 1 \text{ and } 0 \leq \mu_b + \nu_b \leq 1\}$,

$IF\beta O(X) = \{0_{\sim}, 1_{\sim}, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a + \nu_a \leq 1 \text{ and } 0 \leq \mu_b + \nu_b \leq 1\}$,

$IF\beta C(Y) = \{0_{\sim}, 1_{\sim}, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1] / \text{either } \mu_u < 0.6 \text{ or } \mu_v < 0.7 \text{ or both, } 0 \leq \mu_u + \nu_u \leq 1 \text{ and } 0 \leq \mu_v + \nu_v \leq 1\}$,

$IF\beta O(Y) = \{0_{\sim}, 1_{\sim}, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1] / \text{either } \mu_u > 0.4 \text{ or } \mu_v > 0.3 \text{ or both, } 0 \leq \mu_u + \nu_u \leq 1 \text{ and } 0 \leq \mu_v + \nu_v \leq 1\}$.

Then f is both an IF β G continuous mapping and an IF β GOM. Therefore f is an IF β GHM.

Theorem 3.3: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF β GHM, then f is an IF homeomorphism if X and Y are IF $\beta_{ga} T_{1/2}$ space.

Proof: Let B be an IFCS in Y . Then $f^{-1}(B)$ is an IF β GCS in X , by hypothesis. Since X is an IF $\beta_{ga} T_{1/2}$ space, $f^{-1}(B)$ is an IFCS in X . Hence f is an IF continuous mapping. By hypothesis $f^{-1}: (Y, \sigma) \rightarrow (X, \tau)$ is an IF β G continuous mapping. Let A be an IFCS in X . Then $(f^{-1})^{-1}(A) = f(A)$ is an IF β GCS in Y , by hypothesis. Since Y is an IF $\beta_{ga} T_{1/2}$ space, $f(A)$ is

an IFCS in Y . Hence f^{-1} is an IF continuous mapping. Therefore the mapping f is an IF homeomorphism.

Theorem 3.4: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping. If f is an IF β G continuous mapping, then the following are equivalent:

- (i) f is an IF β GOM
- (ii) f is an IF β GHM
- (iii) f is an IF β GCM

Proof: Straight forward.

Remark 3.5: The composition of two IF β GHMs need not be an IF β GHM in general.

Example 3.6: Let $X = \{a, b\}$, $Y = \{c, d\}$ and $Z = \{e, f\}$. Let $G_1 = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b) \rangle$, $G_2 = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b) \rangle$, $G_3 = \langle y, (0.6_c, 0.4_d), (0.4_c, 0.3_d) \rangle$, $G_4 = \langle z, (0.4_e, 0.2_f), (0.6_e, 0.8_f) \rangle$. Then $\tau = \{0_{\sim}, G_1, G_2, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_3, 1_{\sim}\}$ and $\eta = \{0_{\sim}, G_4, 1_{\sim}\}$ are IFTs on X , Y and Z respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = c$ and $f(b) = d$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ by $g(c) = e$ and $g(d) = f$.

Then, $IF\beta C(X) = \{0_{\sim}, 1_{\sim}, \mu_a \in [0,1], \mu_b \in [0,1], v_a \in [0,1], v_b \in [0,1] / \text{either } \mu_a < 0.6 \text{ or } \mu_b < 0.7 \text{ or both, } 0 \leq \mu_a + v_a \leq 1 \text{ and } 0 \leq \mu_b + v_b \leq 1\}$,

$IF\beta O(X) = \{0_{\sim}, 1_{\sim}, \mu_a \in [0,1], \mu_b \in [0,1], v_a \in [0,1], v_b \in [0,1] / \text{either } \mu_a > 0.4 \text{ or } \mu_b > 0.3 \text{ or both, } 0 \leq \mu_a + v_a \leq 1 \text{ and } 0 \leq \mu_b + v_b \leq 1\}$,

$IF\beta C(Y) = \{0_{\sim}, 1_{\sim}, \mu_c \in [0, 1], \mu_d \in [0,1], v_c \in [0,1], v_d \in [0,1] / \text{either } \mu_c < 0.6 \text{ or } \mu_d < 0.4 \text{ or both, } 0 \leq \mu_c + v_c \leq 1 \text{ and } 0 \leq \mu_d + v_d \leq 1\}$,

$IF\beta O(Y) = \{0_{\sim}, 1_{\sim}, \mu_c \in [0,1], \mu_d \in [0,1], v_c \in [0,1], v_d \in [0,1] / \text{either } \mu_c > 0.4 \text{ or } \mu_d > 0.3 \text{ or both, } 0 \leq \mu_c + v_c \leq 1 \text{ and } 0 \leq \mu_d + v_d \leq 1\}$,

$IF\beta C(Z) = \{0_{\sim}, 1_{\sim}, \mu_e \in [0, 1], \mu_f \in [0,1], v_e \in [0,1], v_f \in [0,1] / 0 \leq \mu_e + v_e \leq 1 \text{ and } 0 \leq \mu_f + v_f \leq 1\}$,

$IF\beta O(Z) = \{0_{\sim}, 1_{\sim}, \mu_e \in [0,1], \mu_f \in [0,1], v_e \in [0,1], v_f \in [0,1] / \text{either } \mu_e > 0.6 \text{ or } \mu_f > 0.8 \text{ or both, } 0 \leq \mu_e + v_e \leq 1 \text{ and } 0 \leq \mu_f + v_f \leq 1\}$.

Then f and g are IF β GHMs but $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is not an IF β GHM, since $g \circ f$ is not an IF β G continuous mapping, since $G_4^c = \langle z, (0.6_e, 0.8_f), (0.4_e, 0.2_f) \rangle$ is an IFCS in Z but $(g \circ f)^{-1}(G_4^c) = \langle x, (0.6_a, 0.8_b), (0.4_a, 0.2_b) \rangle$ is not an IF β GCS in X ,

since $(g \circ f)^{-1}(G_4^c) \subseteq G_2$ but $\beta cl((g \circ f)^{-1}(G_4^c)) = 1_{\sim} \notin G_2$.

Definition 3.7: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping. Then f is said to be an *intuitionistic fuzzy $M - \beta$ generalized homeomorphism* (IFM β GHM for short) if f is both an IF β G irresolute mapping and an IFM β GOM.

The family of all IF β GHMs in X is denoted by IFM β GHM(X).

Theorem 3.8: Every IFM β GHM is an IF β GHM but not conversely.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFM β GHM. Let $A \subseteq Y$ be an IFCS. Then A is an IF β GCS in Y . By hypothesis, $f^{-1}(A)$ is an IF β GCS in X . Hence f is an IF β G continuous mapping. Let $B \subseteq X$ be an IFOS. Then B is an IF β GOS in X . By hypothesis, $f(B)$ is an IF β GOS in Y . Hence f is an IF β GOM. Thus f is an IF β GHM.

Example 3.9: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$, $G_2 = \langle y, (0.5_u, 0.3_v), (0.5_u, 0.7_v) \rangle$. Then $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$.

Then, $IF\beta C(X) = \{0_{\sim}, 1_{\sim}, \mu_a \in [0,1], \mu_b \in [0,1], v_a \in [0,1], v_b \in [0,1] / \text{either } \mu_a < 0.5 \text{ or } \mu_b < 0.6 \text{ or both, } 0 \leq \mu_a + v_a \leq 1 \text{ and } 0 \leq \mu_b + v_b \leq 1\}$,

$IF\beta O(X) = \{0_{\sim}, 1_{\sim}, \mu_a \in [0,1], \mu_b \in [0,1], v_a \in [0,1], v_b \in [0,1] / \text{either } \mu_a > 0.5 \text{ or } \mu_b > 0.4 \text{ or both, } 0 \leq \mu_a + v_a \leq 1 \text{ and } 0 \leq \mu_b + v_b \leq 1\}$,

$IF\beta C(Y) = \{0_{\sim}, 1_{\sim}, \mu_u \in [0, 1], \mu_v \in [0,1], v_u \in [0,1], v_v \in [0,1] / 0 \leq \mu_u + v_u \leq 1 \text{ and } 0 \leq \mu_v + v_v \leq 1\}$,

$IF\beta O(Y) = \{0_{\sim}, 1_{\sim}, \mu_u \in [0, 1], \mu_v \in [0,1], v_u \in [0,1], v_v \in [0,1] / 0 \leq \mu_u + v_u \leq 1 \text{ and } 0 \leq \mu_v + v_v \leq 1\}$.

Then f is an IF β GHM but not an IFM β GHM, since $A = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v) \rangle$ is an IF β GCS in Y but $f^{-1}(A)$ is not an IF β GCS in X , since $f^{-1}(A) \subseteq G_1$ but $\beta cl(f^{-1}(A)) = 1_{\sim} \notin G_1$.

Theorem 3.10: The composition of two IFM β GHMs is an IFM β GHM.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \delta)$ be any two IFM β GHMs. Let $A \subseteq Z$ be an IF β GCS in Y . By hypothesis, $g^{-1}(A)$ is an IF β GCS in Y . Again by hypothesis, $f^{-1}(g^{-1}(A))$ is an IF β GCS in X . Therefore $g \circ f$ is an IF β G irresolute mapping. Now let $B \subseteq X$ be an IF β GOS. Then by hypothesis,

$f(B)$ is an IF β GOS in Y and also $g(f(B))$ is an IF β GOS in Z . This implies $g \circ f$ is an IFM β GOM. Hence $g \circ f$ is an IFM β GHM.

Theorem 3.11: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping. If f is an IF β G irresolute mapping, then the following are equivalent:

- (i) f is an IFM β GOM
- (ii) f is an IFM β GHM
- (iii) f is an IFM β GCM

Proof: Straight forward.

Theorem 3.12: The set of all IFM β GHMs in an IFTS (X, τ) is a group under the composition maps.

Proof: Define a binary operation $*$: IFM β GHM(X) \times IFM β GHM(X) \rightarrow IFM β GHM(X) by $f * g = g \circ f$ for every $f, g \in$ IFM β GHM(X) and \circ is the usual operation of composition of maps. Since $g \in$ IFM β GHM(X) and $f \in$ IFM β GHM(X), by Theorem 3.10, $g \circ f \in$ IFM β GHM(X). We know that the composition of maps is associative. The identity map $I: (X, \tau) \rightarrow (X, \tau)$ belonging to IFM β GHM(X) is the identity element. If $f \in$ IFM β GHM(X), then $f^{-1} \in$ IFM β GHM(X). Therefore $f \circ f^{-1} = f^{-1} \circ f = I$ and so the inverse exists for each element of IFM β GHM(X). Hence (IFM β GHM(X), \circ) is a group under the composition of maps.

Theorem 3.13: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFM β GHM, then $\beta gcl(f^{-1}(B)) \subseteq f^{-1}(\beta cl(B))$ for every IFS B in (Y, σ) .

Proof: Let $B \subseteq Y$. Then $\beta cl(B)$ is an IF β GCS in Y . Since f is an IF β G irresolute mapping, $f^{-1}(\beta cl(B))$ is an IF β GCS in X . This implies $\beta gcl(f^{-1}(\beta cl(B))) = f^{-1}(\beta cl(B))$. Now $\beta gcl(f^{-1}(B)) \subseteq \beta gcl(f^{-1}(\beta cl(B))) = f^{-1}(\beta cl(B))$.

Theorem 3.14: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFM β GHM, where (X, τ) and (Y, σ) are IF $\beta_g T_{1/2}$ spaces, then $\beta cl(f^{-1}(B)) = f^{-1}(\beta cl(B))$ for every IFS B in (Y, σ) .

Proof: Since f is an IFM β GHM, f is an IF β G irresolute mapping. Since $\beta cl(f(B))$ is an IF β GCS in Y , $f^{-1}(\beta cl(f(B)))$ is an IF β GCS in X . Since X is an IF $\beta_g T_{1/2}$ space, $f^{-1}(\beta cl(f(B)))$ is an IF β CS in X . Now, $f^{-1}(B) \subseteq f^{-1}(\beta cl(B)) \subseteq \beta cl(f^{-1}(\beta cl(B)))$. We have $\beta cl(f^{-1}(B)) \subseteq \beta cl(f^{-1}(\beta cl(B))) = f^{-1}(\beta cl(B))$. This implies $\beta cl(f^{-1}(B)) \subseteq f^{-1}(\beta cl(B))$ ----- (1). Again since f is an IFM β GHM, f^{-1} is IF β G irresolute mapping. Since $\beta cl(f^{-1}(B))$ is an IF β GCS in X , $(f^{-1})^{-1}(\beta cl(f^{-1}(B))) = f(\beta cl(f^{-1}(B)))$, is an IF β GCS in Y . Now $B \subseteq (f^{-1})^{-1}(f^{-1}(B)) \subseteq (f^{-1})^{-1}(\beta cl(f^{-1}(B)))$. Therefore $\beta cl(B) \subseteq \beta cl(f(\beta cl(f^{-1}(B)))) = f(\beta cl(f^{-1}(B)))$, since Y is an

IF $\beta_g T_{1/2}$ space. Hence $f^{-1}(\beta cl(B)) \subseteq f^{-1}(f(\beta cl(f^{-1}(B)))) \subseteq \beta cl(f^{-1}(B))$. That is $f^{-1}(\beta cl(B)) \subseteq \beta cl(f^{-1}(B))$ ----- (2). Thus from (1) and (2) we get $\beta cl(f^{-1}(B)) = f^{-1}(\beta cl(B))$ and hence the proof.

Theorem 3.15: Let $f: X \rightarrow Y$ be an IFM β GHM. Then f induces an isomorphism from the group IFM β GHM(X) onto the group IFM β GHM(Y).

Proof: Using f , we define a map $\phi_f: h(X) \rightarrow h(Y)$ by $\phi_f(h) = f \circ h \circ f^{-1}$ for every $h \in$ IFM β GHM(X). Then ϕ_f is a bijection. Also for all $h_1, h_2 \in$ IFM β GHM(X), $\phi_f(h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \phi_f(h_1) \circ \phi_f(h_2)$. This implies ϕ_f is a homeomorphism and so ϕ_f is an isomorphism induced by f .

Corollary 3.16: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFM β GHM, where (X, τ) and (Y, σ) are IF $\beta_g T_{1/2}$ spaces, then $\beta cl(f(B)) = f(\beta cl(B))$ for every IFS B in X .

Proof: Since f is an IFM β GHM, f^{-1} is also an IFM β GHM. Therefore by Theorem 3.14 $\beta cl((f^{-1})^{-1}(B)) = (f^{-1})^{-1}(\beta cl(B))$ for every $B \subseteq X$. That is $\beta cl(f(B)) = f(\beta cl(B))$ for every IFS B in X .

Corollary 3.17: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFM β GHM, where (X, τ) and (Y, σ) are IF $\beta_g T_{1/2}$ spaces, then $\beta int(f(B)) = f(\beta int(B))$ for every IFS B in X .

Proof: For any IFS $B \subseteq X$, $\beta int(B) = \beta cl(B^c)^c$. By Corollary 3.16, $f(\beta int(B)) = f(\beta cl(B^c)^c) = (f(\beta cl(B^c)))^c = \beta cl(f(B^c))^c = \beta int(f(B^c)^c) = \beta int(f(B))$.

Corollary 3.18: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFM β GHM, where (X, τ) and (Y, σ) are IF $\beta_g T_{1/2}$ spaces, then $\beta int(f^{-1}(B)) = f^{-1}(\beta int(B))$ for every IFS B in Y .

Proof: Since f is an IFM β GHM, f^{-1} is also an IFM β GHM, the proof directly follows from corollary 3.17.

IV. REFERENCES

- [1] Atanassov, K., Intuitionistic Fuzzy Sets, Fuzzy sets and systems, 1986, 87-96.
- [2] Coker, D., An introduction to intuitionistic fuzzy topological spaces, Fuzzy sets and systems, 1997, 81 - 89.
- [3] Saranya, M., and Jayanthi, D., On Intuitionistic fuzzy β Generalized Closed Sets, International Journal of Computational Engineering Research, 6(3), 2016, 37 - 42.
- [4] Saranya, M., and Jayanthi, D., On Intuitionistic Fuzzy β Generalized Open Sets, International Journal of Engineering Sciences & Management Research, 3(5), 2016, 48 - 53.
- [5] Saranya, M., and Jayanthi, D., On Intuitionistic Fuzzy β Generalized $T_{1/2}$ spaces, Imperial Journal of Interdisciplinary Research, 2(6), 2016, 447 - 451.

- [6] **Saranya, M., and Jayanthi, D.,** On Intuitionistic Fuzzy β Generalized continuous mappings, International Journal of Advance Foundation and Research in Science & Engineering, 2(10), 2016, 42 - 51.
- [7] **Saranya, M., and Jayanthi, D.,** Completely β Generalized Continuous Mappings in Intuitionistic Fuzzy Topological Spaces, (accepted)
- [8] **Saranya, M., and Jayanthi, D.,** Intuitionistic Fuzzy β Generalized Closed Mappings (submitted)