On $p^{\#}$g closed sets in Topological spaces

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Abstract

In this paper we introduce the new concept of $p^{\#}g$ closed sets in topological spaces and a basic properties of $p^{\#}g$-closed set were obtain
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Mathematics Subject Classification: 54A05

Keywords: $p^{\#}g$-closed sets and $p^{\#}$g-open sets.

1 Introduction

The study of generalized closed (briefly g-closed) sets in a topological spaces was initiated by N.Levine in 1970[7] and in 1982 A.S.Mashhour [11] introduced the concept of preopen(briefly p-open) sets in topological spaces. Later in 1998 H.Maki, T.Nori [10] gave a new type of generalized closed sets in topological spaces called generalized pre-closed(briefly gp-closed) sets.

The aim of this paper is to introduce the new type of closed set called $p^{\#}g$ closed set in topological spaces and to continue the study of $p^{\#}g$-closed sets thereby contributing new innovation and concepts, in the field of topology through analytical as well as research works. The notion of $p^{\#}g$-closed sets and its different characterizations are given in this paper.

2 Preliminaries

A subset A of a topological space X is said to be **open** if $A \in \tau$. A subset A of a topological space X is said to be **closed** if the set $X - A$ is open. The **interior** of a subset A of a topological space X is defined as the union

of all open sets contained in A. It is denoted by $int(A)$. The **closure** of a subset A of a topological space X is defined as the intersection of all closed sets containing A. It is denoted by $cl(A)$.
Throughout this paper (X, τ) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. Let $A \subseteq X$,the closure of A and interior of A will be denoted by $cl(A)$ and int(A) respectively.

Definitions 2.1.

1. A subset A of a space (X, τ) is said to be **pre-open** [11] if $A \subseteq int\,(cl\,(A))$ and **pre-closed** if $cl\,(int\,(A)) \subseteq A$.

2. A subset A of a space (X, τ) is said to be **semi open** [6] if $A \subseteq cl\,(int\,(A))$ and **semi closed** if $int\,(cl\,(A)) \subseteq A$.

3. A subset A of a space (X, τ) is said to be **regular-open** [14] if $A = int\,(cl\,(A))$ and **regular-closed** if $A = cl\,(int\,(A))$.

4. A subset A of a space (X, τ) is said to be **semi pre-open** [2] if $A \subseteq cl\,(int\,(cl\,(A)))$ and **semi pre-closed** if $int\,(cl\,(int\,(A))) \subseteq A$.

5. A subset A of a space (X, τ) is said to be α-**open**[13]if $A \subseteq int\,(cl\,(int\,(A)))$ and α-**closed** if $cl\,(int\,(cl\,(A))) \subseteq A$.

The complement of a pre-open (resp.semi-open, α-open) set is called **pre-closed (resp.semi-closed, α-closed)**. The intersection of all pre-closed (resp.semi-closed, α-closed) sets containing A is called the **pre-closure (resp.semi-closure, α-closure)** of A and is denoted by $pcl(A)$(resp. $scl(A)$, α-$cl(A)$).The union of all pre-open (resp.semi-open, α-open) sets contained in A is called the **pre-interior(resp.semi-interior, α-interior)** of A and is denoted by $pint(A)$(resp. $sint(A)$, α-$int(A)$). The family of all semi-open (resp.pre-open, α-open) sets is denoted by $PO(X)$(resp. $SO(X)$, $\alpha - O(X)$). The family of all pre-closed (resp.semi-closed, α-closed) sets is denoted by $PCl(X)$ (resp. $SCl(X)$, α-$Cl(X)$).

Definitions 2.2.

1. A subset A of a space (X, τ) is called **generalized-closed set** [7] (briefly g-closed) if $cl\,(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

2. A subset A of a space (X, τ) is called α **generalized-closed set** [9] (briefly αg-closed) if $\alpha\,(cl\,(A)) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

3. A subset A of a space (X, τ) is called **generalized α-closed set** [8] (briefly $g\alpha$-closed) if $\alpha(cl(A)) \subseteq U$, whenever $A \subseteq U$ and U is α-open in (X, τ).

4. A subset A of a space (X, τ) is called **generalized pre-closed set** [10] (briefly gp-closed) if $pcl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

5. A subset A of a space (X, τ) is called **generalized semi-pre closed set** [3] (briefly gsp-closed) if $spcl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

6. A subset A of a space (X, τ) is called **weekly generalized-closed set** [12] (briefly wg-closed) if $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

7. A subset A of a space (X, τ) is called **semi weekly generalized-closed set** [12] (briefly swg-closed) if $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in (X, τ).

8. A subset A of a space (X, τ) is called **πgeneralized-closed set** [4] (briefly πg-closed) if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is π-open in (X, τ).

9. A subset A of a space (X, τ) is called **πgeneralizedα-closed set** [5] (briefly $\pi g\alpha$-closed) if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is π-open in (X, τ).

10. A subset A of a space (X, τ) is called **weekly-closed set** [15] (briefly w-closed) if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in (X, τ).

11. A subset A of a space (X, τ) is called **πgeneralized semi-closed set** [1] (briefly πgs-closed) if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is π-open in (X, τ).

12. A subset A of a space (X, τ) is called **$\hat{g}$-closed set[16]** if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in (X, τ).

3 $p^{\#}g$-Closed sets in Topological Spaces

In this section the notion of a new class of sets called $p^{\#}g$-closed sets in topological spaces is introduced and their properties were studied.

Definition 3.1 *A subset A of space (X, τ) is called $p^{\#}g$-**closed** if $int(pcl(A)) \subseteq U$, whenever $A \subseteq U$ and U is p-open in X.*

The family of all $p^{\#}g$-closed subsets of the space X is denoted by $P^{\#}GC(X)$.

Definition 3.2 *The intersection of all $p^{\#}g$-closed sets containing a set A is called $p^{\#}g$-**closure** of A and is denoted by $p^{\#}g$-$cl(A)$.*

A set A is $p^{\#}g$-closed set if and only if $p\#g\ Cl\,(A) = A$.

Definition 3.3 *A subset A in X is called $p^{\#}g$-**open** in X if A^c is $p^{\#}g$-closed in X.*

The family of a $p^{\#}g$-open sets is denoted by $P^{\#}GO(X)$.

Definition 3.4 *The union of all $p^{\#}g$-open sets containing a set A is called $p^{\#}g$-**interior** of A and is denoted by $p^{\#}g$-$Int(A)$.*

A set A is $p^{\#}g$-open set if and only if $p^{\#}g\ Int\,(A) = A$.

Theorem 3.5 *Every closed set is a $p^{\#}g$-closed set.*

Proof: *Let A be a closed set in X. Such that $A \subseteq U$, U is p-open. Since A is closed, $cl\,(A) = A$.For every subset A of X, $int\,(pcl\,(A)) \subseteq cl\,(A) = A \subseteq U$ and so we have $int\,(pcl\,(A)) \subseteq U$. Hence A is $p^{\#}g$-closed.*

Remark 3.6 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.7 *Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $A = \{a\}$ is $p^{\#}g$-closed but not a closed set of (X, τ).*

Theorem 3.8 *Every s-closed set is a $p^{\#}g$-closed set.*

Proof: *Let A be a s-closed set in X. Such that $A \subseteq U$, U is p-open. Since A is s-closed, $scl\,(A) = A$. For every subset A of X, $int\,(pcl\,(A)) \subseteq scl\,(A) = A \subseteq U$ and so we have $int\,(pcl\,(A)) \subseteq U$. Hence A is $p^{\#}g$-closed.*

Remark 3.9 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.10 *Let $X = \{a, b, c, d, e\}$ with topology $\tau = \{X, \phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}\}$. Then $A = \{a, b, c, e\}$ is $p^{\#}g$-closed but not a s-closed set of (X, τ).*

Theorem 3.11 *Every α closed set is a $p^{\#}g$-closed set.*

Proof: *Let A be a α-closed set in X. Such that $A \subseteq U$, U is p-open. Since A is α-closed, $\alpha cl\,(A) \subseteq A$. For every subset A of X, $int\,(pcl\,(A)) \subseteq \alpha cl\,(A) \subseteq A \subseteq U$ and so we have $int\,(pcl\,(A)) \subseteq U$. Hence A is $p^{\#}g$-closed.*

Remark 3.12 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.13 *Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $A = \{a, c\}$ is $p\#g$-closed but not α closed set of (X, τ).*

Theorem 3.14 *Every r-closed set is a $p^{\#}g$-closed set.*

Proof: *Let A be a r-closed set in X. Such that $A \subseteq U$, U is p-open. Since A is r-closed, $rcl(A) \subseteq A$. For every subset A of X, $int(pcl(A)) \subseteq rcl(A) \subseteq A \subseteq U$ and so we have $int(pcl(A)) \subseteq U$. Hence A is $p^{\#}g$-closed.*

Remark 3.15 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.16 *Let $X = \{a, b, c, d, e\}$ with topology $\tau = \{X, \phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}\}$. Then $A = \{b, c\}$ is $p^{\#}g$-closed but not a r-closed set of (X, τ).*

Theorem 3.17 *Every $g\alpha$ closed set is a $p^{\#}g$-closed set.*

Proof: *Let A be a $g\alpha$-closed set in X. Such that $A \subseteq U$, U is p-open. Since A is $g\alpha$-closed, $\alpha cl(A) \subseteq A$. For every subset A of X, $int(pcl(A)) \subseteq \alpha cl(A) \subseteq A \subseteq U$ and so we have $int(pcl(A)) \subseteq U$. Hence A is $p^{\#}g$-closed.*

Remark 3.18 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.19 *Let $X = \{a, b, c, d, e\}$ with topology $\tau = \{X, \phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}\}$. Then $A = \{a, d\}$ is $p^{\#}g$-closed but not $g\alpha$ closed set of (X, τ).*

Theorem 3.20 *Every $p^{\#}g$ closed set is a gsp-closed set.*

Proof: *Let A be a $p^{\#}g$-closed set in X. Such that $A \subseteq U$, U is open. Since A is $p^{\#}g$-closed, $int(pcl(A)) \subseteq A$. For every subset A of X, $spcl(A) \subseteq int(pcl(A)) \subseteq A \subseteq U$ and so we have $spcl(A) \subseteq U$. Hence A is gsp-closed.*

Remark 3.21 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.22 *Let $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then $A = \{a, b\}$ is gsp-closed but not $p^{\#}g$ closed set of (X, τ).*

Theorem 3.23 *Every αg closed set is a $p^{\#}g$-closed set.*

Proof: *Let A be a αg-closed set in X. Such that $A \subseteq U$, U is p-open. Since A is αg-closed,$\alpha cl(A) \subseteq A$. For every subset A of X,$int(pcl(A)) \subseteq \alpha cl(A) \subseteq U$ and we have $int(pcl(A)) \subseteq U$. Hence A is $p^{\#}g$-closed.*

Remark 3.24 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.25 *Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $A = \{b, c\}$ is $p^{\#}g$-closed but not αg closed set of (X, τ).*

Theorem 3.26 *Every gp-closed set is a $p^{\#}g$-closed set.*

Proof: *Let A be a gp-closed set in X. Such that $A \subseteq U$, U is p-open. Since A is gp-closed, $pcl(A) \subseteq A$. For every subset A of X, $int(pcl(A)) \subseteq pcl(A) \subseteq A \subseteq U$ and so we have $int(pcl(A)) \subseteq U$. Hence A is $p^{\#}g$-closed.*

Remark 3.27 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.28 *Let $X = \{a, b, c, d, e\}$ with topology $\tau = \{X, \phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}\}$. Then $A = \{a, b, d\}$ is $p^{\#}g$-closed but not a gp-closed set of (X, τ).*

Theorem 3.29 *Every wg-closed set is a $p^{\#}g$-closed set.*

Proof: *Let A be a wg-closed set in X. Such that $A \subseteq U$, U is p-open. Since A is wg-closed, $cl(int(A)) \subseteq A$. For every subset A of X, $int(pcl(A)) \subseteq cl(int(A)) \subseteq A \subseteq U$ and so we have $int(pcl(A)) \subseteq U$. Hence A is $p^{\#}g$-closed.*

Remark 3.30 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.31 *Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $A = \{b, d\}$ is $p^{\#}g$-closed but not a wg-closed set of (X, τ).*

Theorem 3.32 *Every $\hat{g}$-closed set is a $p^{\#}g$-closed set.*

Proof follows from the definition, since every p-open set is semi-open.

Example 3.33 *In example (3.7), the set $\{b, c\}$ is p#g-closed but not a $\hat{g}$-closed set of (X, τ).*

Theorem 3.34 *Every swg-closed set is a $p^{\#}g$-closed set.*

Proof follows from the definition, since every p-open set is semi-open.

Example 3.35 *In example (3.7), the set $\{a, d\}$ is $p^{\#}g$-closed but not a swg-closed set of (X, τ).*

Theorem 3.36 *Every πg-closed set is a $p^{\#}g$-closed set.*

Proof follows from the definition, since every p-open set is π-open.

Example 3.37 *In example (3.22), the set $\{b\}$ is $p^{\#}g$-closed but not a πg-closed set of (X, τ).*

Theorem 3.38 *Every $\pi g\alpha$-closed set is a $p^{\#}g$-closed set.*

Proof follows from the definition, since every p-open set is π-open.

Example 3.39 *In example (3.22), the set $\{a\}$ is $p^{\#}g$-closed but not a $\pi g\alpha$-closed set of (X, τ).*

Theorem 3.40 *Every g-closed set is a $p^{\#}g$-closed set.*

Proof follows from the definition, since every p-open set is open.

Example 3.41 *In example (3.10), the set $\{a, c, d\}$ is $p^{\#}g$-closed but not a g-closed set of (X, τ).*

Theorem 3.42 *Every p-closed set is a $p^{\#}g$-closed set.*

Proof follows from the definition, since every p-open set is open.

Example 3.43 *In example (3.10), the set $\{b, c, d, e\}$ is $p^{\#}g$-closed but not a p-closed set of (X, τ).*

Theorem 3.44 *Every w-closed set is a $p^{\#}g$-closed set.*

Proof follows from the definition, since every p-open set is semi-open.

Example 3.45 *In example (3.7), the set $\{b, d\}$ is $p^{\#}g$-closed but not a w-closed set of (X, τ).*
So the class of $p^{\#}g$-closed sets properly contain the class of $\hat{g}$-closed set, swg-closed set, $\pi g\alpha$-closed set, πg-closed set, g-closed set, w-closed sets.

Theorem 3.46 *Every $p^{\#}g$-closed set is a πgs-closed set.*

Proof follows from the definition, since every π-open set is p-open.

Example 3.47 *In example (3.7), the set $\{a, b, c\}$ is πgs-closed but not a $p^{\#}g$-closed set of (X, τ).*
so the class of ϕgs-closed sets properly contain the class of $p^{\#}g$-closed sets.

Remark 3.48 *Figure 3.1 gives the implication relations of $p^{\#}g$-closed sets based on the above results.*

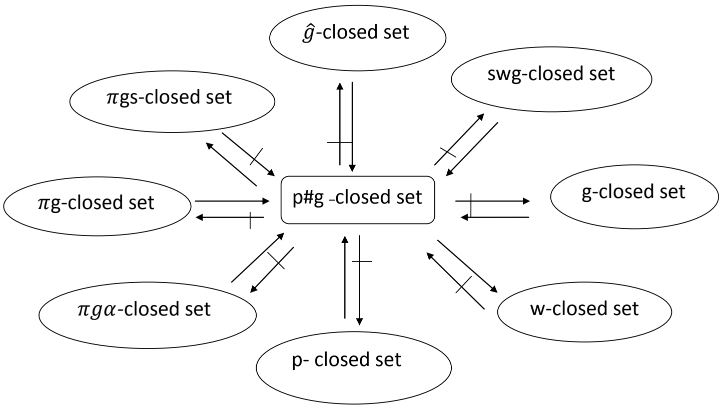

Figure 3.1 Implication of p#g- closed set

Where A $\longrightarrow$ B represents A implies B

A $\not\longrightarrow$ B represents A does not implies B

A $\not\longleftarrow$ B represents B does not implies A

Theorem 3.49 *For each $x \in \{X\}$,either $\{x\}$ p-closed or $\{x\}^c$ is $p^{\#}g$-closed in X.*

Proof: *Suppose that $\{x\}$ is not p-closed, then the only pre-open set containing $\{x\}^c$ in X. Thus pre-closure of $\{x\}^c$ is contained in X. Hence $\{x\}^c$ is $p^{\#}g$-closed in X.*

Theorem 3.50 *The intersection of two $p^{\#}g$-closed subsets of X is also $p^{\#}g$-closed subset of X.*

Proof: *Assume that P and Q are $p^{\#}g$-closed set in X. Let $P \bigcap Q \subseteq U$ and U be p-open in X. Since $P \subset U$ and $Q \subset U$, U is p-open.Then $int\,(pcl\,(P)) \subseteq U$ and $int\,(pcl\,(Q)) \subseteq U$ and we have $int\,(pcl\,(P \bigcap Q)) \subseteq int\,(pcl\,(p)) \bigcap int\,(pcl\,(Q))$ $\subseteq U$. Since U is p-open. Hence $P \bigcap Q$ is $p^{\#}g$-closed set in X.*

Remark 3.51 *The union of two $p^{\#}g$-closed sets in X is generally not $p^{\#}g$-closed set in X.*

Example 3.52 *Let $X = \{a, b, c, d, e\}$ with topology $\tau = \{X, \phi, \{a, b\}, \{c, d\},$ $\{a, b, c, d\}\}$. If $P = \{a, b\}$ and $Q = \{c, d\}$, then P and Q are $p\#g$-closed sets in X, but $P \bigcap Q = \{a, b, c, d\}$ is not a $p^{\#}g$-closed set of X.*

Theorem 3.53 *Let A be a closed subset of $p^{\#}g$-closed set (X, τ) iff $int(cl(A)) - A$ does not contain any nonempty p-closed set in X.*

Proof: *Necessary:*

Let that A is $p^{\#}g$-closed set in X. Suppose F be a p-closed subset of X. Such that $F \subseteq int(pcl(A)) - A$ Now, $F \subseteq int(pcl(A)) \bigcap A^c$. Since A is $p^{\#}g$-closed set. Thus $\Rightarrow F \subseteq int(pcl(A))$ and $F \subseteq A^c \Rightarrow A \subseteq F^c$. Since F^c is preopen and A is $p^{\#}g$-closed set.$int(pcl(A)) \subseteq F^c \Rightarrow F \subseteq (int(pcl(A)))^c$ Thus implies $F \supseteq (int(pcl(A)) \bigcap (int(pcl(A)))^c) = \phi$. This shows that, $F = \phi$. Hence $int(pcl(A)) - A = \phi$ does not contains any nonempty p-closed set in X.

Sufficient:

Let $A \subseteq U$ and U is preopen then $int(pcl(A)) \subseteq U$. Suppose that $int(pcl(A))$ does not contained in U.Then $int(pcl(A)) \bigcap U^c$ is a non-empty preclosed set of $int(pcl(A)) - A$. Which is contradiction Therefore $int(pcl(A)) \subseteq U$ Hence A is $p^{\#}g-$closed.

Theorem 3.54 *If A is $p^{\#}g$-closed set and B is any set such that $A \subset B \subset int(pcl(A))$ then B is $p^{\#}g$-closed set.*

Proof: *Let $B \subset U$ and $U$is pre-open. Given $A \subset B$,then $A \subset U$.Since A is p#g-closed set, $A \subset U$ implies $int(pcl(A)) \subseteq U$. By assumption it follows that $\Rightarrow int(pcl(B)) \subseteq int(pcl(A)) \subseteq U \Rightarrow int(pcl(B)) \subseteq U$ and U is pre-open. Hence B is $p^{\#}g$-closed.*

Theorem 3.55 *If $cl(pint(A)) \subset B \subset A$ and A is $p^{\#}g$-open then B is $p^{\#}g$-closed.*

Proof: *Let $cl(pint(A)) \subset B \subset A$. Thus $(X-A) \subset (X-B) \subset (X - cl(pint(A))) \Rightarrow (X-A) \subset (X-B) \subset (X - int(pcl(X-A)))$ Since $(X-A)$ is $p^{\#}g$-closed. By theorem (3.36) $(X-A) \subset (X-B) \subset (X - int(pcl(X-A)))$ $\Rightarrow X - B$ is $p^{\#}g$-closed.*

Theorem 3.56 *Let (X, τ) be a compact topological spaces. If A is $p^{\#}g$-closed subset of X,then A is compact.*

Proof: *Let $\{U_i\}$ be a open cover of A. Since every open set is pre-open and A is $p^{\#}g$-closed, we get $int(pcl(A)) \subseteq \bigcup U_i$. Since a closed subset of a compact space is compact, $int(pcl(A))$ is compact. Therefore there exists a finite sub-cover, say $\{U_1 \bigcup U_2 \bigcup \cdots \bigcup U_n\}$ of U_i for $int(pcl(A))$. So $A \subseteq int(pcl(A)) \subseteq \{U_1 \bigcup U_2 \bigcup \cdots \bigcup U_n\}$. Therefore A is not compact.*

Theorem 3.57 *Let (X, τ) be a Lindelof [countable compact] and suppose that A is $p^{\#}g$-closed subset of X. Then A is not Lindelof [countable compact].*

Proof: *Let $\{U_i\}$ be a open cover of A. Since every open set is pre-open, $\{U_i\}$ is a countable pre-open cover of $A \bigcup U_i$ is pre-open.Then $int(pcl(A)) \subseteq$*

$\bigcup U_i$ *because* A *is* $p^{\#}g$*-closed. Since a closed subset of a Lindelof space is Lindelof,* $int\,(pcl\,(A))$ *is not Lindelof. Therefore* $int\,(pcl\,(A))$ *has no countable sub-cover, say* $\{U_1 \bigcup U_2 \bigcup \cdots \bigcup U_n\}$ *and it follows that,* $A \subseteq int\,(pcl\,(A)) \subseteq \{U_1 \bigcup U_2 \bigcup \cdots \bigcup U_n\}$ *Hence* A *is not Lindelof.*

Theorem 3.58 *Let* (X, τ) *be a normal space and if* Y *is a*$p^{\#}g$*-closed subset of* (X, τ)*,then the subspace* Y *is normal.*

Proof: *If* G_1 *and* G_2 *are disjoint closed sets in topological space* (X, τ) *such that* $(Y \bigcap G_1) \bigcap (Y \bigcap G_2) = \phi$*. Then* $Y \subseteq (G_1 \bigcap G_2)^c$ *and* $(G_1 \bigcap G_2)^c$ *is pre-open.* Y *is p#g-closed in* (X, τ)*. Therefore* $int\,(pcl\,(A)) \subseteq (G_1 \bigcap G_2)^c$ *.Hence* $(int\,(pcl\,(Y)) \bigcap G_1) \bigcap (int\,(pcl\,(Y)) \bigcap G_2) = \phi$*. Since* (X, τ) *is normal, there exists disjoint open set* A *and* B *such that* $(int\,(pcl\,(Y)) \bigcap G_1) \subseteq A$ *and* $(int\,(pcl\,(Y)) \bigcap G_2) \subseteq B$*.(i.e)* $(Y \bigcap A)$ *and* $(Y \bigcap B)$ *are open set of* Y *such that* $(Y \bigcap G_1) \subseteq (Y \bigcap A)$ *and* $(Y \bigcap G_2) \subseteq (Y \bigcap B)$ *are disjoint open sets of* Y *. Hence* Y *is normal.*

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