On αsg closed sets in Topological spaces

A.Devika, R.Vani and K.Gomathi

Associate Professor,Department of Mathematics,PSG College of Arts & Science,Coimbatore,Tamilnadu.

Assistant Professor,Department of Mathematics with CA,PSG College of Arts & Science,Coimbatore,Tamilnadu.

Research Scholar,Department of Mathematics,PSG College of Arts & Science,Coimbatore,Tamilnadu.

Abstract

In this paper we introduce the new concept of αsg closed sets in topological spaces and a basic properties of αsg-closed sets were obtained.

Mathematics Subject Classification: 54A05

Keywords: αsg-closed sets and αsg-open sets.

1 Introduction

The study of semi-open (briefly s-open) sets in a topological spaces was initiated by N.Levine[7] in 1963 and also he generalized the concept of closed sets to generalized closed(briefly g-closed) sets[8] in 1970. Bhattacharya and Lahiri[3] generalized the concept of closed sets to semi-generalized closed(briefly sg-closed) sets in 1987. O.Njastad[14] introduced αsets(called as α-closed sets).

The aim of this paper is to introduce the new type of closed set called αsg closed set and to continue the study of αsg-closed sets thereby contributing new innovation and concept, in the field of topology through analytical as well as research works. The notion of αsg-closed sets and its different characterizations are given in this paper.

2 Preliminaries

A subset A of a topological space X is said to be **open** if $A \in \tau$. A subset A of a topological space X is said to be **closed** if the set $X - A$ is open. The **interior** of a subset A of a topological space X is defined as the union of all open sets contained in A. It is denoted by $int(A)$. The **closure** of a subset A of a topological space X is defined as the intersection of all closed sets containing A. It is denoted by $cl(A)$.

Throughout this paper (X, τ) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. Let $A \subseteq X$,the closure of A and interior of A will be denoted by $cl(A)$ and int(A) respectively.

Definitions 2.1.

1. A subset A of a space (X, τ) is said to be **semi open** [7] if $A \subseteq cl\,(int\,(A))$ and **semi closed** if $int\,(cl\,(A)) \subseteq A$.
2. A subset A of a space (X, τ) is said to be α**-open** [14] if $A \subseteq int\,(cl\,(int\,(A)))$ and α**-closed** if $cl\,(int\,(cl\,(A))) \subseteq A$.
3. A subset A of a space (X, τ) is said to be **semi pre-open** [1] if $A \subseteq cl\,(int\,(cl\,(A)))$ and **semi pre-closed** if $int\,(cl\,(int\,(A))) \subseteq A$.
4. A subset A of a space (X, τ) is said to be **regular-open** [17] if $A = int\,(cl\,(A))$ and **regular-closed** if $A = cl\,(int\,(A))$.
5. A subset A of a space (X, τ) is said to be **pre-open** [12] if $A \subseteq int\,(cl\,(A))$ and **pre-closed** if $cl\,(int\,(A)) \subseteq A$.

The complement of a semi-open (resp.pre-open, α-open) set is called **semi-closed (resp.pre-closed, α-closed)**. The intersection of all semi-closed (resp.pre-closed, α-closed) sets containing A is called the **semi-closure (resp.pre-closure, α-closure)** of A and is denoted by $scl(A)$(resp. $pcl(A)$, α-$cl(A)$) . The union of all semi-open (resp.pre-open, α-open) sets contained in A is called the **semi-interior(resp.pre-interior, α-interior)** of A and is denoted by $sint(A)$(resp. $pint(A)$, α-$int(A)$). The family of all semi-open (resp.pre-open, α-open)sets is denoted by $SO(X)$(resp. $PO(X)$, $\alpha - O(X)$). The family of all semi-closed (resp.pre-closed, α-closed)sets is denoted by $SCl(X)$ (resp. $PCl(X)$, α-$Cl(X)$).

Definitions 2.2.

1. A subset A of a space (X, τ) is called **generalized-closed set** [8] (briefly g-closed) if $cl\,(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).
2. A subset A of a space (X, τ) is called **generalized semi-closed set** [2] (briefly gs-closed set) if $scl\,(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

3. A subset A of a space (X, τ) is called **semi-generalized closed set** [3] (briefly sg-closed set) if $scl\,(A) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in (X, τ).

4. A subset A of a space (X, τ) is called α **generalized-closed set** [10] (briefly αg-closed) if $\alpha\,(cl\,(A)) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

5. A subset A of a space (X, τ) is called **generalized** α**-closed set** [9] (briefly $g\alpha$-closed) if $\alpha\,(cl\,(A)) \subseteq U$, whenever $A \subseteq U$ and U is α-open in (X, τ).

6. A subset A of a space (X, τ) is called **generalized pre-closed set** [11] (briefly gp-closed) if $pcl\,(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

7. A subset A of a space (X, τ) is called **generalized semi-pre closed set** [4] (briefly gsp-closed) if $spcl\,(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

8. A subset A of a space (X, τ) is called **semi weekly generalized-closed set** [13] (briefly swg-closed) if $cl\,(int\,(A)) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in (X, τ).

9. A subset A of a space (X, τ) is called **star generalized-closed set** [20] (briefly $*g$-closed) if $cl\,(A) \subseteq U$, whenever $A \subseteq U$ and U is g-open in (X, τ).

10. A subset A of a space (X, τ) is called **weekly-closed set** [18] (briefly w-closed) if $cl\,(A) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in (X, τ).

11. A subset A of a space (X, τ) is called **generalized-closed set** [19] (briefly $\hat{g}$-closed) if $cl\,(A) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in (X, τ).

12. A subset A of a space (X, τ) is called **weekly generalized-closed set** [13] (briefly wg-closed) if $cl\,(int\,(A)) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

13. A subset A of a space (X, τ) is called π**generalized-closed set** [5] (briefly πg-closed) if $cl\,(A) \subseteq U$, whenever $A \subseteq U$ and U is π-open in (X, τ).

14. A subset A of a space (X, τ) is called π**generalized**α**-closed set** [6] (briefly $\pi g\alpha$-closed) if $\alpha cl\,(A) \subseteq U$, whenever $A \subseteq U$ and U is π-open in (X, τ).

15. A subset A of a space (X, τ) is called **m generalized-closed set** [16] (briefly mg-closed) if $cl\,(int\,(A)) \subseteq U$, whenever $A \subseteq U$ and U is g-open in (X, τ).

16. A subset A of a space (X, τ) is called **generalized-closed set** [15] (briefly $\tilde{g}$-closed) if $cl\,(A) \subseteq U$, whenever $A \subseteq U$ and U is $\#gs$open in (X, τ).

3 αsg-Closed sets in Topological Spaces

In this section the notion of a new class of sets called αsg-closed sets in topological spaces is introduced and their properties were studied.

Definition 3.1 *A subset A of space (X, τ) is called αsg-**closed** if $int\,(scl\,(A)) \subseteq U$, whenever $A \subseteq U$ and U is α-open in X.*

The family of all αsg-closed subsets of the space X is denoted by $\alpha SGC(X)$.

Definition 3.2 *The intersection of all αsg-closed sets containing a set A is called αsg-**closure** of A and is denoted by αsg-$cl(A)$.*

A set A is αsg-closed set if and only if $\alpha sg\ Cl\,(A) = A$.

Definition 3.3 *A subset A in X is called αsg-**open** in X if A^c is αsg-closed in X.*

The family of a αsg-open sets is denoted by $\alpha SGO(X)$.

Definition 3.4 *The union of all αsg-open sets containing a set A is called αsg-**interior** of A and is denoted by αsg-$Int(A)$.*

A set A is αsg-open set if and only if $\alpha sg\ Int\,(A) = A$.

Theorem 3.5 *Every closed set is a αsg-closed set.*

Proof: *Let A be a closed set in X. Such that $A \subseteq U$, U is α-open. Since A is closed, $cl\,(A) = A$.For every subset A of X, $int\,(scl\,(A)) \subseteq cl\,(A) = A \subseteq U$ and so we have $int\,(scl\,(A)) \subseteq U$. Hence A is αsg-closed.*

Remark 3.6 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.7 *Let $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then $A = \{a\}$ is αsg-closed but not a closed set of (X, τ).*

Theorem 3.8 *Every p-closed set is a αsg-closed set.*

Proof: *Let A be a p-closed set in X. Such that $A \subseteq U$,U is α-open.Since A is p-closed, $pcl(A) = A$.For every subset A of X, $int(scl(A)) \subseteq pcl(A) = A \subseteq U$ and so we have $int(scl(A)) \subseteq U$. Hence A is αsg-closed.*

Remark 3.9 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.10 *Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $A = \{a\}$ is αsg-closed but not a p-closed set of (X, τ).*

Theorem 3.11 *Every α closed set is a αsg-closed set.*

Proof: *Let A be a α-closed set in X. Such that $A \subseteq U$, U is α-open.Since A is α-closed, $\alpha cl(A) \subseteq A$.For every subset A of X, $int(scl(A)) \subseteq \alpha cl(A) \subseteq A \subseteq U$ and so we have $int(scl(A)) \subseteq U$.Hence A is αsg-closed.*

Remark 3.12 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.13 *Let $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then $A = \{b\}$ is αsg-closed but not α closed set of (X, τ).*

Theorem 3.14 *Every r-closed set is a αsg-closed set.*

Proof: *Let A be a r-closed set in X. Such that $A \subseteq U$, U is α-open.Since A is r-closed, $rcl(A) \subseteq A$.For every subset A of X, $int(scl(A)) \subseteq rcl(A) \subseteq A \subseteq U$ and so we have $int(scl(A)) \subseteq U$.Hence A is αsg-closed.*

Remark 3.15 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.16 *Let $X = \{a, b, c, d, e\}$ with topology $\tau = \{X, \phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}\}$. Then $A = \{b, c, e\}$ is αsg-closed but not a r-closed set of (X, τ).*

Theorem 3.17 *Every $g\alpha$ closed set is a αsg-closed set.*

Proof: *Let A be a $g\alpha$-closed set in X. Such that $A \subseteq U$, U is α-open.Since A is $g\alpha$-closed, $\alpha cl(A) \subseteq A$.For every subset A of X, $int(scl(A)) \subseteq \alpha cl(A) \subseteq A \subset U$ and so we have $int(scl(A)) \subseteq U$.Hence A is αsg-closed.*

Remark 3.18 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.19 *Let $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then $A = \{b\}$ is αsg-closed but not $g\alpha$ closed set of (X, τ).*

Theorem 3.20 *Every g closed set is a αsg-closed set.*

Proof: *Let A be a g-closed set in X. Such that $A \subseteq U$, U is α-open.Since A is g-closed, $cl(A) \subseteq A$.For every subset A of X, $int(scl(A)) \subseteq cl(A) \subseteq A \subset U$ and so we have $int(scl(A)) \subseteq U$.Hence A is αsg-closed.*

Remark 3.21 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.22 *Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $A = \{a, c\}$ is αsg-closed but not g- closed set of (X, τ).*

Theorem 3.23 *Every αg closed set is a αsg-closed set.*

Proof: *Let A be a αg-closed set in X. Such that $A \subseteq U$, U is α-open.Since A is αg-closed, $\alpha cl(A) \subseteq A$.For every subset A of X, $int(scl(A)) \subseteq \alpha cl(A) \subseteq U$ and we have $int(scl(A)) \subseteq U$.Hence A is αsg-closed.*

Remark 3.24 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.25 *Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $A = \{a, d\}$ is αsg-closed but not αg closed set of (X, τ).*

Theorem 3.26 *Every $*g$-closed set is a αsg-closed set.*

Proof: *Let A be a $*g$-closed set in X. Such that $A \subseteq U$, U is α-open.Since A is $*g$-closed, $cl(A) \subseteq A$.For every subset A of X, $int(scl(A)) \subseteq cl(A) \subseteq A \subseteq U$ and so we have $int(scl(A)) \subseteq U$.Hence A is αsg-closed.*

Remark 3.27 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.28 *Let $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then $A = \{a\}$ is αsg-closed but not a $*g$-closed set of (X, τ).*

Theorem 3.29 *Every w-closed set is a αsg-closed set.*

Proof: *Let A be a w-closed set in X.Such that $A \subseteq U$, U is s-open.Since A is w-closed, $cl(A) \subseteq A$.For every subset A of X, $int(scl(A)) \subseteq cl(A) \subseteq A \subseteq U$ and so we have $int(scl(A)) \subseteq U$.Hence A is αsg-closed.*

Remark 3.30 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.31 *Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $A = \{a, c\}$ is αsg-closed but not a w-closed set of (X, τ).*

Theorem 3.32 *Every swg-closed set is a αsg-closed set.*

Proof: *Let A be a swg-closed set in X. Such that $A \subseteq U$, U is α-open.Since A is swg-closed, $cl(int(A)) \subseteq A$.For every subset A of X, $int(scl(A)) \subseteq cl(int(A)) \subseteq A \subseteq U$ and so we have $int(scl(A)) \subseteq U$.Hence A is αsg-closed.*

Remark 3.33 *The converse of the above theorem need not be true as seen from the following example.*

Example 3.34 *Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $A = \{a\}$ is αsg-closed but not a swg-closed set of (X, τ).*

Theorem 3.35 *The union of two αsg-closed subsets of X is also αsg-closed subset of X.*

Proof: *Assume that P and Q are p#g-closed set in X. Let $P \bigcup Q \subseteq U$ and U be α-open in X. Since $P \subset U$ and $Q \subset U$, U is α-open.Then $int(scl(P)) \subseteq U$ and $int(scl(Q)) \subseteq U$ and we have $int(scl(P \bigcup Q)) \subseteq int(scl(p)) \bigcup int(scl(Q)) \subseteq U$. Since U is α-open. Hence $P \bigcup Q$ is αsg-closed set in X.*

Remark 3.36 *The intersection of two αsg-closed sets in X is generally not αsg-closed set in X.*

Example 3.37 *Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. If $P = \{b, c\}$ and $Q = \{b, d\}$, then P and Q are αsg-closed sets in X, but $P \bigcup Q = \{b\}$ is not a αsg-closed set of X.*

Theorem 3.38 *Every $\hat{g}$-closed set is a αsg-closed set.*

Proof follows from the definition, since every α-open set is semi-open.

Example 3.39 *In example (3.10), the set $\{b, c\}$ is αsg-closed but not a $\hat{g}$-closed set of (X, τ).*

Theorem 3.40 *Every wg-closed set is a αsg-closed set.*

Proof follows from the definition, since every α-open set is open.

Example 3.41 *In example (3.10), the set $\{a, d\}$ is αsg-closed but not a wg-closed set of (X, τ).*

Theorem 3.42 *Every πg-closed set is a αsg-closed set.*

Proof follows from the definition, since every α-open set is π-open.

Example 3.43 *In example (3.13), the set $\{b\}$ is αsg-closed but not a πg-closed set of (X, τ).*

Theorem 3.44 *Every $\pi g\alpha$-closed set is a αsg-closed set.*

Proof follows from the definition, since every α-open set is π-open.

Example 3.45 *In example (3.13), the set $\{a\}$ is αsg-closed but not a $\pi g\alpha$-closed set of (X, τ).*

Theorem 3.46 *Every mg-closed set is a αsg-closed set.*

Proof follows from the definition, since every α-open set is g-open.

Example 3.47 *In example (3.16), the set $\{b, c, e\}$ is αsg-closed but not a mg-closed set of (X, τ).*

Theorem 3.48 *Every gp-closed set is a αsg-closed set.*

Proof follows from the definition, since every α-open set is open.

Example 3.49 *In example (3.10), the set $\{b\}$ is αsg-closed but not a gp-closed set of (X, τ).*

So the class of αsg-closed sets properly contain the class of $\hat{g}$-closed set, wg-closed set, $\pi g\alpha$-closed set, πg-closed set, gp-closed set and mg-closed sets.

Remark 3.50 *The concept of αsg-closed set is independent of the following classes of sets namely gs-closed set and $\tilde{g}$-closed set.*

Example 3.51 *Consider the topological space $X = \{a, b, c, d, e\}$,with topology $\tau = \{X, \phi, \{a, b\}, \{c, d\}, \ \{a, b, c, d\}\}$.In this space,the set $\{a, b\}$ is αsg-closed set but not gs-closed set and the set $\{a, e\}$ is gs-closed set but not αsg-closed set.*

Example 3.52 *Consider the topological space $X = \{a, b, c, d\}$,with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \ \{a, b, c\}, \{a, b, d\}\}$.In this space,the set $\{a\}$ is αsg-closed set but not $\tilde{g}$-closed set and the set $\{a, b\}$ is $\tilde{g}$-closed set but not αsg-closed set.*

Remark 3.53 *Figure 3.1 gives the implication relations of αsg-closed sets based on the above results.*

On αsg closed sets in Topological spaces

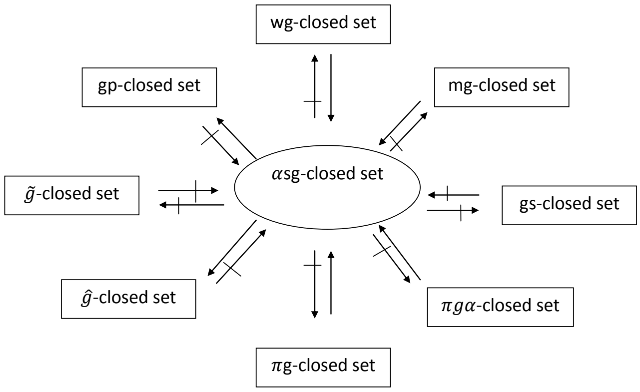

Figure 3.1 Implication of αsg- closed set

Where A ⟶ B represents A implies B

A ⟶ B represents A does not implies B

A ⟵ B represents B does not implies A

Theorem 3.54 *For $x \in X$, the set $X - \{x\}$ is αsg-closed or α-open.*

Proof: *Suppose $X - \{x\}$ is not α-open. Then X is the only α-open set containing $X - \{x\}$. $\Rightarrow int\,(scl\,(X - \{x\})) \subseteq X$. Then $X - \{x\}$ is αsg-closed in X*

Theorem 3.55 *Let $A \subseteq Y \subseteq X$ and suppose that A is αsg-closed in X, then A is αsg-closed relative to Y.*

Proof: *Given that $A \subseteq Y \subseteq X$ and A is αsg-closed in X. To show that A is αsg-closed relative to Y. where U is α-open in X. Since A is αsg-closed, $A \subseteq U$, implies that $int\,(scl\,(A)) \subseteq U$,It follows that $Y \bigcap int\,(scl\,(A)) \subseteq Y \bigcap U$. Thus A is αsg-closed relative to Y.*

Theorem 3.56 *If A is αsg-closed and $A \subseteq B \subseteq int\,(scl\,(A))$. Then B is αsg-closed.*

Proof: *Let U be a α-open set of X, such that $B \subseteq U$. Then $A \subseteq U$ and since A is αsg-closed, we have, $int\,(scl\,(A)) \subseteq U$ Now,$int\,(scl\,(B)) \subseteq int\,(scl\,(int\,(scl\,(A)))) = int\,(scl\,(A)) \subseteq U$ Hence B is αsg-closed set.*

Theorem 3.57 *If a subset A of (X, τ) is α-open and αsg-closed, then A is semi-closed in (X, τ).*

Proof: *If a subset A of (X,τ) is α-open and αsg-closed. Then $int\,(scl\,(A)) \subseteq U \subseteq A$. Hence A is semi-closed in (X,τ).*

Theorem 3.58 *If a set A is αsg-closed, then $int\,(scl\,(A)) - A$ contains no non-empty α-closed set.*

Proof: *Let F be a non-empty α-closed set such that $F \subseteq int\,(scl\,(A)) - A$, then $F \subseteq int\,(scl\,(A))$ and $A \subseteq X - F$, we have $int\,(scl\,(A)) \subseteq int\,(X - F)$. $\Rightarrow int\,(scl\,(A)) \subseteq X - cl(A) \Rightarrow cl(A) \subseteq X - int\,(scl\,(A))$. Therefore $F \subseteq int\,(scl\,(A)) \bigcap (X - int\,(scl\,(A))) = \phi$. Hence $int\,(scl\,(A)) - A$ contains no non-empty α-closed set.*

Theorem 3.59 *Let A be α-closed in (X,τ), then A is semi-closed iff $int\,(scl\,(A)) - A$ is α-closed.*

Proof: *Necessity:*

Let A be semi-closed, then $scl(A) = A$. Hence $int\,(scl\,(A)) - A = \{\phi\}$. Which is α-closed.

Sufficiency:

Suppose $int\,(scl\,(A)) - A$ is α-closed. Since A is αsg-closed by theorem(), $int\,(scl\,(A)) - A = \{\phi\}$. Then $int\,(scl\,(A)) = A$. This means that A is semi-closed.

References

[1] D. Andrijevic, semipre-open sets, *Mat. Vesnik,* **38(1)**(1986), 24-32.

[2] S.P. Arya, T.M. Tour, Characterization of s-normal spaces, *Indian J.Pure-Appl.Math,* **21(8)**(1990), 717-719.

[3] P. Bhattacharya, B.K. Lahari, semi-generalized closed sets in topology, *Indian.J.Math.,* **29(3)**(1987), 375-382.

[4] J. Dontchev, On generalizing semi-pre open sets, *Mem.Fac.Sci.Kochi. Univ.ser.A.Math,* **16**(1995), 35-48.

[5] J. Dontchev and T.Noiri, Quasi-normal spaces and πg-closed sets, *Acta Math.Hungar,***89(3)**(2000),211-219.

[6] C. Janaki, Studies on $\pi g\alpha$-closed sets in Topology, *Ph.D Thesis,Bharathiar University ,Coimbatore.*(2009).

[7] N. Levine, semi-open sets and semi-continuity in topological spaces, *American Mathematical Monthly*, **70**(1963), 36-41.

[8] N.Levine, Generalized closed sets in topological spaces,*Rend.circ.mat.palermo,vol 19(2),(1970)* 89-96.

[9] H. Maki, R. Devi, K. Balachandran, Generalized α-closed sets in topology, *Bull.Fukuoka Univ, Ed.partIII.,* **42**(1993), 13-21.

[10] H. Maki, R. Devi, K. Balachandran, Associated topologies of generalized α-closed sets and α-generalized closed sets, *Mem.Fac.Sci.Kochi Univ.sev.A.Math.,* **15**(1994), 51-63.

[11] H. Maki, J. Umehera, T. Noiri, Every Topological space is pre-t1/2, *Mem.Fac.Sci.Kochi.Univ.ser.A.Math.,* **17**(1996), 33-42.

[12] A.S. Mashlour, M.E. Abd.EI- Monsef, S.N.EI. Deeb, On pre-continuous and weak pre-continuous mappings, *proc.Math,phys.soc.Egypt,* **53**,(1982), 47-53.

[13] N. Nagaveni,Studys on generalizations of homeomorphism in topological spaces, *Ph.D.,Thesis,Bharathiar University,Coimbatore*(1999).

[14] O. Njastad, On some classes of nearly open sets, *Pacific.J.Math.,* **15**(1965), 961-970.

[15] T. Noiri,S.Jafari,N.Rajesh and M.L.Thivagar. Another generalization of closed sets, *Kochi J.Math*,**3**(2008)25-38.

[16] J.K.Park and J.H.Park, Mildly generalized closed set, almost normal and mildly normal space, *Chaos.Solution and Fractal*,**20**(2004),1103-1111.

[17] M.H.Stone,Application of the theory of Boolean rings to general topology, *Trans.Amer.Math.Soc.*,**41**(1937),375-381.

[18] P. Sundaram,and M.shiek john.,On w-closed sets in topology, *Act aciencia indica*,(2000),389-392.

[19] M.K.R.S.Veerakumar,$\hat{g}$-closed sets and GIC-functions, *Indian J.Math.*,**43(2)**(2001),231-247.

[20] M.K.R.S.Veerakumar ,Between closed sets and g-closed sets, *Mem.Fac.Sci.Kochi Univ.(Math).*,**21**(2000),1-19.