Properties of Almost Contra Continuous Functions in Fuzzy Topology

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Abstract — *The theme of this paper is about new notion of contra continuity called almost contra θ-generalized semi-continuous functions in fuzzy topological spaces and to investigate properties and relationships among fuzzy functions. Furthermore, fuzzy θgs-compact, fuzzy θgs-connceted and fuzzy θgs-normal spaces are also introduced*

Keywords- *Fuzzy θgs-closed set, Fuzzy almost contra θgs-continuous, fuzzy θgs-compact space, fuzzy θgs-connected space, fuzzy θgs-normal space.*

I. INTRODUCTION

Many of our real life problems in engineering, medical and social science, economics involve imprecise data and their solutions involve the mathematical concepts based on uncertainty. To handle such uncertainties, L.A. Zadeh [16] introduced the notion of fuzzy sets and fuzzy operations. The analytical part of fuzzy set theory was practically presented by C.L. Chang [3] who introduced fuzzy topological spaces. In 1981, Azad [1] introduced some weaker forms of continuity in fuzzy topological space. He introduced fuzzy semi-open, fuzzy semi-closed, fuzzy semi-continuous functions, fuzzy almost continuous functions in fuzzy topological spaces.

In the year 1970, Levine [8] gave the notion of generalized closed set in general topology. In 1997, Balasubramanian and Sundaram [2] defined the concepts of fuzzy generalized closed set in fuzzy topological spaces. Later El-Shafei [6] introduced semi-generalized closed sets and semi-generalized continuous functions in fuzzy topological spaces and some of their properties. In [10] authors introduced the concept of θ-generalized-semi-closed set in topology.

In 2013, Zabidin Salleh et al [14] introduced and studied the notion of θ-semi-generalized-closed sets in fuzzy topological spaces. In this paper, fuzzy almost contra θ-generalized-semi-continuous functions are introduced. Further, fuzzy θ-generalized-semi-compact and connected spaces are introduced and discussed their characterizations.

II. PRELIMINARIES

In this paper X be a set and I the unit interval. A fuzzy set in X is an element of the set of all functions from X to I. The family of all fuzzy sets in X is denoted by I^X. A fuzzy singleton x_α is a fuzzy set in X define by $x_\alpha(x) = \alpha$, $x_\alpha(y) = 0$ for all $y \neq x$, $x \in (0, 1]$. The set of all fuzzy singletons in X is denoted by S(X). For every $x_\alpha \in S(X)$ and $\mu \in I^X$, we define $x_\alpha \in \mu$ if and only if $x_\alpha \leq \mu(x)$. The members of τ are called fuzzy open sets and their complements are fuzzy closed sets. Spaces (X, τ) and (Y, σ) (or simply, X and Y) always mean fuzzy topological spaces in the sense of Chang [3]. By 1_X and 0_X , we mean fuzzy sets with constant function 1 (unit function) and 0 (zero function), respectively.

For a fuzzy set μ of X, fuzzy closure and fuzzy interior of μ denoted by Cl(μ) and Int(μ), respectively. The operators fuzzy closure and fuzzy interior are defined by $Cl(\mu) = \Lambda\{ \lambda : \lambda \geq \mu, 1-\mu \in \tau\}$ where λ is fuzzy closed set in X and $Int\ \mu = \Lambda\{\eta: \eta \leq \mu, \eta \in \tau \}$ [15] where η is fuzzy open set in X. Fuzzy semi-closure [15] of μ denoted by $scl(\mu) = \Lambda\{\eta:\mu \leq \eta, \eta \in FSC(X)\}$ and fuzzy θ-closure of μ denoted by $Cl_\theta = \Lambda\{ Cl(\eta): \mu \leq \eta, \eta \in \tau \}$[4].

Definition 2.1[1]: A fuzzy subset A of a space X is called

(1) Fuzzy semi-open (briefly, Fs-open) set if $A \leq Cl(Int(A))$.

(2) Fuzzy semi-closed (briefly, Fs-closed) set if $Int(Cl(A)) \leq A$.

(3) Fuzzy regular closed if $Cl(Int(A))= A$ and fuzzy regular open if $Int(Cl(A)) = A$.

The family of all fuzzy semi open, fuzzy semi closed in X will be denoted by FSO(X), FSC(X), respectively.

Definition 2.2[14]: Let X be a fuzzy topological space and μ be a fuzzy set of X. Then the operators semi-θ-closure of μ denoted by $Scl_\theta(\mu)$ and fuzzy semi-θ-interior of μ is denoted by $Sint_\theta(\mu)$ are defined as follows,

$Scl_\theta(\mu) = \Lambda \{scl(\eta): \mu \leq \eta, \eta \in FSO(X)\}$,

$Sint_\theta \mu) = V \{ sint(\eta): \mu \geq \eta, \eta \in FSC(X)\}$.

Definition 2.3 [6]: A fuzzy set μ in X is called fuzzy semi-generalized closed set (briefly, fsg-closed set) if $scl(\mu) \leq \eta$ whenever $\mu \leq \eta$ and η is fuzy semi open.

Definition 2.4 [14]: A fuzzy subset μ of X is said to be fuzzy θ-generalized-semi closed set (briefly, Fθgs-closed set) if $Scl_\theta(\mu) \leq \eta$ whenever $\mu \leq \eta$ and η is fuzzy open.

The complement of fuzzy θ-generalized-semi closed set is fuzzy θ-generalized-semi open set (in short, Fθgs-open set). The family of all Fθgs-closed sets in X are denoted by FθGSC(X) and The family of all f-θgs-open sets in X are denoted by FθGSO(X).

Definition 2.5[14]: A function f: X→Y is said to be
(1) fuzzy θ-generalized semi continuous (briefly, f-θsg-continuous) if $f^{-1}(\lambda)$ is Fθgs-closed in X for each fuzzy semi-closed set λ in Y.
(2) fuzzy θ-generalized semi irresolute(briefly, f-θgs-irresolute) if $f^{-1}(\lambda)$ is Fθgs-closed set in X for each Fθgs-closed set λ in Y.

III. FUZZY ALMOST *θ*-CONTRA GENERALIZED-SEMI CONTINUOUS FUNCTIONS.

Definition 3.1: Let X and Y be fuzzy topological spaces. A fuzzy function f: X → Y is said to be fuzzy almost θ-generalized-semi continuous (briefly, fuzzy almost contra θgs-continuous) if inverse image of each fuzzy regular open set in Y is Fθgs-closed in X.

Theorem 3.2: For a fuzzy function f: X → Y, the following statements are equivalent:
(i) f is fuzzy almost contra θgs-continuous.
(ii) For every fuzzy regular closed set μ in Y, $f^{-1}(\mu)$ is fuzzy θgs-open.
Proof: (i) → (ii). Let μ be a fuzzy regular closed set in Y, then Y-F is fuzzy regular open set in Y. By (i) $f^{-1}(Y-\mu) = X - f^{-1}(\mu)$ is Fθgs-closed set in X. This implies that $f^{-1}(\mu)$ is Fθgs-open set in X. Therefore (ii) holds.
(ii) → (i). Let G be a fuzzy regular open set of Y. Then Y- G be a fuzzy regular closed set in Y. By (ii) $f^{-1}(Y-G)$ is Fθgs-open set in X. This implies that X - $f^{-1}(G)$ is Fθgs-open in X, which implies $f^{-1}(G)$ is Fθgs-closed set in X. Therefore (i) holds.

Theorem 3.3 : Let f: X → Y be a fuzzy function and let g: X→ X x Y be the fuzzy graph function of f defined by $g(x_p) = (x_p , f(x_p))$ for every $x_p \in X$. If g is fuzzy almost contra θgs-continuous, then f is fuzzy almost contra θgs-continuous.
Proof: Let μ be a fuzzy regular closed set in Y, then (X x μ) is fuzzy regular closed set in X x Y. Since g is fuzzy almost contra θgs-continuous, then $f^{-1}(\mu) = g^{-1}(X \times \mu)$ is Fθgs-open in X. Thus, f is fuzzy almost contra θgs-continuous.

Definition 3.4: A fuzzy filter base λ is said to be fuzzy θgs-convergent to a fuzzy singleton x_p in X if for any Fθgs-open set μ in X containing x_p, there exists a fuzzy set η ∈ λ such that η ≤ μ .
Definition 3.5: A fuzzy filter base λ is said to be fuzzy rc-convergent [4] to a fuzzy singleton x_1 in X if for any fuzzy regular closed set μ in X containing x_p, there exists a fuzzy set η ∈ λ such that η ≤ μ.

Theorem 3.6: If a fuzzy function f: X → Y is fuzzy almost contra θgs-continuous, then for each fuzzy singleton $x_p \in X$ and each filter base λ in X θgs-converging to x_p, the fuzzy filter base f(λ) is fuzzy rc-convergent to $f(x_p)$.
Proof: Let $x_p \in X$ and λ be any fuzzy filter base in fuzzy θgs-converging to x_p . Since f is fuzzy almost contra θgs-continuous, then for any fuzzy regular closed set λ in Y containing $f(x_p)$, there exists a Fθgs-open set μ ∈ X containing x_p such that f(μ) ≤ λ . Since λ is fuzzy θgs-converging to x_p, there exists A ∈ λ such that A ≤ μ. This means that f(A) ≤μ and therefore the fuzzy filter base f(λ) is fuzzy rc-convergent to $f(x_p)$.

IV. CHARACTERIZATION OF FUZZY ALMOST CONTRA *θ*-GENERALIZED-SEMI CONTINUOUS FUNCTIONS.

In this section fuzzy θ-generalized semi-connected and fuzzy θ-generalized semi-normal spaces are introduced and characterization of fuzzy almost contra θ-generalized semi-continuous functions is done .

Definition 4.1: A fuzzy topological space X is called fuzzy θgs-connected if X is not the union of two disjoint nonempty θgs-open sets.

Definition 4.2: A fuzzy space X is called fuzzy connected [13] if X is not the union of two disjoint nonempty fuzzy open sets.

Theorem 4.3: If f: X → Y is fuzzy almost contra θgs-continuous surjection and X is fuzzy θgs-connected, then Y is fuzzy connected.
Proof: Suppose Y is not fuzzy connected. Then there exist nonempty disjoint fuzzy open sets P and Q such that Y = P Λ Q. Therefore, P and Q are fuzzy clopen in Y. Since f is fuzzy almost contra θgs-continuous, $f^{-1}(P)$ and $f^{-1}(Q)$ are Fθgs-open in X. Moreover, $f^{-}(P)$ and $f^{-1}(Q)$ are nonempty disjoint and X= $f^{-1}(P)$ Λ $f^{-1}(Q)$. This shows that X is not fuzzy θgs-connected. This contradicts the fact that Y is not Fuzzy connected assumed. Hence Y is fuzzy connected.

Definition 4.4: A fuzzy space X is said to be fuzzy θgs-normal if every pair of nonempty disjoint fuzzy closed sets can be separated by disjoint fuzzy θgs-open sets.

Definition 4.5: A fuzzy space X is said to be fuzzy strongly θgs-normal if every pair of nonempty disjoint fuzzy closed sets A and B there exist disjoint

Fθgs-open sets U and V such that $A \leq U$, $B \leq V$ and $Cl(A) \wedge Cl(B) = \phi$.

Theorem 4.6: If Y is fuzzy strongly θgs-normal and f: X → Y is fuzzy almost contra θgs-continuous closed surjection, then X is fuzzy θgs-normal.

Proof: Let A and B be disjoint nonempty fuzzy closed sets of X. Since f is injective and closed, f(A) and f(B) are disjoint fuzzy closed sets. Since Y is fuzzy strongly θgs-normal, then there exist Fθgs-open sets P and Q such that $f(A) \leq P$ and $f(B) \leq Q$ and $Cl(A) \wedge Cl(B) = \phi$. Then, since Cl(A) and Cl(B) are regular closed and f is fuzzy almost contra θgs-continuous, $f^{-1}(Cl(A))$ and $f^{-1}(Cl(B))$ are Fθgs-open sets. Since, $P \leq f^{-1}(Cl(A))$, $Q \leq f^{-1}(Cl(B))$ and $f^{-1}(Cl(A))$ and $f^{-1}(Cl(B))$ are disjoint, X is fuzzy θgs-normal.

Definition 4.7: A fuzzy space X is said to be fuzzy θgs-T_1 if for each pair of distinct fuzzy singletons x and y in X, there exist Fθgs-open sets U and V containing x and y respectively, such that $y \notin U$ and $x \notin V$.

Definition4.8: A fuzzy space X is said to be fuzzy θ gs-T_2 if for each pair of distinct fuzzy points x and y in X, there exist Fθgs-open set U containing x and Fθgs-open set V containing y such that $U \wedge V = \phi$.

Theorem 4.9: If f: X → Y is a fuzzy almost contra θgs-continuous injection and Y is fuzzy Urysohn, then X is fuzzy θgs-T_2.

Proof: Let Y is fuzzy Urysohn. By the injectivity of f, it follows that $f(x) \neq f(y)$ for any distinct fuzzy singletons x and y in X. Since Y is fuzzy Urysohn, then there exist fuzzy open sets U and V such that $f(x) \in U$ and $f(y) \in V$ and $Cl(U) \wedge Cl(V) = \phi$. Since f is fuzzy almost contra θgs-continuous, then there exist fuzzy open sets W and Z in X containing x and y, respectively, such that $f(W) \leq Cl(U)$ and $f(Z) \leq Cl(V)$. Hence $W \wedge Z = \phi$. This shows that X is fuzzy θgs-T_2.

Definition 4.10: A fuzzy space X is said to be fuzzy weakly T_2 [4] if each element of X is an intersection of fuzzy regular closed sets.

Theorem 4.11: If f: X → Y is a fuzzy almost contra θgs-continuous injection and Y is fuzzy weakly T_2, then X is fuzzy θgs-T_1.

Proof: Suppose that Y is fuzzy weakly T_2. For any distinct points x and y in X, there exist fuzzy regular closed sets U , V in Y such that $f(x) \in U$, $f(y) \notin U$, $f(x) \notin V$ and $f(y) \in V$. Since f is fuzzy almost contra θgs-continuous, by Theorem 3.2(ii), $f^{-1}(U)$ and $f^{-1}(V)$ are Fθgs-open subsets of X such that $x \in f^{-1}(U)$, $y \notin f^{-1}(U)$ and $x \notin f^{-1}(V)$, $y \in f^{-1}(V)$. This shows that X is fuzzy θgs-T_1

Definition 4.12: The fuzzy graph G(f) of a fuzzy function f : X → Y is said to be fuzzy strongly contra-θgs-closed if for each $(x, y) \in (X \times Y) - G(f)$, there exist a fuzzy θgs-open set U in x containing x and a fuzzy regular closed set V in Y containing y, such that $(U \times V) \wedge G(f) = \phi$.

Lemma 4.13: The following properties are equivalent for the fuzzy graph G(f) of a fuzzy function f:

(i) G(f) is fuzzy strongly contra-θgs-closed.

(ii) For each $(x, y) \in (X \times Y)-G(f)$, there exist a Fθgs-open set U in X containing x and a fuzzy regular closed set V containing y such that $f(U) \wedge V = \phi$.

Theorem 4.14: If f: X → Y is fuzzy almost contra θgs-continuous and Y is fuzzy Urysohn, G(f) is fuzzy strongly contra-θgs-closed set in X x Y.

Proof: Let Y is fuzzy Urysohn. Let $(x, y) \in (X \times Y) - G(f)$. It follows that $f(x) \neq y$. Since Y is fuzzy Urysohn, then there exist fuzzy open sets U and V such that $f(x) \in U$, $y \in V$ and $Cl(U) \wedge Cl(V) = \phi$. Since f is fuzzy almost contra θgs-continuous, then there exists a fuzzy θgs-open set μ in X containing x such that $f(\mu) \leq Cl(U)$. Therefore, $f(\mu) \wedge Cl(V) = \phi$ and G(f) is fuzzy strongly contra-θgs-closed in (X x Y).

Theorem 4.15: Let f: X → Y is fuzzy strongly contra-θgs-closed graph. If f is injective, then X is fuzzy θgs-T_1.

Proof: Let x and y be any two distinct points of X. Then, we have $(x, f(y)) \in (X \times Y) - G(f)$. By Lemma 4.13, there exist a Fθgs-open set μ containing x and a fuzzy regular closed set η in Y containing f(y) such that $f(\mu) \wedge \eta = \phi$;hence $\mu \wedge f^{-1}(\eta) = \phi$. Therefore, we have $y \notin \mu$. This implies that X is fuzzy θgs-T_1.

V. FUZZY θ-GENARALIZED –SEMI COMPACTNESS

In this section we introduce Fuzzy θ-generalized semi-compact space and the relationship between fuzzy almost contra θgs-continuous functions and the other forms are investigated.

Definition 5.1:.A space X is said to be fuzzy θgs-compact if every fuzzy θgs-open cover of X has a finite subcover.

Definition 5.2:.A space X is said to be fuzzy θgs-closed-compact if every fuzzy θgs-closed cover of X has a finite subcover.

Definition 5.3[7]: A space X is said to be fuzzy nearly compact if every fuzzy regular open cover of X has a finite subcover.

Theorem 5.4: The fuzzy almost contra-θgs-continuous images of fuzzy θgs-closed-compact spaces are fuzzy nearly compact.

Proof: Suppose that f: X → Y is a fuzzy almost contra-θgs-continuous surjection. Let $\{\eta_i : i \in I\}$ be any fuzzy regular open cover of Y. Since f is fuzzy almost contra-θgs-continuous, then $\{f^{-1}(\eta_i): i \in I\}$ fuzzy is a fuzzy θgs-closed cover of X. Since X is fuzzy θgs-closed-compact, there exists a finite subset I_o of I such that $X = \Lambda \{f^{-1}(\eta_i: i \in I_o\}$. Thus, we have $Y = \Lambda \{\eta_i : i \in I_o\}$ and Y is nearly compact.

Definition 5.5: A fuzzy function f: X → Y is called fuzzy weakly almost contra-θgs-continuous if for each x ∈ X and each fuzzy regular closed set η of Y containing f(x), there exists Fθgs-open set μ in X containing x, such that int(f(μ)) ≤ η.

Definition 5.6: A fuzzy function f: X → Y is called fuzzy (θgs, s)-open if the image of each Fθgs-open set is Fs-open.

Theorem 5.7: If fuzzy function f: X → Y is fuzzy weakly almost contra-θgs-continuous and fuzzy (θgs, s)-open, then f is fuzzy almost contra-θgs-continuous.

Proof: Let x ∈ X and η be a fuzzy regular closed set containing f(x). Since f is fuzzy weakly almost contra-θgs-continuous, there exists a Fθgs-open set μ in X containing x such that Int(f(μ)) ≤ η. Since f is fuzzy (θgs, s)-open, f(μ) is a Fs-open set in Y and f(μ) ≤ Cl(Int(f(μ))) ≤ η. This shows that f is fuzzy almost contra-θgs-continuous.

Definition 5.8[4]: A fuzzy space is said to be fuzzy P_α if for any fuzzy open set μ of X and each $x_\alpha \in \mu$, there exists fuzzy regular closed set ρ containing x_p such that $x_p \in \rho \leq \mu$.

Theorem 5.9: Let f: X → Y ba a fuzzy function. Then, if f is fuzzy almost contra-θgs-continuous and Y is fuzzy P_α, then f is fuzzy almost contra-θgs-continuous.

Proof: Let μ be a fuzzy open set in Y. Since Y is fuzzy P_α, there exists a family Ψ whose members are fuzzy regular closed set of Y such that $\mu = \Lambda \{ \rho : \rho \in \Psi \}$. Since f is fuzzy almost contra-θgs-continuous, $f^{-1}(\rho)$ is Fθgs-open in X for each $\rho \in \Psi$ and $f^{-1}(\mu)$ is Fθgs-open in X. Therefore, f is fuzzy almost contra-θgs-continuous.

Definition 5.10 [4]: A fuzzy space is said to be fuzzy weakly P_α if for any fuzzy regular open set μ of X and each $x_\alpha \in \mu$, there exists fuzzy regular closed set ρ containing x_p such that $x_p \in \rho \leq \mu$.

Definition 5.11: A fuzzy function f : X → Y is said to be fuzzy almost θgs-continuous at $x_p \in \mu$ if for each fuzzy open set η containing $f(x_p)$, there exists a Fθgs-open set μ containing x_p such that $f(\mu) \leq$ Int (Cl(η)).

Theorem 5.12: Let f: X →Y be a fuzzy almost contra-θgs-continuous function. If Y is fuzzy weakly P_α then f is fuzzy almost θgs-continuous.

Proof: Let μ be any fuzzy regular open set of Y. Since Y is fuzzy weakly P_α, there exists a family Ψ whose members are fuzzy regular closed set of Y such that $\mu = \Lambda \{ \rho : \rho \in \Psi \}$. Since f is fuzzy almost contra-θgs-continuous, $f^{-1}(\mu)$ is fuzzy θgs-open in X. Hence, f is fuzzy almost θgs-continuous.

Theorem 5.13: Let X, Y, Z be fuzzy topological spaces and let f: X → Y and g: Y→ Z be fuzzy functions. If f is f- θgs-irresolute and g is fuzzy almost contra-θgs-continuous, then g o f: X → Z is a fuzzy almost contra-θgs-continuous function.

Proof: Let μ ≤ Z be any fuzzy regular closed set and let (g o f) $(x_p) \in \mu$. Then $g(f(x_p)) \in \mu$. Since g is fuzzy almost contra-θgs-continuous fuction, it follows that there exists a Fθgs-open set ρ containing $f(x_p)$ such that $g(\rho) \leq \mu$. Since f is f-θgs-irresolute function, it follows that there exists a Fθgs-open set η containing x_p such that $f(\eta) \leq \rho$. From here we obtain that (g o f) (η) = g(f(η)) ≤ g(ρ) ≤ μ. Thus we have shown that (g o f) is fuzzy almost contra-θgs-continuous function.

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