On sgp-Closed and sgp-Open Functions in Topology

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Abstract: *We continue the study on sgp-closed set [12] , by introducing the new classes of functions called sgp-closed and sgp-open functions in topological spaces and some of their properties are discussed by giving counter examples wherever necessary.*

Keywords: sgp-closed set, sgp-open set, sgp-closed function and sgp-open function, sgp-homeomorphism

I. INTRODUCTION

In 1970, Levine [6] initiated the study of generalized closed(g-closed) sets, that is , a subset A of a topological space X is g-closed if the closure of A included in every open superset of A and defined a $T_{1/2}$ space to be one in which the closed sets and g-closed sets coincide. The notion has been studied extensively in recent years by many topologists.

In 1982, Malghan [9] introduced and studied the concept of generalized closed functions. After that several topologists introduced and studied α-open functions, gp-closed functions, gs-closed functions, gpr-closed functions and ω-open functions.

The concept of homeomorphism has been generalized by many topologists. Crossley and Hildebrand [2] have introduced and studied semi-homeomorphisms which are strictly weaker than homeomorphisms in topological spaces. Maki et al [7] have introduced and studied g-homeomorphisms and gc-homeomorphisms in topological spaces. Recently many researchers like Devi [3], Gnanambal [4], Sheik John [16] have introduced and investigated several types of homeomorphisms in topological spaces.

Recently Navalagi and Mahesh Bhat [12] introduced the notion of sgp-closed set utilizing pre-closure operator. The notions of sgp-open sets, sgp-continuity are introduced in [12]. In this paper we continue the study of sgp-closed sets, with introducing and characterizing sgp-open and sgp-closed functions. We have also introduced sgp-homeomorphism and studied some of its basic properties

II. PRELIMINARIES

In the entire paper (X,τ) and (Y,σ) (or X and Y) represents topological spaces on which no separation axioms are assumed unless explicitly stated. If A is any subset of space X, then Cl(A) and Int(A) denote the closure of A and the interior of A in X respectively.

We use some definitions:

Definition 2.1: A subset A of space X is called
(i) a semi-open set [5] if $A \subseteq Cl(Int(A))$
(ii) a semi-closed set [2] if $Int(Cl(A)) \subseteq A$
(iii) pre-open [10] , if $A \subseteq Int(Cl(A)))$
(iv) α-open set [14] if $A \subseteq Int(Cl(Int(A)))$
(v) ω-closed [17] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.
The complements of these sets are their respective closed sets in the space X.

Definition 2.2 [10]: The pre closure of a subset A of X is the intersection of all pre-closed sets containing A in X and is denoted by pCl(A).

Definition 2.3: A subset A of a space X is called
(i) generalized-closed (in brief, g-closed) set [6] if $Cl(A) \subseteq U$ and U is open in X. The complement of g-closed set is g-open set.
(ii) semi generalized pre closed (in short, sgp-closed) set [12] if $pCl(A) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in X. The complement of sgp-closed set is sgp-open set and the family of all sgp-open sets of X is denoted by SGPO(X).

Definition 2.4[12]: A space X is said to be ${}_{sgp}T_c$-space if every sgp-closed set is closed set in it.

Definition 2.5: A function $f: X \to Y$ is called
(i) ω-open [16] if f(G) is ω-open in Y for every open set G in X.

(ii) gp-closed [15] if f(A) is gp-closed in Y for every closed set A in X.

(iii) gpr-open [4] if f(E) is gpr-open in Y for every open set E in X.

(iv) α*-homeomorphism [3] if f and f^{-1} are α-irresolute functions.

(v) generalized homeomorphism (in short, g-homeomorphism) [7] if f is both g-continuous and g-open.

III. PROPERTIES OF sgp-CLOSED AND sgp-OPEN FUNCTIONS

Definition 3.1: A function f: X → Y is called semi-generalized pre-open (in short, sgp-open) function if the image of every open set in X is sgp-open in Y.

Definition 3.2: A function f: X → Y is called semi-generalized pre-closed (briefly, sgp-closed) function if the image of every closed set in X is sgp-closed in Y.

Theorem 3.3: Every open function is sgp-open function but not conversely.
Proof: Assume f: X → Y to be an open function. Let G be an open set in X. Then f(G) is open in Y. And therefore f(G) is sgp-open in Y. Hence f is sgp-open function.

Example 3.4: Let X = {p, q, r}, τ = {X, ϕ, {p}, {p, q}} and σ = {Y, ϕ, {p},{q, r}}. Then the identity function f: X → Y is not an open function, since for the open set {p, q} in X, f({p, q}) = {p, q} is not an open in Y. However f is sgp-open function.

Theorem 3.5: If f: X → Y is sgp-open function and Y is a $_{sgp}T_c$-space, then f is an open function.

Proof: Let K be an open set in X. Then f(K) is sgp-open in Y, since f is sgp-open function. Then f(K) is open in Y as Y is $_{sgp}T_c$-space. Hence f is open function.

Theorem 3.6: Every α-open map is sgp–open but the converse is need not be true.
Proof: Let f: X → Y be an α-open map. Let G be an open set in X. Then f(G) is α-open set in Y. Therefore f(G) is sgp -open in Y. Hence f is sgp -open.

Example 3.7: Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {b}. {a, b}} and σ = {Y, ϕ, {a}, {b, c}}. Let f: X → Y be the identity map. Then f is sgp -open but not α-open, since for the open set {b} in X, f({b}) = {b} is not α-open in Y but it is sgp -open in Y.

Theorem 3.8: If f: X → Y is sgp-open and Y is $_{sgp}T_c$ -space, then f is α-open map.
Proof: Let E be an open set in X. Then f(E) is sgp -open in Y as f is sgp -open. Since Y is $_{sgp}T_c$ -space, f(E) is open and so α-open in Y. Hence f is α-open map.

Theorem 3.9: Every ω-open function is sgp-open but the converse need not be true as seen from the following example.
Proof: Let f: X → Y be an ω-open function. Let G be an open set in X. Then f(G) is ω-open set in Y. Therefore f(G) is sgp -open in Y. Hence f is sgp -open.

Example 3.10: Let X = Y = {p, m, n}, τ = {X, ϕ, {p}, {p, m}} and σ = {Y, ϕ, {p}}. Let f: X → Y be the identity function. Then f is sgp -open but not ω-open, since for the open set {p, m} in X, f({p, m}) = {p, m} is not ω-open in Y but it is sgp -open in Y.

Theorem 3.11: If a function f: X → Y is sgp-open, then f is gp-open but not conversely.
Proof: Let f: X → Y is a sgp –open function. Let H be an open set in X. Then f(H) is sgp-open in Y. As every sgp-open set is gp-open, therefore f(H) is gp-open. Hence f is gp-open.

Example 3.12: Let X = {a, b, c} = Y, τ = {X, ϕ, {a}, {b}, {a, b}} and σ = {Y, ϕ, {a}}. Let f: X→Y be the identity function. Then f is gp-open but not sgp -open, since for the open set {b} in X, f({b}) = {b} is not sgp -open in Y but it is gp-open in Y.

Theorem 3.13: If f: X → Y is sgp-open function, then f is gpr-open.
Proof: Let G be an open set in X. Then f(G) is sgp -open in Y. Therefore f(G) is gpr -open. Hence f is gpr –open function.

Remark 3.14: The following example makes clear that converse of the above theorem need not be true.

Example 3.14: Let X = {a, b, c} = Y, τ = {X, ϕ, {a}, {b}, {a, b}} and σ = {Y, ϕ, {a}}. Let f: X → Y is the identity function. Then f is gpr-open but not sgp -open, since for the open set {b} in X, f({b}) = {b} is not sgp -open in Y but it is gpr-open in Y.

Theorem 3.15: Every sgp-open function is gsp-open function, but not conversely.
Proof: Let f: X → Y be a sgp-open function. Let W be an open set in X. Then f(W) is sgp -open in Y. Therefore f(W) is gsp -open. Hence f is gsp-open.

Example 3.16: Let X = {a, b, c} = Y, τ = {X, ϕ, {a}, {b, c}} and σ = {Y, ϕ, {a}, {b}, {a, b}}. Let f: X → Y be the identity function. Then f is gsp-open but not sgp-open, since for the open set {b, c} in X, f({b, c}) = {b, c} is not sgp -open in Y but it is gsp-open in Y.

Theorem 3.17: Every closed function is sgp-closed function but the converse need not be true.
Proof: Let f: X → Y is a closed function. Let Q be a closed set in X. Then f(Q) is closed in Y. Therefore f(Q) is sgp-closed in Y. Hence f is sgp-closed function.

Example 3.18: Let X = {a, b, c} = Y, τ = {X, ϕ, {a}, {b}, {a, b}} and σ = {Y, ϕ, {a}}. Let f: X → Y is the identity function. Then f is sgp-closed but not closed function as the closed set {c} in X, f({c}) = {c} is sgp-closed set but not closed set in Y.

Theorem 3.19: If f: X $\rightarrow$ Y is sgp-closed function and Y is $_{sgp}T_c$-space, then f is a closed map.
Proof: Let f: X $\rightarrow$ Y is a sgp-closed function. Let G be a closed set in X. Then f(G) is sgp-closed set in Y. Since Y is $_{sgp}T_c$-space, f(G) is closed set in Y. Hence f is a closed map.

Corollary 3.20: (i) If f: X$\rightarrow$Y is sgp-closed function, then f is gp-closed function.
(ii) Converse of (i) is true if Y is $_{sgp}T_c$ -space.

(iii) If f: X $\rightarrow$ Y is sgp-closed function, then f is gp-closed function.
(iv) Converse of (iii) is true if Y is $_{sgp}T_c$ -space.

Theorem 3.21: A function f: X $\rightarrow$ Y is sgp-closed if and only if for each subset S of Y and for each open set U containing $f^{-1}(S)$ there is a sgp-open set V of Y such that S $\subseteq$ V and $f^{-1}(V) \subseteq U$.
Proof: Suppose f is sgp-closed. Let S be a subset of Y and U be an open set of X containing $f^{-1}(S)$, that is $f^{-1}(S) \subseteq U$. Now X – U is closed in X. Then f(X – U) is sgp-closed set in Y since f is sgp-closed function. Then V = Y – f(X – U) is a sgp -open set containing S such that $f^{-1}(V) \subseteq U$.

Conversely suppose that F is a closed set of X. Then $f^{-1}(Y–(f (F)) \subseteq X – F$ and X – F is open set. By hypothesis, there is a sgp-open set V of Y such that Y – f(F) $\subseteq$ V and $f^{-1}(V) \subseteq X – F$. Therefore $F \subseteq X – f^{-1}(V)$. Hence $Y–V \subseteq f(F) \subseteq f(X – f^{-1}(V)) \subseteq Y – V$ which implies f(F) = Y – V. Since Y – V is sgp-closed set, f(F) is sgp-closed set and thus f is sgp -closed function.

Remark 3.22: The composition of two sgp-closed functions need not be sgp-closed function as seen from the following example.

Example 3.23: Let X = Y = Z = {a, b, c}, τ = {X, ϕ, {a}, {a, c}, X}, σ = {Y, ϕ, {a}} and η = {Z, ϕ, {b}, {a, b}}. Let f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z as identity maps. Then f and g are both sgp-closed maps but the composition gof: X $\rightarrow$ Z is not a sgp-closed function, since for the closed set {b} in X, (gof)({b}) = g(f({b}) = g({b}) = {b} is not sgp-closed set in Z.

Theorem 3.24: Composition of closed function and sgp-closed function is again sgp-closed function.
Proof: Let f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z are sgp-closed functions. Let F be a closed set in X. Then f(F) is closed in Y since f is a closed map. Again since g is sgp-closed, g(f(F)) is sgp-closed in Z. That is g(f(F)) = (gof)(F) is sgp-closed in Z. Hence gof is sgp-closed function.

Theorem 3.25: If f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z are sgp-closed functions and Y is $_{sgp}T_c$-space, then gof: X $\rightarrow$ Z is sgp-closed.

Proof: Let F be a closed set in X. Then f(F) is sgp-closed set in Y as f is sgp-closed function. Since Y is $_{sgp}T_c$-space, f(F) is closed set in Y. Again since g is sgp-closed function, g(f(F)) is sgp-closed set in Z. That is g(f(F)) = (gof)(F) is sgp-closed set in Z. Hence gof is sgp-closed function.

Theorem 3.26: If f: X $\rightarrow$ Y is sgp-closed g: Y $\rightarrow$ Z is gp-closed and Y is $_{sgp}T_c$-space, then gof: X $\rightarrow$ Z is is gp-closed function.
Proof: Let O be a closed set in X. Then f(O) is sgp-closed set in Y as f is sgp-closed function. Since Y is $_{sgp}T_c$-space, f(O) is closed set in Y. Also since g is gp-closed, g(f(O)) is gp-closed set in Z. That is g(f(O)) = (gof)(O) is gp-closed set in (Z, γ). Hence gof is gp-closed function.

Theorem 3.27: For f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z two functions
(i) If gof: X $\rightarrow$ Z is sgp-closed function and f: X $\rightarrow$ Y is continuous and surjection, then g is sgp-closed function.
(ii) If gof: X $\rightarrow$ Z is sgp-closed function and g is sgp-irresolute and injective, then f is sgp-closed function.
(iii) If gof: X $\rightarrow$ Z is sgp-closed function and f is gp-continuous, surjective and X is $_{sgp}T_c$-space, then g is sgp-closed function..

Proof: (i) Let K be a closed set in Y. Then $f^{-1}(K)$ is closed in X since f is continuous. So $(gof)(f^{-1}(K))$ is sgp-closed set in Z as gof is sgp-closed function. That is $(gof)(f^{-1}(K)) = g(f(f^{-1}(K)) = g(K)$ is sgp-closed set in Z. Hence g is sgp-closed function.

(ii) Let A be a closed set in X. Since gof is sgp-closed, (gof)(A) is sgp-closed in Z. Since g is sgp-irresolute $g^{-1}(gof)(A)$ is sgp-closed set in Y. Since g is injective $g^{-1}(gof)(A) = f(A)$ is sgp-closed set in Y. Hence f is sgp-closed function.

(iii) Let N be a closed set in Y. Since f is gp-continuous, $f^{-1}(N)$ is gp-closed set in X. Again since gof is sgp-closed function, $(gof)(f^{-1}(N)) = g(N)$ is sgp-closed set in Z. Hence g is sgp-closed function.

Theorem 3.28: Let (X,τ) and (Y,σ) be any topological spaces. If f: (X,τ) $\rightarrow$ (Y,σ) is sgp-closed function and A is closed set of (X,τ), then its restriction f_A: (A,τ_A) $\rightarrow$ (Y, σ) is sgp-closed function.
Proof: Let B be a closed of (A, τ_A). Then B = A $\cap$ F for some closed set F of (X, τ) and so B is closed in (X, τ). Since f is sgp-closed function, f(B) is sgp-closed set in (Y, σ). But f(B) = f_A(B) and therefore f_A is a sgp-closed function.

Theorem 3.30: For any bijection f: X→ Y the following statements are equivalent:
(i) f^{-1}: Y→X is sgp-continuous
(ii) f is sgp-open function.
(iii) f is sgp-closed function.
Proof: (i) ⇒ (ii): Let K be an open set of X. By (i), $(f^{-1})^{-1}(K)$ = f(K) is sgp-open in Y and so f is sgp-open function.
(ii) ⇒ (iii): Let G be a closed set of X. Then X – G is open set of X. By hypothesis f(X– G) is sgp-open in Y. That is f(X– G) = Y – f(G) is sgp-open set in Y and therefore f(G) is sgp-closed set in Y. Hence f is sgp-closed function.

(iii) ⇒(i): Let D be closed set of X. Then by hypothesis f(D) closed set in Y. But f(D) = $(f^{-1})^{-1}$(D) is sgp-closed set in Y. Therefore f^{-1} is sgp-continuous function.

Theorem 3.31: A function f: X → Y is sgp-open if and only if for any subset S of Y and for any closed set F containing f^{-1}(S), there exists sgp-closed set K of Y containing S such that $f^{-1}(K) \subseteq F$.
Proof: Suppose f: X → Y is sgp-open function. Let S be a subset of Y and F be a closed set of X containing f^{-1}(S). Then K = Y – f(X- F) is a sgp-closed set containing S such that $f^{-1}(K) \subseteq F$.
Conversely, suppose that U is an open set of X. Then $f^{-1}(Y- f(U)) \subseteq X - f^{-1}[f(U)] \subseteq X - U$ and X – U is closed. By hypothesis, there is a sgp-closed set K of Y such that Y-f(U) ⊆ K and $f^{-1}(K) \subseteq X - V$.
Therefore $U \subseteq X - f^{-1}(K)$. Hence Y- K ⊆ f(U) ⊆ f $[X - f^{-1}(K)] \subseteq$ Y-K. Which implies f(U) ⊆ Y – K . Since Y – K is sgp-open set, f(U) is sgp-open set and hence f is sgp-open function.

IV. sgp-HOMEOMORPHISMS IN TOPOLOGICAL SPACES

In this section we introduce the concept of sgp-homeomorphisms in topological spaces and obtained some of their properties.

Definition 4.1: A bijective function f: X→Y is called sgp-homeomorphism if f and f^{-1} are sgp-continuous functions.

Example 4.2: Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {a, c} } and σ = {Y, ϕ, {a, b}}. Define a function f: X→Y by f(a)= a, f(b)= c and f(c) = b. Then the function f is bijective, sgp- continuous and sgp-open function. Hence f is a sgp-homeomorphism.

Theorem 4.3: Every homeomorphism is a sgp-homeomorphism, but not conversely.
Proof: Let X→Y be a homeomorphism. Then f and f^{-1} are continuous functions. Since every continuous function is sgp-continuous Therefore f and f^{-1} are sgp-continuous functions. Hence f is sgp-homeomorphism.

Example 4.4: In Example 4.2, the function f is sgp-homeomorphism but not a homeomorphism.

Theorem 4.5: If f: X→ Y is ω-homeomorphism then f is sgp-homeomorphism but the converse is not true.
Proof: Let f:X→Y is ω-homeomorphism. Then f and f^{-1} are ω-continuous functions. Since every ω-continuous is sgp-continuous. Therefore f and f^{-1} are sgp-continuous functions. Hence f is sgp-homeomorphism.

Example 4.6: Let X = Y = {x, y, z}, τ = {X,ϕ, {x}} and σ = {Y, ϕ, {x}, {x, y}}. Let f: X→ Y be a identity function. Then f is sgp-homeomorphism but not a ω-homeomorphism, since for the closed set {z} in Y, f^{-1}({z}) = {z} is not ω-closed in X but it is sgp-closed in X.

Theorem 4.7: Every α*-homeomorphism is sgp-homeomorphism but not conversely.
Proof: Suppose f: X→Y is α*-homeomorphism. Then f and f^{-1} are α-continuous functions. Since every α-continuous function is sgp-continuous function, if follows that, f and f^{-1} are sgp-continuous functions. Hence f is sgp-homeomorphism.

Example 4.8: Let X = {p, q, r} = Y, τ = {X, ϕ, {p}, {q, r}} and σ = {Y, ϕ, {p}, {q}, {p, q}}. Then the identity function f: X→ Y is sgp-homeomorphism but not α*-homeomorphism, since for the closed set {r} in Y, f^{-1}({r}) = {r} is not α-closed in X but it is sgp-closed in X.

Remark 4.9: The concepts of sgp-homeomorphisms and g-homeomorphisms are independent of each other as seen from the following examples.

Example 4.10: Let X = {a, b, c} = Y, τ = {X, ϕ, {a}, {a, c}} and σ = {Y, ϕ, {a}, {a, b}}. Then the identity function f: X→Y is sgp-homeomorphism but not g-homeomorphism, since for the open set {a, c} in X, f({ a, c}) = {a, c} is not g-open in Y but it is sgp-open in Y

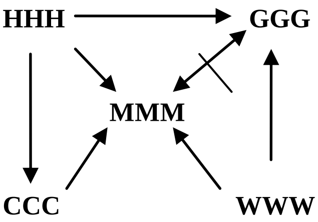
Example 4.11: Let X = {a, b, c} = Y, τ = {X, ϕ, {a}} and σ = {Y, ϕ, {b}}. Define a map f : X→ Y by f(a) = c, f(b) = a and f(c) = b. Then the function f is g-homeomorphism but not a sgp-homeomorphism, since for the open set {a} of X, f ({a}) = {c} is not a sgp-open in Y but it is g-open in Y.

Theorem 4.12: Let f: X→ Y be a bijective and sgp-continuous function. Then the following statements are equivalent.

(i) f is sgp-open.
(ii) f is sgp-homeomorphism.
(iii) f is sgp-closed

Proof: The proof follows from Theorem 3.30.

Remark 4.13: From the above results we have the following diagrams



Where

HHH--- homeomorphism, GGG--- g-homeomorphism

CCC---α*-homeomorphism, MMM-- sgp-homeomorphism

WWW--ω-homeomorphism

Remark 4.14: The composition of two sgp-homeomorphisms need not be a sgp-homeomorphism as seen from the following example.

Example 4.15: Let X = Y = Z = {a, b, c}, τ = {ϕ, {a}, {b}, {a, b}, X}, σ = {ϕ, {a}, Y) and η = {ϕ, {a}, {a, b}, Z}. Define functions f : (X, τ) → (Y, σ) by f(a) = b, f(b) = a and f(c) = c and g : (Y, σ) → (Z, η) is defined as g(a) = b, g(b) = c and g(c) = a. Then f and g are both sgp-homeomorphisms but the composition gof is not a sgp-homeomorphism, since for the open set {a, b} in (Z, η), $(gof)^{-1}$({a, b}) = $f^{-1}(g^{-1}$({a, b})) = f^{-1}({a, c}) = {b, c} is not an sgp-open in (X,τ).

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