MATLAB Programming for Simulation of an SIR Deterministic Epidemic Model

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ABSTRACT- *In this paper, we consider a deterministic SIR epidemic model with the goal of disclosing a simulation method, a mathematical model was implemented in MATLAB function that allows simulating the spread of an infection disease. The graphical interface shown in this paper is performed using the MATLAB software version 7.10.0.*

Key words: *Epidemiology, simulation.*

2010 Mathematics Subject Classification: *92D30, 92D25, 93A30, 65L07.*

I. INTRODUCTION

In the deterministic epidemic model, individuals in the population are assigned to different subgroups or compartments, these compartments are defined with respect to disease status of the epidemic. The simplest model, which was described by Kermack and McKendrick [3] in 1927, consists of three compartments: susceptible (S) , infected (I) and recovered (R) .

Susceptible (S) : - Individuals those who have never had the illness and can catch it.

Infected (I) : - Individuals those who currently have the illness and are contagious.

Recovered (R) : - Individuals those who have already had the illness and are immune.

So, the total population $(N) = (S) + (I) + (R)$.

The SIR model is easily written using ordinary differential equations (ODEs), which imply a deterministic model with continuous time. In this paper, we consider a SIR epidemic model with non-monotonic incidence rate proposed by [4] with the initial conditions: $S(0) = S_0 > 0$, $I(0) = I_0 > 0$, $R(0) = R_0 > 0$.
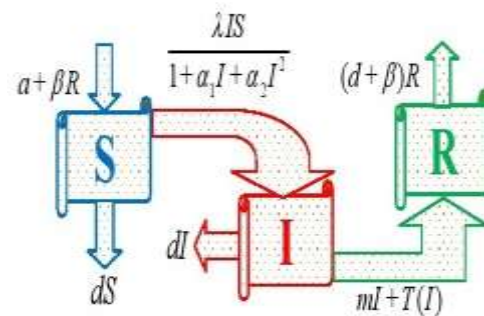
II. SIR EPIDEMIC MODEL

Fig.1: The Model Diagram

$$\left.\begin{aligned}
\frac{dS}{dt} &= a - dS - \frac{\lambda IS}{1 + \alpha_1 I + \alpha_2 I^2} + \beta R \\
\frac{dI}{dt} &= \frac{\lambda IS}{1 + \alpha_1 I + \alpha_2 I^2} - (d + m)I - T(I) \\
\frac{dR}{dt} &= mI - (d + \beta)R + T(I)
\end{aligned}\right\} \quad (1)$$

where, a is a recruitment rate of the population, d is the natural death rate of the population, λ is the proportionality constant, β is the rate at which recovered individuals lose immunity and return to susceptible class, m is the natural recovery rate of the infective individuals, α_1 and α_2 are the parameter measures of the psychological or inhibitory effect, the incidence rate $\frac{\lambda SI}{1 + \alpha_1 I + \alpha_2 I^2}$, where λI measures the infecting force of the disease and $\frac{1}{1 + \alpha_1 I + \alpha_2 I^2}$ describes the psychological or inhibitory effect of the behavioral change to the susceptible individuals when the number of infective individuals is very large. The treatment rate of a disease is
$$T(I) = \begin{cases} rI, & 0 \le I \le I_0, \\ K_1, & I > I_0. \end{cases}$$ proposed by Wang [5].

III. THE SIR MODEL AS FUNCTION IN MATLAB.

Case I. When the treatment rate is proportional to the infective so that $0 < I \le I_0$, choose the

parameters in the model as follows: $a = 3$, $d = 0.1$, $\lambda = 0.3$, $\beta = 0.1$, $m = 0.01$, $\alpha_1 = 1$, $\alpha_2 = 0.1$, $r = 0.2$ .The basic reproduction number on above

parameters is $R_0 = \dfrac{\lambda a}{d(d+m+r)} = 29.0322 > 1$.

Write the following in Function editor.

function SIR_01(S0, I0, R0, a, d, lambda, beta, m, alpha1, alpha2, r, tmax)

```
S=zeros(1,tmax);
I=zeros(1,tmax);
R=zeros(1,tmax);
S(1)=S0;
I(1)=I0;
R(1)=R0;
T(1)=0;

for t=1:1:tmax
T(t+1)=t;
S(t+1)=S(t)+(a-d*S(t)-
(lambda*S(t)*I(t)/(1+alpha1*I(t)+
alpha2*I(t)^2))+beta*R(t));
I(t+1)=I(t)+((lambda*S(t)*I(t)/(1+alpha1*I(t)+alph
a2*I(t)^2))-(d+m+r)*I(t));
R(t+1)=R(t)+(m*I(t)-(d+beta)*R(t)+r*I(t));

End
plot(T,S,T,I,T,R);
title('SIR Model');
legend('S(t)','I(t)','R(t)');
xlabel('time, t');
ylabel('Population');
```

Write the following in command window:
>> SIR_01(S0, I0, R0, a, d, lambda, beta, m, alpha1, alpha2, r, tmax) with the initial values of S, I, R and value of all parameters; like as

SIR_01(20,15,10,3,0.1,0.3,0.1,0.01,1,0.1,0.2,200)
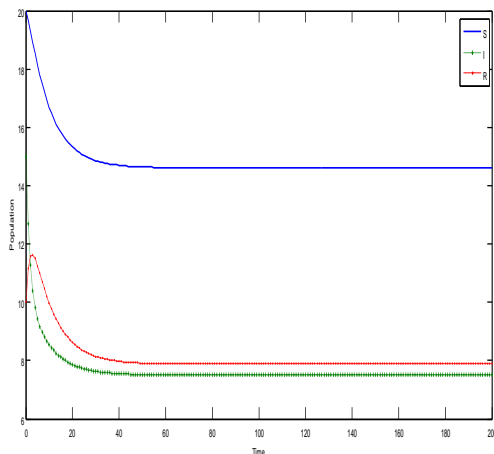
which yields: figure. 2

Fig. 2

Case II. When $I > I_0$, choose the parameters $a = 2.8$, $d = 0.0453$, $\lambda = 0.4$, $\beta = 0.13$, $m = 0.01$, $\alpha_1 = 1$, $\alpha_2 = 2$, $k1 = 0.2$. The basic reproduction number on above parameters is

$R_0 = \dfrac{\lambda a}{d(d+m+K_1)} = 96.84317 > 1$.

Write the following in Function editor.

function SIR_02(S0,I0,R0,a,d,lambda,beta,m, alpha1, alpha2,k1,tmax)

```
S=zeros(1,tmax);
I=zeros(1,tmax);
R=zeros(1,tmax);
S(1)=S0;
I(1)=I0;
R(1)=R0;
T(1)=0;

for t=1:1:tmax
T(t+1)=t;
S(t+1)=S(t)+(a-d*S(t)-
(lambda*S(t)*I(t)/(1+alpha1*I(t)+alpha2*I(t)^2))+
beta*R(t));
I(t+1)=I(t)+((lambda*S(t)*I(t)/(1+alpha1*I(t)+alph
a2*I(t)^2))-(d+m+r)*I(t));
R(t+1)=R(t)+(m*I(t)-(d+beta)*R(t)+k1*I(t));

End
plot(T,S,T,I,T,R);
title('SIR Model');
legend('S(t)','I(t)','R(t)');
xlabel('time, t');
ylabel('Population');
```

Write the following in command window:
>> SIR_01(S0, I0, R0, a, d, lambda, beta, m, alpha1, alpha2, k1, tmax) with the initial values of S, I, R and value of all parameters; like as

>> SIR_02(20,15,10,2.8,0.0453,0.4,0.13,0.01,1,2,0.2,200)
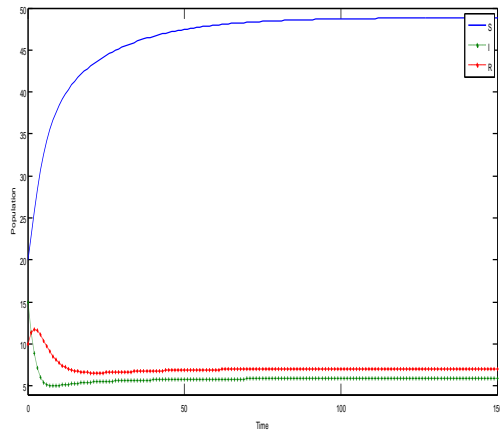
which yields: figure. 3

Fig. 2

IV. CONCLUSIONS

This MATLAB programming allows simulating and analyzing the dispersion of an SIR epidemic disease model with different parameters. It is valid for the teaching in a simple, affordable and practical way.

ACKNOWLEDGMENT

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