Completion Time in a Metagraph with Fuzzy Duration for Edges

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Abstract— *In this paper, it is supposed that a project can be presented as a Metagraph. In real world, activity durations are often indefinite. In the projects, nondeterministic times can be shown as random variables or fuzzy numbers. Here, we have assumed that the activity times are fuzzy numbers with discrete membership functions. In fuzzy Metagraphs, a new algorithm is developed to execute the forward computations. In forward computations, for defining the maximum number among all the fuzzy numbers, one of the ranking methods of fuzzy numbers is used. Finally, by using the forward computations, critical path(critical edges and critical elements) are defined. Also, completion time of Metagraph, is computed as a fuzzy number. Two examples are solved by using the proposed algorithm.*

Keywords— *Fuzzy Metagraph, Project Completion Time, Critical Path.*

I. Introduction

In the recent years, Metagraphs have been used as a tool to analyze the engineering systems. One of its applications is project planning and control. Management of a model using the Petri network and metagraphs has been illustrated in [1]. The metagraph and its features have been described in [2, 3]. Integration of models has been considered and it has been shown that the model integration has become much easier by applying some of the metagraph's properties [4].

The application of metagraphs in decision support systems has been shown in [5, 6]. Integrating the constraints in metagraphs has been described in [7].

Metagraphs are used in workflow management [8]. Metagraphs can also be used to model the interactions among workflow tasks and thus provide a comprehensive tool for modelling the various types of information managed by a DSS [9]. Constrained resource allocation in fuzzy metagraphs has been investigated in [10]. Constrained renewable resource allocation in fuzzy metagraph via Min-Slack has been studied in [11].

Completion time of stochastic metagraphs by sampling from edge time and using the conditional Monte-Carlo simulation have been developed in [12]. A new method for allocation of constrained Non-Renewable resource in fuzzy metagraphs has been described in [13].

The application of metagraphs in hierarchical modelling has been expressed in [14]. Metagraphs and some of their applications have been described in [15].

In this paper, the metagraph is considered as a tool for "project planning and control". The deterministic projects have been shown as metagraphs in [16], but in real world, the duration of project's activities are not deterministic. This uncertainty can be explained by either fuzzy numbers or random variables. Majority of recent researchers have defined the fuzzy characters for the projects, since the fuzzy models are closer to reality and simpler to use [17]. A fuzzy project can be shown as a metagraph with fuzzy characters. Forward and backward computations and determination of critical path and critical edges in fuzzy metagraphs have been described in [18] when the activity times are trapezoidal fuzzy numbers.

Ranking the fuzzy numbers is an essential tool in forward computations. Many studies have been conducted to rank the fuzzy numbers. An approximate approach for ranking fuzzy numbers based on left and right dominance is studied in [19]. The process of fuzzy numbers' ranking is considered in [20] in which the new ranking index is proposed not only for measuring the quantities but also incorporate the quantity factor into consideration for the need of general decision-making problems. For comparison of fuzzy subsets with a α-cut dependent index, a method is explained in [21].

In this paper, we intend to obtain the completion time of a project which has been shown with metagraphs. It is assumed that the execution time for each activity is expressed as fuzzy numbers with discrete membership function. To determine the maximum number among the fuzzy numbers, we use the fuzzy number ranking methods. Then critical path, critical activities and elements will be identified.

II. METAGRAPH

In this section, some basic definitions about metagraph cited in [11], are introduced.

A. *Metagraph Definition*

The finite set of elements of $X=\{x_i, i=1,2,\dots,I\}$is called a generating set. A metagraph on X is pairs of (X,E) that

$E = \{e_j, j = 1,2,\dots,J\}$ is a set of edges. In fact, e_j is an ordered pair as (v_j, w_j) that $v_j \subset x$ is called the invertex of e_j and $w_j \subset x$ is called the outvertex of e_j .in addition, for each $j = 1,2,\dots,J$ there must be $\forall j, V_j \cap W_j = \emptyset$.

To have a clear undrestanding of above mentioned concepts, fig. 1 is described as follows :

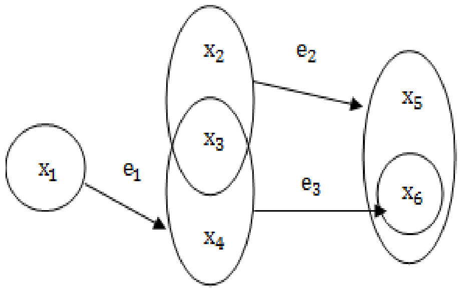

Fig. 1 A metagraph

$E = \{e_1, e_2, e_3\}$
$X = \{x_1, x_2, \dots, x_6\}$
$e_1 = (\{x_1\}, \{x_3, x_4\})$
$e_2 = (\{x_2, x_3\}, \{x_5, x_6\})$
$e_3 = (\{x_3, x_4\}, \{x_6\})$

B. Path

An element $x \in X$ is connected to element $x' \in X$ if a sequence of edges $(e'_k, k = 1,\dots,K')$ exist such that $x \in V'_1, x' \in W'_{k'}$ and $W'_k \cap V'_{k+1} \neq \emptyset, \forall k = 1,\dots,K' - 1$. This sequence of elements is called a simple path from x to x'. K' is the path length. Sometimes x is called Source and x' is called Target. The set of metagraph paths are shown with P.

C. Metapath

A metapath is the set of edges that is shown with $M(B, C)$. B is the source set and C is the target set. B is the invertex elements that are not also outvertex elements. C is the outvertex elements that are not also invertex elements.

Metagraph in Fig. 2 is a metapath with $B = \{x_1, x_2, x_4, x_5\}$ and $C = \{x_7\}$.

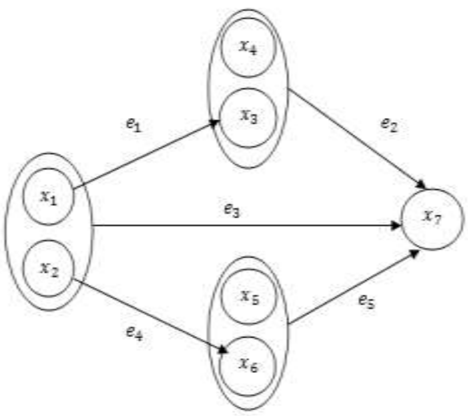

Fig. 2 A metagraph

III. Definition Related to the Fuzzy Set

Reference [22] introduces the operations on fuzzy numbers. Some of them are as follows.

A. Fuzzy Set

If X is a set of elements in which each element is shown with x, then the fuzzy set $\tilde{A}$ in X will be the set of ordered pairs as follows:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \epsilon X\}$$

$\mu_{\tilde{A}}(x)$ is the degree of membership or membership function of X in $\tilde{A}$.

B. Discrete Fuzzy Set

When the universe of discourse X is discrete and finite for a fuzzy set of $\tilde{A}$, it will be shown as the following:

$$\tilde{A} = \sum_{i=1}^{n} \frac{\mu_{\tilde{A}}(x_i)}{x_i}$$

C. Continuous Fuzzy Set

When the universe X is continuous and infinite, the fuzzy set $\tilde{A}$ is denoted by:

$$\tilde{A} = \int \frac{\mu_{\tilde{A}}(x_i)}{x_i}$$

D. Bounded Sum

The bounded sum $\tilde{C} = \tilde{A} \oplus \tilde{B}$ is defined as

$$\tilde{C} = \{(x, \mu_{\tilde{A}\oplus\tilde{B}}(x)) \mid x \in X\}$$

IV. RANKING FUZZY NUMBERS

It is not always necessary to change the fuzzy numbers to crisp numbers in a fuzzy state of decision making. We can rank the fuzzy sets instead and then make a decision. So the selected method is described in [19].

For a fuzzy number A, the $\alpha - Cuts$ (level sets) $A_\alpha = \{x \in \mathcal{R} | \mu_A(x) \geq \alpha\}, \alpha \in [0,1]$, are the convex set of $\mathcal{R}$. The lower and upper limits of the K^{th} $\alpha - cut$ for the fuzzy number of A_i are defined as :

$$l_{i,k} = inf\{x | \mu_A(x) \geq \alpha_k\}$$

$$r_{i,k} = sup\{x | \mu_A(x) \geq \alpha_k\}$$

Respectively, where $l_{i,k}$ and $r_{i,k}$ are left and right spreads. When two fuzzy numbers A_i and A_j are compared, the left (right) dominance $(D^R_{i,j})D^L_{i,j}$ of A_i over A_j is defined as the average difference of the left (right) spreads at some $\alpha - levels$. They are formulated as :

$$D_{i,j}^{L}=\frac{1}{n+1}\sum_{k=0}^{n}\left(l_{i,k}-l_{j,k}\right)$$

$$D_{i,j}^{R}=\frac{1}{1+n}\sum_{k=0}^{n}\left(r_{i,k}-r_{j,k}\right)$$

When $n+1,\alpha-Cuts$ are used to calculate the dominance, Let α_k denote the K^{th} $\alpha-level$, and $\alpha_k={}^{k}/_{n},\ k\in\{0,1,\dots,n\}$. Therefore, the distance between each to adjacent $\alpha-levels$ is equal: $\alpha_k-\alpha_{k-1}={}^{1}/_{n},\ K\geq 1$.

The total dominance of A_i over A_j with the index of optimism $\beta\in[0,1]$ can be defined as the convex combination of $D_{i,j}^{L}$ and $D_{i,j}^{R}$ by:

$$D_{i,j}(\beta)=\beta D_{i,j}^{R}+(1-\beta)D_{i,j}^{L}$$
$$=\beta\left[\frac{1}{n+1}\sum_{k=0}^{n}\left(r_{i,k}-r_{j,k}\right)\right]+(1-\beta)\left[\frac{1}{n+1}\sum_{k=0}^{n}\left(l_{i,k}-l_{j,k}\right)\right]$$

$$=\frac{1}{n+1}\left\{\left[\beta\sum_{k=0}^{n}r_{i,k}+(1-\beta)\sum_{k=0}^{n}l_{i,k}\right]-\left[\beta\sum_{k=0}^{n}r_{i,k}+(1-\beta)\sum_{k=0}^{n}l_{i,k}\right]\right\}$$

A decision maker can rank each pair of fuzzy numbers, A_i and A_j , using $D_{i,j}(\beta)$ based on the following rules:

1) If $D_{i,j}(\beta)>0$, then $A_i>A_j$;
2) If $D_{i,j}(\beta)=0$, then $A_i=A_j$;
3) If $D_{i,j}(\beta)<0$, then $A_i<A_j$.

This ranking can also be used for the fuzzy numbers with discrete membership function pay attention to the following example:

$$\tilde{A}=\{(4,1),(5,0.75),(6,0.5),(7,0.25)\}$$
$$\tilde{B}=\{(4,0.5),(5,1),(6,0.5)\}$$

Table 1. The lower and upper limits of $\boldsymbol{K^{th}}$ $\boldsymbol{\alpha-cut}$ *for the fuzzy numbers of* ***A*** *and* ***B***

α	0	0.25	0.5	0.75	1
$L_{A,K}$	4	4	4	4	4
$R_{A,K}$	7	7	6	5	4
$L_{B,K}$	4	4	4	5	5
$R_{B,K}$	6	6	6	5	5

The ranking result:

By substituting the numbers in the final formula, for $n=4,\beta=0.5$, there will be $A<B$.

For the ease of calculation, a program has been written in MATLAB. To increase the accuracy of fuzzy numbers ranking, we can increase the number of $\alpha-Cuts$. In order to obtain a more accurate and reliable answer, the written program ranks the set of fuzzy numbers in terms of the number of $\alpha-Cut$ and number of different βs.

V. CALCULATION OF CRITICAL PATH IN DETERMINISTIC METAGRAPH

Suppose that the duration of edges e_j is deterministic and known. It is shown by d_j. Then, by using the following algorithm [16], completion time of metagraph and its critical path and other specification such as the earliest and latest times of starting and finishing and critical edges can be determined.

A. Phase 1: Forward Computation

For each element $x_i\in B$ set $Q_i=0$ and mark it. For other elements, set $Q_i=0$ until $E=M(B,C)\neq\emptyset$ for each edges of e_j in E that (in which) invertex elements are marked, then set : $ES_j=max\{Q_i\}\ ,x_i\in v_j$

ES_j is the earliest start time of edge e_j.

For each $x_k\in W_j$ set $Q_k=max\{Q_k,(ES_j+d_j)\}$ and mark it, set $E\leftarrow E-\{e_j\}\leftarrow E=E/\{e_j\}$ then repeat the above steps until $E=\emptyset$, in this case T^e is obtained as earliest completion time of the metagraph. Set $T^e=T^l=max\{Q_i\}\ ,\forall\ x_i\in X$.

B. Phase 2 : Backward Computation

For each element $x_i\in C$ set $L_i=T^l$ and mark it. For other elements set $L_i=T^l$ until $E_0=M(B,C)\neq\emptyset$ for each edge e_j in E_0 , if all the elements in outvertex are marked. Set $LF_j=Min\{L_i\},x_i\in W_j$.

For each $x_k\in V_j$ put $L_k=min\{L_k,(LF_j-d_j)\}$ and mark it. Set $E_0\leftarrow E_0-\{e_j\}\leftarrow E_0=E_0/\{e_j\}$ and repeat the above steps. Then, we will have the ordered pair of (Q_i,L_i) for each element after execution of the algorithm.

VI. THE PROPOSED ALGORITHM FOR FUZZY METAGRAPH

Suppose that the execution time for each activity (edge) e_j is a fuzzy number with discrete membership function and it is shown with :

$$d_j = \sum_{i=0}^{n} \left\{ \left(x_i, \mu_{\tilde{d}_j}(x_i) \right) \middle| x_i \epsilon X \right\}$$

Here the symbol $\sum$ means the discrete fuzzy set nit the sum of symbols. Then phase 1 and 2 calculations of the critical paths will be as follows:

A. *The Algorithm of Forward Path*

Step1: Based on data received from the real worlds, we assume that the completion time of activities are fuzzy numbers with discrete membership function.

Step 2: For each element of $x_i \in B$ set $\tilde{Q}_i = \{(0,0)\}$ and mark it.

Step 3: For other elements set $\tilde{Q}_\mathrm{i} = \{(0,0)\}$ until $E = M(B,C) \neq \emptyset$.

Step 4: Set $\widetilde{ES}_j = max\{\tilde{Q}_i\}, x_i \in v_j$ and obtain the earliest start time for edge e_j using the fuzzy number ranking.

Step 5: For each $x_k \in w_j$ set $\tilde{Q}_k = max\{\tilde{Q}_k, (\widetilde{ES}_j + d_j)\}$ and obtain $\tilde{Q}_k$ using the ranking method and mark it.

Step 6: Put $E \leftarrow E - \{e_j\}$ and then repeat the above steps until $E = \emptyset$. In this case $\tilde{T}^e$ is obtained as the earliest completion time of the metagraph.

In order to calculate the backward path, we have to set $T^e = T^l$, then we should calculate the backward path based on the appropriate fuzzy subtraction and computation of minimum values in order to identify the critical path and critical activities of the project.

An algorithm is proposed to determine the critical path and critical activities considering the forward path.

B. *Algorithm to Identify the Critical Path*

Step 1: For each element $Q_i \in C$ set $Q_i = T^l$.

Step 2: If $ES_k + d_k = max\{Q_n, ES_j + d_j\}, n$ is belong to target and j is index of edges that elements Q_n are at their outvertex, then e_k is on the critical path.

Step 3: If $ES_k = max\{Q_i\} = Q_{k'}$, i is index of elements which are located in outvertex e_k, then the element k' is on the critical path.

Step 4: If the element k' is located in the initial invertex of the metagraph, w have to stop, because $ES_k = max\{Q_i\}$, otherwise $C = \{Q_i | Q_i \; \epsilon \; outvertex \; e_k\}$ then go to step 2.

VII. EXAMPLE

A. *Example 1*

Consider the metagraph of Fig. 3. The duration of each edge is shown in table 2.

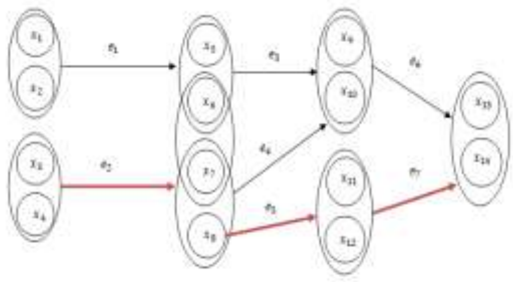

Fig. 3 Metagraph of example 1

Table 2. Edge times of example 1

e_1	$\{(1,0.2), (2,0.5), (3,0.6)\}$
e_2	$\{(1,0.5), (2,0.8), (3,1)\}$
e_3	$\{(1,0.7), (2,1), (3,0.5), (4,0.4)\}$
e_4	$\{(1,0.3), (2,1), (3,0.5), (4,0.7)\}$
e_5	$\{(1,1), (2,0.6), (3,0.4), (4,0.2)\}$
e_6	$\{(1,0.8), (2,1), (3,0.3)\}$
e_7	$\{(1,0.5), (2,0.7), (3,0.8), (4,1)\}$

By using the algorithm, the completion time of the project is: $\{(1,1), (2,1), (3,1), (4,1)\}$

The critical path is $e_2 - e_5 - e_7$ and the critical elements are $x_3, x_4, x_7, x_8, x_{11}, x_{12}, x_{13}, x_{14}$.

The ranking of paths is shown in table 3.

Table 3. The paths ranking for example 1

1	$e_1 + e_3 + e_6$	$\{(1,1), (2,1), (3,1), (4,0.4)\}$
2	$e_2 + e_4 + e_6$	$\{(1,1), (2,1), (3,1), (4,0.7)\}$
3	$e_2 + e_5 + e_7$	$\{(1,1), (2,1), (3,1), (4,1)\}$

Path 3 > path 2 > path 1.

B. *Example 2*

Consider the metagraph of Fig. 4. The duration of each edge is shown in table 4.

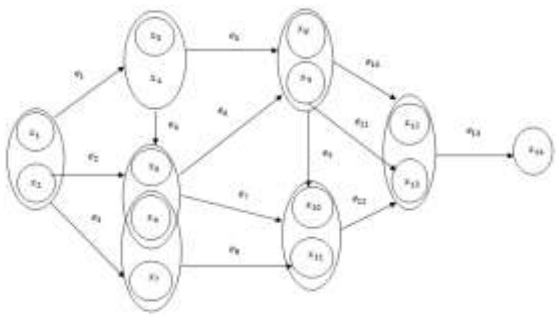

Fig. 4. Metagraph of example 2

Table 4. Edges times of example 2

e_1	$\{(1,0.5),(2,0.3),(3,0.8)\}$
e_2	$\{(1,0.2),(2,0.8),(3,1)\}$
e_3	$\{(1,0.3),(2,0.5),(3,0.7),(4,1)\}$
e_4	$\{(1,0.2),(2,0.3),(3,0.4)\}$
e_5	$\{(1,0.8),(2,1),(3,0.5),(4,0.1)\}$
e_6	$\{(1,0.4),(2,0.6),(3,0.8)\}$
e_7	$\{(2,0.5),(3,0.7),(4,1)\}$
e_8	$\{(1,0.5),(2,0.3),(4,0.7)\}$
e_9	$\{(1,0.3),(2,0.5),(3,0.8),(4,0.1)\}$
e_{10}	$\{(1,0.2),(2,0.5),(3,0.3),(4,0.2)\}$
e_{11}	$\{(1,0.8),(2,0.3),(3,0.1)\}$
e_{12}	$\{(1,0.5),(2,0.8),(3,1)\}$
e_{13}	$\{(1,0.7),(2,0.5),(3,0.4),(4,0.3)\}$

By using the algorithm, the completion time of the project is: $\{(1,1),(2,1),(3,1),(4,1)\}$.

The critical path is $e_3 - e_7 - e_{12} - e_{13}$ and the critical elements are $x_{14}, x_{13}, x_{12}, x_{11}, x_{10}, x_6, x_2, x_1$.

The ranking of paths is shown in table 5.

Table 5. The paths ranking for example 2

1	$e_1+e_5+e_{10}+e_{13}$	$\{(1,1),(2,1),(3,1),(4,0.6)\}$
2	$e_1+e_5+e_9+e_{12}+e_{13}$	$\{(1,1),(2,1),(3,1),(4,0.5)\}$
3	$e_1+e_5+e_{11}+e_{13}$	$\{(1,1),(2,1),(3,1),(4,0.4)\}$
4	$e_1+e_4+e_6+e_9+e_{12}+e_{12}+e_{13}$	$\{(1,1),(2,1),(3,1),(4,0.4)\}$
5	$e_1+e_4+e_6+e_{11}+e_{13}$	$\{(1,1),(2,1),(3,1),(4,0.3)\}$
6	$e_1+e_4+e_7+e_{12}+e_{13}$	$\{(1,1),(2,1),(3,1),(4,1)\}$
7	$e_1+e_4+e_8+e_{12}+e_{13}$	$\{(1,1),(2,1),(3,1),(4,1)\}$
8	$e_2+e_6+e_{10}+e_{13}$	$\{(1,1),(2,1),(3,1),(4,0.5)\}$
9	$e_2+e_6+e_{11}+e_{13}$	$\{(1,1),(2,1),(3,1),(4,0.3)\}$
10	$e_2+e_6+e_9+e_{12}+e_{13}$	$\{(1,1),(2,1),(3,1),(4,0.4)\}$
11	$e_2+e_7+e_{12}+e_{13}$	$\{(1,1),(2,1),(3,1),(4,1)\}$
12	$e_3+e_7+e_{12}+e_{13}$	$\{(1,1),(2,1),(3,1),(4,1)\}$
13	$e_3+e_8+e_{12}+e_{13}$	$\{(1,1),(2,1),(3,1),(4,1)\}$

Ranking of the paths:

Path 13 = Path 12 = Path 11 = Path 7 = Path 6 > Path 1 > Path 8 = Path 2 > Path 3 = Path 4 = Path 10 > Path 9 = Path 5.

VIII. CONCLUTION

In this research, the fuzzy metagraphs are proposed for displaying the projects which have activities with fuzzy time, and it is assumed that these activities have discrete membership functions. An algorithm has been developed to determine the completion time of the project. It performs the forward path calculations for fuzzy numbers with discrete membership functions. Finally, the completion time of the project will be obtained. This algorithm has been designed such that the critical elements and activities of the metagraph are determined by performing the calculation of forward path only. The assumption that activity time is a fuzzy number with discrete membership function will let the project managers determine an appropriate fuzzy number to describe the execution time for the activities with different natures. Whenever it is necessary to determine the maximum fuzzy number among several fuzzy numbers in performing the forward calculations, we have used the $Chen - Lu$ ranking method. For the ease of implementation, a computer program has been developed using $MATLAB$ software. The development of this computer program helps us to increase the number of applied cuts in fuzzy number ranking and do the ranking with a higher accuracy.

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