

# An Appropriate Approach for Transforming and Optimizing Multi-Objective Quadratic Fractional Programming Problem

Huma Akhtar<sup>1\*</sup>, Geeta Modi<sup>2</sup> and Sushma Duraphe<sup>3</sup>

<sup>\*</sup>Research Scholar Govt. M.V.M. Bhopal, India

<sup>2</sup>Professor & Head, Department of Mathematics, Govt. M.V.M. Bhopal, India

<sup>3</sup>Associate Professor, Department of Mathematics, Govt. M.V.M. Bhopal, India

**Abstract:** - This paper presents an advanced optimal average of maximin & minimax technique to solve multi-objective quadratic fractional programming problem (MOQFPP) to single objective quadratic fractional programming problem (SOQFPP) and suggested an algorithm for it. This can be illustrate with the help of numerical example. The numerical result in this paper indicates that our technique is promising.

**Keywords:** - MOQFPP, Advanced optimal average of maximin & minimax technique.

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**1.Introduction:-** Quadratic fractional programming problems(i.e. ratio of objectives that have numerator and denominator) has attracted considerable research and interest since they have been utilized in production planning, financial and corporative planning, health care and hospital planning. Several methods to solve such problems are proposed in (1962) [2], their method depends on transforming this linear fractional programming problem (LFPP) to an equivalent linear program. In (1983) [3], Chandra Sen. defined the multi-objective linear programming problem, and suggested an approach to construct the multi-objective function under the limitation that the optimal values of individual problems are greater than zero. Sulaiman and Abdulrahim in (2013) [4], using transforming technique to solve MOLFPP. Also in (2014) [7], Sulaiman, Sadiq & Abdulrahim studied the new arithmetic average technique to solve multi-objective linear fractional programming problem and it is comparison with other techniques.

Akhtar Huma, et al (2017) [1], transforming and optimizing multi-objective quadratic fractional programming problem. Sulaiman & nawkhash (2013)[6], transforming and solving multi-objective quadratic fractional programming problems by optimal average of maximin & minimax technique.

In order to extend this work we have defined a MOQFPP and investigated the algorithm to solve quadratic fractional programming problem for multi-objective function, irrespective of the number of objectives with less computational burden and suggest an advanced optimal average of maximin & minimax technique ( $O_{AV_s}$ ) to generate the best optimal solution.

## 2. Quadratic Programming:-

If the optimization problem assumes the form

$$\text{Max. } z \text{ (or Min. } z) = \delta + C^T X + \frac{1}{2} X^T G X$$

Subject to:

$$AX \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} b \\ X \geq 0$$

Where  $A = (a_{ij})_{m \times n}$  is a matrix of coefficients,

$\forall i = 1,2,3, \dots, m$  and  $j = 1,2,3, \dots, n$ ,

$$b = (b_1, b_2, \dots, b_m)^T, \quad X = (x_1, x_2, \dots, x_n)^T, \\ C^T = (c_1, c_2, \dots, c_n)^T.$$

and  $G = (g_{ij})_{n \times n}$  is a positive definite(negative definite) or positive semi definite(negative semi definite) symmetric square matrix and  $\delta$  is scalar, moreover the constraints are linear and the objective function is quadratic. Such optimization problem is said to be a quadratic programming problem (QPP).

**3. Mathematical form of QFPP:-**

The mathematical form of QFP problem is given as follows:

$$\text{Max. } z = \frac{c^T X + \delta + \frac{1}{2} X^T G X}{C^T X + \gamma}$$

Subject to:

$$AX \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} b$$

$$X \geq 0$$

Where  $G$  are  $(n \times n)$  matrix of coefficients with  $G$  are symmetric matrices,  $X$  is an  $n$ -dimensional column vector of decision variables,  $c, C$  are  $n$ -dimensional column vector of constants,  $A$  is an  $(m \times n)$  matrix and  $b$  is an  $m$ -dimensional column vector of constants,  $\gamma, \delta$  are scalars.

In this paper the problem that has objective function is tried to be solved can be represented as follows.

$$\text{Max. } z = \frac{(c_1^T X + \alpha)(c_2^T X + \beta)}{C^T X + \gamma}$$

Subject to:

$$AX \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} b$$

$$X \geq 0$$

$A$  is an  $m \times n$  matrix, all vectors are assumed to be column vectors unless transposed(T). Where  $X$  is an  $n$ -dimensional column vector of decision variables,  $c_1, c_2, C$  are the  $n$ -dimensional column vector of constants,  $b$  is an  $m$ -dimensional column vector of constants,  $\alpha, \beta, \gamma$  are scalars.

**3.1 Algorithm for solving QFPP by New Modified Simplex Method:-**

An algorithm to solve QFPP by New Modified Simplex Method is presented in reference [5] p. 3756 and p. 3757.

**4. Multi-Objective Quadratic Fractional Programming Problem:-**

Multi-Objective functions are the ratio of two objective functions that have quadratic objective function in numerator and linear objective function in

denominator, this is said to be MOQFPP then can be defined:

$$\left. \begin{aligned} \text{Max. } z_1 &= \frac{(c_{11}^T X + \alpha_1)(c_{21}^T X + \beta_1)}{C_1^T X + \gamma_1} \\ \text{Max. } z_2 &= \frac{(c_{12}^T X + \alpha_2)(c_{22}^T X + \beta_2)}{C_2^T X + \gamma_2} \\ \text{Max. } z_r &= \frac{(c_{1r}^T X + \alpha_r)(c_{2r}^T X + \beta_r)}{C_r^T X + \gamma_r} \\ \text{Min. } z_{r+1} &= \frac{(c_{1r+1}^T X + \alpha_{r+1})(c_{2r+1}^T X + \beta_{r+1})}{C_{r+1}^T X + \gamma_{r+1}} \\ &\vdots \\ \text{Min. } z_s &= \frac{(c_{1s}^T X + \alpha_s)(c_{2s}^T X + \beta_s)}{C_s^T X + \gamma_s} \end{aligned} \right\} \quad (4.1)$$

Subject to:

$$AX = b \quad (4.2)$$

$$X \geq 0 \quad (4.3)$$

Where  $b$  is an  $m$ -dimensional column vector of constants,  $X$  is an  $n$ -dimensional column vector of decision variables,  $r$  is number of objective functions to be maximized,  $s$  is the number of objective functions to be maximized and minimized and  $(s-r)$  is the number of objective functions that is minimized.  $A$  is an  $m \times n$  matrix of constants, all vectors are assumed to be column vectors unless transposed(T).  $c_{1i}, c_{2i}, C_i$  (where  $i = 1, 2, \dots, s$ ) are  $n$ -dimensional vectors of constants,  $\alpha_i, \beta_i, \gamma_i$  (where  $i = 1, 2, \dots, s$ ) are scalars.

**5. Solving MOQFPP by Using the Following Technique:-**

**5.1. Advanced Optimal Average of Maximine & Minimax ( $O_{AV_s}$ ) Technique:-**

An algorithm for obtaining the optimal solution for the MOQFPP is as follows:

**Step1:** Write the standard form of the problem, by introducing slack and artificial variables to constraints, and write starting simplex table.

**Step2:** calculate the  $\Delta_j$  by the following formula  $\Delta_j = Z_2 \Delta \xi_{1j} - Z_1 \Delta \xi_{2j}$  [2], then write it in the starting simplex table.

**Step3:** Find the solution of first objective problem by using simplex process.

**Step4:** Check the solution for feasibility in step3, if it is feasible then go to step5, otherwise use dual simplex method to remove infeasibility.

**Step5:** Check the solution for optimality if all  $\Delta_j \geq 0$ , then go to step6, otherwise back to step3.

**Step6:** Assign a name to the optimum value of the maximum objective function  $z_i$  say  $\varphi_i$  where  $\forall i = 1, 2, \dots, r$ . And assign a name to the optimum value of the minimum objective function  $z_i$  say  $\varphi_i$  where  $\forall i = r+1, \dots, s$ .

**Step7:** Repeat process from the step3; for  $i = 1, 2, \dots, s$  to be include all the objective functions.

**Step8:** Select  $y_1 = \min\{\varphi_i\}$ ,  $\forall i = 1, 2, \dots, r$  and  $y_2 = \max\{\varphi_i\}$ ,  $\forall i = r+1, \dots, s$ , then calculate

$$O_{AV_s} = \frac{(|y_1|+|y_2|)}{s}$$

**Step9:** Optimize the combined objective function under the same constraints (4.2) and (4.3) as:

$$\text{Max. } Z = \frac{(\sum_{i=1}^r \text{Max. } z_i - \sum_{i=r+1}^s \text{Min. } z_i)}{O_{AV_s}} \quad (5.1.1)$$

**6. Numerical Example:-**

**Example 6.1.**

$$\text{Max. } z_1 = \frac{(2x_1+x_2+1)(2x_1+x_2+2)}{(2x_1+2x_2+2)}$$

$$\text{Max. } z_2 = \frac{(4x_1+2x_2+2)(6x_1+3x_2+6)}{(3x_1+3x_2+3)}$$

$$\text{Max. } z_3 = \frac{(4x_1+2x_2+2)(6x_1+3x_2+6)}{(6x_1+6x_2+6)}$$

$$\text{Min. } z_4 = \frac{(-8x_1-4x_2-4)(6x_1+3x_2+6)}{(5x_1+5x_2+5)}$$

$$\text{Min. } z_5 = \frac{(-4x_1-2x_2-2)(10x_1+5x_2+10)}{(2x_1+2x_2+2)}$$

Subject to:

$$x_1 + 2x_2 \leq 4, \quad 3x_1 + x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

**Solution:** After finding the value of each of individual objective functions, the results are given below :

Table 1

$i$	$x_i$	$\varphi_i$
1	(2,0)	5
2	(2,0)	20
3	(2,0)	10
4	(2,0)	-24
5	(2,0)	-50

$$\text{Max. } Z = \frac{(\sum_{i=1}^r \text{Max. } z_i - \sum_{i=r+1}^s \text{Min. } z_i)}{O_{AV_s}}$$

$$O_{AV_s} = \frac{(|y_1|+|y_2|)}{s} = \frac{|5|+|-24|}{5} = \frac{29}{5}$$

$$\text{Max. } Z = \frac{(218x_1+109x_2+109)(2x_1+x_2+2)}{(10x_1+10x_2+10)} / \left(\frac{29}{5}\right)$$

$$\text{Max. } Z = \frac{(218x_1+109x_2+109)(2x_1+x_2+2)}{(58x_1+58x_2+58)}$$

After solving the Max. Z by given subject to the same constraints as before, we find the optimal solution:

$$\text{Max. } Z = 18.739, \quad x_1 = 2, \quad x_2 = 0.$$

**7) Comparison of the Numerical Results:-** Comparison of the numerical results which are obtained from the example 6.1 is shown in the following table:

Table 2

Techniques		Example 6.1
Chandra Sen. Technique		5
Mean Technique		5
Median Technique		5.5
Average Technique	Mean Technique	4.479
	Median Technique	4.638
Optimal average of maximine and minimax technique		7.517
New Average Technique	Mean Technique	11.199
	Median Technique	11.596
Advanced optimal average of maximin and minimax technique		18.739

In the above table, it is clear that the result obtained in example 6.1 when using advanced optimal average of maximin and minimax technique is better than other results.

## **8) Conclusion:-**

In this paper, we have defined an Advanced optimal average of maximin and minimax technique and compare it with other techniques namely Chandra Sen., mean & median, average mean & median, new average mean & median and optimal average of maximin & minimax techniques.

The comparisons of these techniques are based on the value of the objective functions. After solving the numerical example, we found that Max. Z which obtained by our technique (Advanced optimal average of maximin and minimax technique) is better than other techniques (Chandra Sen., mean & median, average mean & median new average mean & median and optimal average of maximin & minimax techniques).

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## **10) References:-**

- [1] Akhtar Huma, Modi Geeta and Duraphe Sushma, Transforming and Optimizing MOQFPP, International Journal of Statistics and Applied Mathematics, Vol. 2, Issue 1 (2017), 66-70
- [2] Carnes and W.W. Cooper, Programming with Linear Fractional Functional, Naval Research Logistics Quarterly, Vol. 9 (1962), 181-186.
- [3] Sen. Chandra, A new approach for multi-objective rural development planning, The Indian Economic Journal, Vol. 30, No. 4(1983), 91-96.
- [4] Sulaiman, N. A. & Abdulrahim Basiya K., Using transformation technique to solve multi-objective linear fractional programming problem, IJRRAS Vol.14, No.3 (2013), 559-567.
- [5] Sulaiman, N. A. & Nawkhass Maher A., A newmodified simplex method and compared it to a traditional simplex method by using pseudoaffinity of quadratic fractional function, Applied Mathematical Sciences, Vol. 7(2013), no. 76, 3749-3764
- [6] Sulaiman, N. A. & Nawkhass Maher A., Transforming and solving multi-objective quadratic fractional programming problems by optimal average of maximin & minimax techniques, American Journal of Operations Research, Vol. 3, Issue 3(2013), 92-98.
- [7] Sulaiman, N. A., Sdiq Gulnar W. & Abdulrahim Basiya K., "New arithmetic average technique to solve multi-objective linear fractional programming problem and it is comparison with other techniques", IJRRAS, Vol. 18, Issue 2 (2014), 122-131.