On a Basic Analogue of Generalized H-function with the help of fractional q-integral operator of Kober type

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Abstract. *In this paper, our objective is to investigate the basic analogue of a new hypergeometric function, which is a generalization of the basic I-function. In this regard, the application of Kober type q-integral operator with new hypergeometric function has been discussed. Similar result obtained by other authors follows as special cases of our findings.*

Keywords: *Basic analogues of H and I-function, Basic hypergeometric function, Fractional q-integral operators.*

1 Introduction:

In the past century, many authors have generalized H-function. In a recent paper, sudland et al.[10] have introduced a generalization of saxena's I-function[9], which is also a generalization of Fox's H-function. This function is known as Aleph function. In their paper, sexena and pogany [7] have studied fractional integration formulae for the Aleph functions.

Südland et al. [11] studied the generalized fractional driftless Fokker-Planck equation with power law coefficient. As a result, a special function was found, which is a particular case of the Aleph function. The Aleph function has been defined by means of Mellin- Barnes type contour integrals as

$$\aleph[z] = \aleph^{m,n}_{u_i,v_i,\tau_i;r}\left[z\left|\begin{matrix}(a_j,A_j)_{1,n}, & [\tau_j(a_{ji},A_{ji})]_{n+1,u_i;r}\\ (b_j,B_j)_{1,m}, & [\tau_j(b_{ji},B_{ji})]_{m+1,v_i;r}\end{matrix}\right.\right] \quad (1.1)$$

$$\aleph[z] = \frac{1}{2\pi\omega}\int_L \Omega^{m,n}_{u_i,v_i,\tau_i;r}(\xi)\, z^{\xi}\, d\xi$$

For all $z \neq 0$, $\omega = \sqrt{-1}$ and ·

$$\Omega^{m,n}_{u_i,v_i,\tau_i;r}(\xi) = \frac{\prod_{j=1}^{m}\Gamma(b_j - B_j\xi)\prod_{j=1}^{n}\Gamma(1-a_j+A_j\xi)}{\sum_{i=1}^{r}\tau_i\left[\prod_{j=m+1}^{v_i}\Gamma(1-b_{ji}+B_{ji}\xi)\prod_{j=n+1}^{u_i}\Gamma(a_{ji}-A_{ji}\xi)\right]} \quad (1.2)$$

The parameters, u_i, v_i are non-negative integers satisfying the inequality, and $0 \leq n \leq u_i$, $1 \leq m \leq v_i$ and $\tau_i \geq 0$; i=1, . . . The parameters A_j, B_j, A_{ji}, B_{ji} are positive real numbers and are complex numbers and a_j, b_j, a_{ji}, b_{ji} are complex numbers . The $L = L_{\omega\gamma\infty}$ is a suitable contour of the Mellin-Barnes type in the complex ξ -plane which runs from $\gamma - \omega\infty$ *to* $\gamma + \omega\infty$ with $\in \Re$, such that the poles of $\Gamma(b_j - B_j\xi)$, j= 1,. . . separating from those of $\Gamma(1 - a_j + A_j\xi)$, j=1 , ... All the poles of the integrand (1.2) are assumed to be simple, and empty products are interpreted as unity. For the existence conditions,

$\varphi_l > 0, |arg\ z| < (\varphi_l\,\pi)/2$, for all $i = 1,2, \ldots, r$ or,

$\varphi_l \geq 0, |arg\ z| \leq (\varphi_l\,\pi)/2$, *and*

$Re(\xi_l + 1) < 0$ for all $l = 1,2, \ldots, r$

Where

$$\varphi_l = \sum_{j=1}^{n} A_j + \sum_{j=1}^{m} B_j - c_l\left(\sum_{j=n+1}^{u_l} A_{jl} + \sum_{j=m+1}^{v_l} B_{jl}\right),$$

$\xi_l = \sum_{j=1}^{n} a_j - \sum_{j=1}^{m} b_j + c_l\left(\sum_{j=n+1}^{u_l} a_{jl} - \sum_{j=m+1}^{v_l} b_{jl}\right) + \frac{1}{2}(u_l - v_l) \quad l = 1,2,..$

The basic analogue of this ℵ-function has been defined by Dutta and Arora [13] in term of Mellin-Barnes type contour integrals in the following manner

$$\aleph[z;q] = \aleph_{u_i,v_i,\tau_i;r}^{m,n}\left[z\left|\begin{matrix}(a_j,A_j)_{1,n}, & [\tau_j(a_{ji},A_{ji})]_{n+1,u_i;r}\\(b_j,B_j)_{1,m}, & [\tau_j(b_{ji},B_{ji})]_{m+1,v_i;r}\end{matrix}\right.\right]$$

(1.3)

$$\aleph[z;q] = \frac{1}{2\pi\omega}\int_L \Omega_{u_i,v_i,\tau_i;r}^{m,n}(\xi;q)\pi z^{\xi}\, d\xi$$

where $\omega = \sqrt{-1}$

$$\Omega_{u_i,v_i,\tau_i;r}^{m,n}(\xi;q) = \frac{\prod_{j=1}^{m} G(q^{b_j - B_j\xi})\prod_{j=1}^{n} G(q^{1-a_j+A_j\xi})}{\sum_{i=1}^{r}\{\tau_i\prod_{j=m+1}^{v_i} G(q^{1-b_{ji}+B_{ji}\xi})\prod_{j=n+1}^{u_i} G\left(q^{a_{ji}-A_{ji}\xi}\right)G(q^{1-s})sin\pi\xi\}}$$

(1.4)

$$G(q^{\delta}) = \{\prod_{j=0}^{\infty}(1-q^{\delta+j})\}^{-1} = \frac{1}{(q^{\delta};q)_{\infty}}$$

(1.5)

The parameters, u_i, v_i are non-negative integers satisfying the inequality, and $0 \le n \le u_i$, $1 \le m \le v_i$ and $\tau_i \ge 0$; i=1, . . . is finite.The parameters A_j, B_j, A_{ji} , B_{ji} are positive real numbers and are complex numbers and a_j , b_j , a_{ji} , b_{ji} are complex numbers . The $C = C_{\omega\gamma\infty}$ is a suitable contour of the Mellin-Barnes type in the complex ξ -plane which runs from $\gamma - \omega\infty$ to $\gamma + \omega\infty$ with ∈ ℜ , such that the poles of $G(q^{b_j - B_j\xi})$ separating from those of $G(q^{1-a_j+A_j\xi})$,j=1,. . . . All the poles of the integrand (3.1.4) are assumed to be simple and empty products are interpreted as unity. The integral converges if $\Re[\xi\log(z) - \log\sin\pi\xi]<0$ for large value of | ξ | on the contour, that is if $|\arg(z) - \rho_2\rho_1^{-1}\log|z||<\pi$, where, 0<|q|< 1, $\log q = -\rho = -(\rho_1 + \rho\rho_2)$, ρ, ρ_1 are definite quantities, ρ_1 and ρ_2 being real.

When all $\tau_i = 1$; (1.3) yields the q -analogue of the I-function due to Saxena and Kumar [6].

Again, when r = 1, $u_i = u$, $v_i = v$ and $c_i = 1$; (1.3) yields the q -analogous of the H -function due to Saxena et al. [7].

2 Definitions and Preliminaries:

In this section, we use the following definitions and fundamental facts of basic analogue of special function and integral operator.

Definition: Basic Analogue of Aleph Function:

(i) We shall make use of $\aleph q(z;q)$ notation for basic analogue of Aleph function using the notation defined earlier by Jain et al. [14] . It is defined as

$\aleph q(z;q)$

$$= \left[\aleph^{m,n}_{p_i,q_i,\tau_i;r}\left[z;q\left|\begin{matrix}(a_j,A_j)_{1,n},\dots.[\tau_j(a_{ji},A_{ji})]_{n+1,p_i}\\(b_j,B_j)_{1,m},\dots.[\tau_j(b_{ji},B_{ji})]_{m+1,q_i}\end{matrix}\right.\right]\right]$$

$$\aleph q(z;q) = \left[\frac{1}{2\pi i}\int_L \Omega^{m,n}_{p_i,q_i,\tau_i;r}(s;q)\, z^s\, ds\right]$$

(2.1)

$$\Omega^{m,n}_{p_i,q_i,\tau_i;r}(s;q) = \frac{\prod_{j=1}^{m}\Gamma_q(b_j - B_j s)}{\sum_{i=1}^{r}\tau_i\left[\prod_{j=m+1}^{q_i}\Gamma_q(1-b_{ji}+B_{ji}s)\right]}$$

$$\times\frac{\Gamma_q(1-a_j+A_j s)}{\prod_{j=n+1}^{p_i}\Gamma_q(a_{ji}-A_{ji}s)\times\Gamma_q s\times\Gamma_q(1-s)\sin\pi s}$$

And $z\neq 0, i=\sqrt{-1}$.

Here $a_i, b_j \epsilon C > 0; p_i > 0, q_j > 0;\ 1 + \sum_{j=1}^{q} B_j - \sum_{i=1}^{p} A_i \geq 0;\ a\epsilon R$ for suitably bounded value of |z|.

The integral converges if $Re[s\log(z) - \log\sin\pi s] < 0$, on the contour C, where 0<|q|< 1.

(ii) The fractional q-calculus is the q-extension of the ordinary calculus. Agarwal [1], introduced the q-basic analogue of Kober fractional integral operator in the following form:

$$I_q^{\eta,\mu}f(x) = \frac{x^{-\eta-?}}{\Gamma_q(?)}\int_0^x (x-qt)_{?-1}t^{\eta}f(t)\,d_q(t)$$

(2.2)

where $Re(\mu) > 0, |q| < 1,$

(iii) In the theory of q-calculus, $0 < |q| < 1$, the q-shifted factorial is defined as

$$[a;q]_k = \begin{Bmatrix}\prod_{j=0}^{k-1}(1-aq^j)\ ,if\ k>0\\ \prod_{j=0}^{\infty}(1-aq^j)\ \ ,if\ k\dashrightarrow\infty\end{Bmatrix} \qquad (2.3)$$

Moreover, $[a;q]_0 = 1,\ and\ if\ k = 0$

Or equivalently

$$(a;q)_k = \frac{(a;q)_\infty}{(aq^k;q\ )_\infty}$$

Further if α is any complex number, then the definition can be stated as follows

$$(a;q)_\alpha = \frac{(a;q)_\infty}{(aq^\alpha;q)_\infty}$$

$$(a;q)_\infty = \frac{\Gamma_q(\alpha)}{(1-q)^\alpha G(q)^\alpha} \qquad (2.4)$$

The q-analogue of power function is defined and denoted as

$$(a-b)_\alpha = a^\alpha\left(\frac{b}{a};q\right) = a^\alpha\prod_{j=0}^{\infty}\frac{\left(1-\frac{b}{a}\right)q^j}{\left(1-\frac{b}{a}\right)q^{j+\alpha}} = a^\alpha\frac{\left(\frac{b}{a};q\right)_\infty}{(q^\alpha\frac{b}{a};q)_\infty} \qquad (2.5)$$

The q- gamma function is defined by

$$\Gamma_q(\alpha) = \frac{G(q^\alpha)}{G(q)}(1-q)^{1-\alpha} = (1-q)_{\alpha-1}(1-q)^{1-\alpha};\ \alpha \in R(0,-1,-2,\dots) \qquad (2.6)$$

where $G(q)^\alpha = \frac{1}{(q^\alpha;q)_\infty}$

The q-derivatives of a function f(x) is given as follows:

$D_q f(x) = \frac{f(x)-f(qx)}{(1-q)x}, (x)\neq 0)$ and $(D_q f)\ 0=0$

(2.7)

$D_q \dashrightarrow \frac{d}{d(x)}$ as $q \dashrightarrow 1$.

We have

$D_q^n(x^{\mu}) = \frac{\Gamma_q(\mu+1)}{\Gamma_q(\mu-n+1)} x^{\mu-n}$, $R(\mu) + 1 > 0$

The q-integral of a function is defined as :

$\int_0^x f(t)\, d_q(t) = x(1-q) \sum_{k=0}^{\infty} q^k f(xq^k)$

(2.8)

$\int_x^{\infty} f(t)\, d_q(t) = x(1-q) \sum_{k=1}^{\infty} q^{-k} f(xq^{-k})$

(2.9)

$\int_0^{\infty} f(t)\, d_q(t) = (1-q) \sum_{k=-\infty}^{\infty} q^k f(xq^k)$ (2.10)

The q- binomial theorem is given as follows

$1\phi 0\ [\alpha;\ q;\ x] \quad = \sum_{n=0}^{\infty} \frac{(a;q)_n}{(q;q)_n} z^n = \frac{(\alpha x;q)_{\infty}}{(x;q)_{\infty}}$

(2.11)

The q- analogues of Gauss summation theorem is given by

$2\phi 1 \quad [q^a\ ; q^b; q^c;\ \ q; q^{c-a-b}] \quad = \frac{\Gamma_q(c)\Gamma_q(c-a-b)}{\Gamma_q(c-a)\Gamma_q(c-b)}$

(2.12)

Agarwal [1] introduced the q-analogue of the Reimann-Liouville fractional integral operator as follows.

$I_q^{\mu} f(x^{\mu}) = \frac{1}{\Gamma_q(\mu)} \int_0^{\infty} (1-qy)_{\mu-1} f(y)\, d_q(y)$

(2.13)

where μ is an arbitrary order of integration such that Re (μ)> 0. From equation (2.8) and (2.13), we get

$I_q^{\mu} f(x) = x^{\mu}(1-q)^{\mu} \sum_{k=0}^{\infty} \frac{q^k (q^{\mu}\ ; q)_k}{(q;q)_k} f(xq^k)$

(2.14)

Agarwal [1], introduced the basic analogue of Kober fractional operator as follows.

$I_q^{\eta,\mu} f(x) \ = \frac{x^{-\eta-\mu}}{\Gamma_q(\mu)} \int_0^x (x - qt)_{\mu-1} t^{\eta} f(t)\, d_q(t)$

(2.15)

Again from equation (2.12) and (2.14), we get

$I_q^{\eta,\mu} f(x) \ = \ (1-q)^{\mu} \sum_{k=0}^{\infty} \frac{q^{k(1+\eta)}(q^{\mu}\ ; q)_k}{(q;q)_k} f(xq^k)$

(2.16)

For the basic concept of q-calculus we refer to reader to [3]

3 Main Result:

In this section, we will establish the following fractional q-integral operator of Kober type involving basic analogue of Aleph-function.

Theorem (3.1) Let $\Re(\alpha) > 0, |q| < 1$ *and* $I_q^{\eta,\mu}$ be the q-analogue of the Kober fractional integral operator (2.2) ,then following results holds:

$$I_q^{\eta,\mu}\left\{ z^{\rho-1} \aleph_{p_i,q_i,\tau_i;r}^{m,n} \left[\sigma z^{\lambda}; q \left| \begin{matrix} (a_j, A_j)_{1,n};\ [\tau_j (a_{ji}, A_{ji})]_{n+1,p_i} \\ (b_j, B_j)_{1,m};\ [\tau_j (b_{ji}, B_{ji})]_{m+1,q_i} \end{matrix} \right. \right] \right\}$$

$$= (1-q)^{\mu} \times z^{\rho-1} \left\{ \begin{matrix} \aleph_{p_i+1,q_i+1,\tau_i;r}^{m,n} [\sigma z^{\lambda};] \\ ;\ \text{for}\ \lambda \geq 0 \end{matrix} \quad \times \right\}$$

$$q\left|\begin{matrix}(1-\eta-\rho,\lambda)(a_j,A_j)_{1,n}; & [\tau_j(a_{ji},A_{ji})]_{n+1,p_i}\\ (b_j,B_j)_{1,m}; & [\tau_j(b_{ji},B_{ji})]_{m+1,q_i}(1-\mu-\eta-\rho,\lambda)\end{matrix}\right.$$

; for $\lambda \geq 0$

$$= (1-q)^{\mu}\times z^{\rho-1}\{\aleph^{m,n}_{p_i+1,q_i+1,\tau_i;r}$$

$$\times\left[\sigma z^{\lambda};q\left|\begin{matrix}(a_j,A_j)_{1,n}; & [\tau_j(a_{ji},A_{ji})]_{n+1,p_i}(\mu+\eta+\rho,-\lambda)\\ (\eta+\rho,-\lambda)(b_j,B_j)_{1,m}; & [\tau_j(b_{ji},B_{ji})]_{m+1,q_i}\end{matrix}\right.\right]$$

; for $\lambda < 0$

Where $\Re[\xi\log\eta-\log\sin\pi\xi]<0$ and $\rho>0$.

Proof: To prove theorem (3.1) when $\lambda\geq 0$, we apply (2.16) and (2.1) to the left side, we get

$$I_q^{\eta,?}\left\{z^{\rho-1}\aleph^{m,n}_{p_i,q_i,\tau_i;r}\left[\sigma z^{\lambda};q\left|\begin{matrix}(a_j,A_j)_{1,n}; & [\tau_j(a_{ji},A_{ji})]_{n+1,p_i}\\ (b_j,B_j)_{1,m}; & [\tau_j(b_{ji},B_{ji})]_{m+1,q_i}\end{matrix}\right.\right]\right\}$$

$$=(1-q)^{\mu}\times\frac{1}{2\pi i}\int_L\sum_{k=0}^{\infty}\frac{q^{k(1+\eta)}(q^{\mu};q)_k}{(q;q)_k}(zq^k)^{\rho-1}$$

$$\times\frac{\prod_{j=1}^{m}\Gamma_q(b_j-B_js)\prod_{j=1}^{n}\Gamma_q(1-a_j+A_js)\,(\sigma z^{\lambda}q^{k\lambda})^s\,ds}{\sum_{i=1}^{r}\tau_i\left[\prod_{j=m+1}^{q_i}\Gamma_q(1-b_{ji}+B_{ji}s)\prod_{j=n+1}^{p_i}\Gamma_q(a_{ji}-A_{ji}s)\times\Gamma_q s\times\Gamma_q(1-s)\sin\pi s\right]}$$

This implies

$$I_q^{\eta,?}\left\{z^{\rho-1}\aleph^{m,n}_{p_i,q_i,\tau_i;r}\left[\sigma z^{\lambda};q\left|\begin{matrix}(a_j,A_j)_{1,n}; & [\tau_j(a_{ji},A_{ji})]_{n+1,p_i}\\ (b_j,B_j)_{1,m}; & [\tau_j(b_{ji},B_{ji})]_{m+1,q_i}\end{matrix}\right.\right]\right\}$$

$$=(1-q)^{\mu}\times z^{\rho-1}$$

$$\frac{1}{2\pi i}\int_L\frac{\prod_{j=1}^{m}\Gamma_q(b_j-B_js)\prod_{j=1}^{n}\Gamma_q(1-a_j+A_js)\,(\sigma z^{\lambda})^s}{\sum_{i=1}^{r}\tau_i\left[\prod_{j=m+1}^{q_i}\Gamma_q(1-b_{ji}+B_{ji}s)\prod_{j=n+1}^{p_i}\Gamma_q(a_{ji}-A_{ji}s)\times\Gamma_q s\times\Gamma_q(1-s)\sin\pi s\right]}$$

$$\times\ {}_1\Phi_0\,[q^{?};q;q^{(\rho+\eta+\lambda s)}]ds$$

On summing the inner ${}_1\Phi_0$ series with help of (2.10), we get

$$I_q^{\eta,?}\left\{z^{\rho-1}\aleph^{m,n}_{p_i,q_i,\tau_i;r}\left[\sigma z^{\lambda};q\left|\begin{matrix}(a_j,A_j)_{1,n}; & [\tau_j(a_{ji},A_{ji})]_{n+1,p_i}\\ (b_j,B_j)_{1,m}; & [\tau_j(b_{ji},B_{ji})]_{m+1,q_i}\end{matrix}\right.\right]\right\}$$

$$(1-q)^{\mu}$$

$$\times z^{\rho-1}\frac{1}{2\pi i}\int_L\frac{\prod_{j=1}^{m}\Gamma_q(b_j-\beta_js)\times\prod_{j=1}^{n}\Gamma_q(1-a_j+\alpha_js)}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{qi}\Gamma_q 1-b_{ji}+\beta_{ji}s\right)}$$

$$\times\frac{\Gamma_q(\rho+\eta+\lambda s)(\sigma z^{\lambda})^s}{\prod_{j=n+1}^{pi}\Gamma_q(a_{ji}-\alpha_{ji}s)\times\Gamma_q s\times\Gamma_q(1-s)\sin\pi s\,\Gamma_q(\rho+\mu+\eta+\lambda s)}ds$$

Now using (2.1) we get

$$I_q^{\eta,?}\left\{z^{\rho-1}\aleph^{m,n}_{p_i,q_i,\tau_i;r}\left[\sigma z^{\lambda};q\left|\begin{matrix}(a_j,A_j)_{1,n}; & [\tau_j(a_{ji},A_{ji})]_{n+1,p_i}\\ (b_j,B_j)_{1,m}; & [\tau_j(b_{ji},B_{ji})]_{m+1,q_i}\end{matrix}\right.\right]\right\}$$

$$=(1-q)^{\mu}\times z^{\rho-1}\aleph^{m,n}_{p_i+1,q_i+1;\tau_i,r}$$

$$\times\left\{\left[\sigma z^{\lambda};q\left|\begin{matrix}(1-\eta-\rho,\lambda)(a_j,A_j)_{1,n}; & [\tau_j(a_{ji},A_{ji})]_{n+1,p_i}\\ (b_j,B_j)_{1,m}; & [\tau_j(b_{ji},B_{ji})]_{m+1,q_i}(1-\mu-\eta-\rho,\lambda)\end{matrix}\right.\right]\right\}$$

, for $\lambda\geq 0$

Also when $\lambda<0$ in theorem (3.1) we get a known result due to Yadav etal [15] as follows

$$I_q^{\eta,?}\left\{z^{\rho-1}\aleph^{m,n}_{p_i,q_i,\tau_i;r}\left[\sigma z^{\lambda};q\left|\begin{matrix}(a_j,A_j)_{1,n}; & [\tau_j(a_{ji},A_{ji})]_{n+1,p_i}\\ (b_j,B_j)_{1,m}; & [\tau_j(b_{ji},B_{ji})]_{m+1,q_i}\end{matrix}\right.\right]\right\}$$

$$=(1-q)^{\mu}\times z^{\rho-1}\aleph^{m,n}_{p_i+1,q_i+1;\tau_i,r}$$

$$\left\{\left[\sigma z^{\lambda}; q \left| \begin{matrix} (a_j, A_j)_{1,n}; & [\tau_j(a_{ji}, A_{ji})]_{n+1,p_i} \ (\mu+\eta+\rho, \ -\lambda) \\ (\eta+\rho, -\lambda)(b_j, B_j)_{1,m}; & [\tau_j(b_{ji}, B_{ji})]_{m+1,q_i} \end{matrix} \right.\right], \text{for } \lambda < 0 \right\}$$

Where $\Re[\xi \log \eta - \log \sin \pi\xi] < 0$ and $\rho > 0$.

4 Special Cases:

The q-extension of Aleph function defined by (1.3) in terms of Mellin-Barnes type of basic integrals is most general in nature, which includes a number of basic analogues of special functions. In this section we discuss only the case involving $I_q(.)$ function and $H_q(.)$ function.

(i) if we set $\tau_j = 1, i = 1, \ldots$ in above theorem we obtain the following formula for $I_q(.)$ function

Corollary 4.1 Let $\Re(\alpha) > 0, |q| < 1$, then the following results holds:

$$I_q^{\eta,\mu}\left\{z^{\rho-1} I_{p_i,q_i;r}^{m,n}\left[\sigma z^{\lambda}; q \left| \begin{matrix} (a_j, A_j)_{1,n}; (a_{ji}, A_{ji})_{n+1,p_i} \\ (b_j, B_j)_{1,m}; (b_{ji}, B_{ji})_{m+1,q_i} \end{matrix} \right.\right]\right\}$$

$$= (1-q)^{\mu} \times z^{\rho-1} I_{p_i+1,q_i+1;r}^{m,n}$$

$$\left\{\left[\sigma z^{\lambda}; q \left| \begin{matrix} (1-\eta-\rho, \lambda)(a_j, A_j)_{1,n}; & (a_{ji}, A_{ji})_{n+1,p_i} \\ (b_j, B_j)_{1,m}; & (b_{ji}, B_{ji})_{m+1,q_i} (1-\mu-\eta-\rho, \lambda) \end{matrix} \right.\right] ; \text{ for } \lambda \geq 0 \right\}$$

$$= (1-q)^{\mu} \times z^{\rho-1} I_{p_i+1,q_i+1;r}^{m,n}$$

$$\left\{\left[\sigma z^{\lambda}; q \left| \begin{matrix} (1-\eta-\rho, \lambda)(a_j, A_j)_{1,n}; & (a_{ji}, A_{ji})_{n+1,p_i} \\ (b_j, B_j)_{1,m}; & (b_{ji}, B_{ji})_{m+1,q_i} (1-\mu-\eta-\rho, \lambda) \end{matrix} \right.\right] ; \text{ for } \lambda < 0 \right\}$$

(ii) if we set $r = 1$, $\tau_j = 1, , p_i = p$, $q_i = q$ $a_{ji} = a_j$ $b_{ji} = b_j$, $A_{ji} = A_j$, $B_{ji} = B_j$ in above theorem we obtain the following formula for $H_q(.)$ function.

Corollary 4.2 Let $\Re(\alpha) > 0, |q| < 1$, then the following results holds:

$$I_q^{\eta,\mu}\left\{z^{\rho-1} H_{p,q}^{m,n}\left[\sigma z^{\lambda}; q \left| \begin{matrix} (a_j, A_j)_{1,p} \\ (b_j, B_j)_{1,q} \end{matrix} \right.\right]\right\}$$

$$= (1-q)^{\mu}$$

$$\times z^{\rho-1}\left\{ H_{p+1,q+1}^{m,n}\left[\sigma z^{\lambda}; q \left| \begin{matrix} (1-\eta-\rho, \lambda)(a_j, A_j)_{1,p} \\ (b_j, B_j)_{1,q}, (1-\mu-\eta-\rho, \lambda) \end{matrix} \right.\right] ; \text{ for } \lambda \geq 0 \right\}$$

$$= (1-q)^{\mu}$$

$$\times z^{\rho-1}\left\{ H_{p_i+1,q_i+1,;r}^{m,n}\left[\sigma z^{\lambda}; q \left| \begin{matrix} (a_j, A_j)_{1,p} \ (\mu+\eta+\rho, \ -\lambda) \\ (\eta+\rho, -\lambda)(b_j, B_j)_{1,q} \end{matrix} \right.\right] ; \text{ for } \lambda < 0 \right\}$$

Where $\Re[\xi \log \eta - \log \sin \pi\xi] < 0$ and $\rho > 0$.

5 Conclusion:

Since most of the special function can be expressed in term of the q-extension of Aleph function defined by (1.3). in this paper We have evaluated Kober's fractional integral operator for the basic analogue for Aleph function. The result are quite general in nature and reduce to corresponding results for basic analogues of G , H and I functions and their several special cases. In this connection one can refer to the work of Yadav and Purohit

[12]. Thus these results can be applied to various problems of mathematical physics.

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