

# SP-Sasakian Manifolds Admitting Special Semi - Symmetric Recurrent Metric Connection

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**ABSTRACT;** *In this paper, we study special semi -symmetric metric recurrent connection in SP-Sasakian manifold. Some results related to this connection are studied and obtained .Also some curvature properties of SP-sasakian manifolds admitting special semi - symmetric recurrent metric connection are studied.*

**KEYWORD;** *sp-sasakian manifold, semi -symmetric metric recurrent connection, concircular curvature tensor ,projective curvature tensor.*

**MSC2010;** 53C07, 53C25

## 1. INTRODUCTION

In 1975, Golab[1] initiated the study of quarter-symmetric linear connection on a differentiable manifold  $M^n$ . A linear connection  $\tilde{\nabla}$  in an  $n$  dimensional differentiable manifold  $M^n$  is said to be a Quarter -symmetric connection if torsion tensor  $T$  is of the form

$$\begin{aligned} T(X,Y) &= \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X,Y] \\ &= \eta(Y)\varphi X - \eta(X)\varphi Y \end{aligned} \quad (1.1)$$

where  $\eta$  is a 1-form and  $\varphi$  is a tensor of type (1,1). In addition, if a quarter-symmetric linear connection  $\tilde{\nabla}$  satisfies the condition  $\tilde{\nabla}_X \xi = 0$  for all  $X, Y, Z \in TM^n$ , where  $TM$  is a lie algebra of vectors fields of the differentiable manifold  $M^n$ . Then  $\tilde{\nabla}$  is said to be quarter-symmetric metric connection. Quarter -symmetric metric connection is also studied by Biswas and De [10] De and Mondal[16] Singh and pandey[15] Mishra and Pandey[5], Rastogi[7], Yano and Imai [6], Sular, S [17] and many others.

In particular, if  $\varphi X = X$  and  $\varphi Y = Y$ , then the quarter-symmetric connection reduces to a semi-symmetric connection . The semi symmetric metric connection is generalized case of quarter- symmetric metric connection.

The semi-symmetric metric connection in an SP-Sasakian manifold have been studied by Sinha and Kalpna[4]. In the present paper we have studied with quarter-symmetric metric connection in an SP-Sasakian manifold . We also discuss the different type of curvatures with a quarter- symmetric metric connection in an SP-Sasakian manifold . This paper organized as follows after preliminaries . In section 1, Introduction . In section 2, We discuss the preliminary results of sp-sasakian manifold and also special semi-symmetric recurrent metrics connection  $\tilde{\nabla}$ . In section 3, existence of special semi-symmetric recurrent metric connection . In section 4, curvature tensor of the manifold with respect to special semi-symmetric recurrent metrics connection  $\tilde{\nabla}$ . In section 5, concircular curvature tensor of sp-sasakian manifold with respect to the special semi-symmetric recurrent metric connection  $\tilde{\nabla}$ . In section 6, projective curvature tensor of sp-sasakian manifold with respect to special semi-symmetric recurrent metric connection  $\tilde{\nabla}$ . In section 7, projective ricci curvature tensor of sp-sasakian manifold with respect to special semi-symmetric recurrent metric connection  $\tilde{\nabla}$  and In section 8, m-projective curvature tensor of sp-sasakian manifold with respect to special semi-symmetric recurrent metric connection  $\tilde{\nabla}$ .

## 2. PRELIMINARIES

An  $n$ -dimensional smooth manifold  $M^n$  is called an almost para-contact manifold [12], if it admits an almost para-contact structure  $(\varphi, \xi, \eta)$  consisting of a (1,1) tensor field  $\varphi$ , vector field  $\xi$ , and 1-form  $\eta$  satisfying

$$\varphi^2 X = X - \eta(X)\xi \quad (2.1)$$

$$\eta(\xi) = 1 \quad (2.2)$$

$$\varphi \circ \xi = 0 \quad (2.3)$$

$$\eta \circ \varphi = 0 \quad (2.4)$$

$$\text{Rank}(\varphi) = n-1 \quad (2.5)$$

for all  $X \in TM^n$

Let  $g$  be Riemannian metric satisfying

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y) \quad (2.6)$$

$$g(\varphi X, Y) = g(X, \varphi Y) \quad (2.7)$$

for all  $X, Y \in TM^n$ . Then  $M^n$  becomes an almost para-contact Riemannian manifold equipped with the almost para-contact Riemannian structure  $(\varphi, \xi, \eta, g)$  [2].

If we define

$$g(X, \xi) = \eta(X), \quad g(\xi, \xi) = 1 \quad (2.8)$$

$$\Phi(X, Y) = g(\varphi X, Y) \quad (2.9)$$

then in addition to the above equation, we have

$$\Phi(X, Y) = \Phi(Y, X) \quad (2.10)$$

$$\Phi(\varphi X, \varphi Y) = \Phi(X, Y) \quad (2.11)$$

An almost para-contact Riemannian manifold is called a P-Sasakian manifold [4] if it satisfies

$$(\nabla_X \varphi)Y = -g(X, Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi \quad (2.12)$$

for all  $X, Y \in TM^n$  where  $\nabla$  denotes the covariant differentiation with respect to Riemannian metric  $g$ . from the above equation it follows that

$$\nabla_X \xi = \varphi X \quad (2.13)$$

$$(\nabla_X \eta)(Y) = g(X, \varphi Y) = (\nabla_Y \eta)(X) \quad (2.14)$$

Especially, a P-Sasakian manifold  $M^n$  is called a special para-Sasakian manifold or briefly an SP-Sasakian manifold .If  $M^n$  admits a 1-form  $\eta$  satisfying

$$(\nabla_X \eta)(Y) = -g(X, Y)\xi + \eta(X)\eta(Y) \quad (2.15)$$

For SP-Sasakian manifold we have [4]

$$\Phi(X, Y) = -g(X, Y)\xi + \eta(X)\eta(Y) \quad (2.16)$$

In an  $n$ -dimensional P-Sasakian manifold  $M^n$ , the curvature tensor  $R$ , the Ricci tensor  $S$  satisfy [3],[2],[9]

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X \quad (2.17)$$

$$R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi \quad (2.18)$$

$$R(\xi, X)\xi = X - \eta(X)\xi \quad (2.19)$$

$$\eta(R(X, Y)U) = g(X, U)\eta(Y) - g(Y, U)\eta(X) \quad (2.20)$$

$$\eta(R(X, Y)\xi) = 0 \quad (2.21)$$

$$\eta(R(\xi, X)Y) = \eta(X)\eta(Y) - g(X, Y) \quad (2.22)$$

$$S(X, \xi) = -(n-1)\eta(X) \quad (2.23)$$

$$QX=-(n-1)X \tag{2.24}$$

$$S(\varphi X, \varphi Y)=S(X, Y)+(n-1)\eta(X)\eta(Y) \tag{2.25}$$

$$S(X, \varphi Y)=S(\varphi X, Y) \tag{2.26}$$

Let  $(M, g)$  be an manifold with Levi-Civita connection  $\tilde{\nabla}$  in a SP-Sasakian Manifold can be defined by

$$\tilde{\nabla}_X Y = \nabla_X Y - \eta(X)Y \tag{2.27}$$

The curvature tensor  $\tilde{R}$  of  $M^n$

$$\begin{aligned} \tilde{R}(X, Y)Z &= R(X, Y)Z + 3g(X, Z)Y - 3g(Y, Z)X - 2g(X, Z)\eta(Y)\xi \\ &- 2\eta(X)\eta(Z)Y + 2g(Y, Z)\eta(X)\xi + 2\eta(Y)\eta(Z)X \end{aligned} \tag{2.28}$$

The Ricci tensor  $S$  and scalar curvature  $r$  of  $M^n$  with respect to quarter -symmetric metric connection  $\tilde{\nabla}$  is defined by

$$\tilde{S}(Y, Z) = S(Y, Z) - (3n-5)g(Y, Z) + 2(n-2)\eta(Y)\eta(Z) \tag{2.29}$$

$$\tilde{r} = r - (n-1)(3n-4) \tag{2.30}$$

Where  $\tilde{S}$  and  $S$  are the Ricci tensor of the connection  $\nabla$  and  $\tilde{\nabla}$  respectively . Similarly  $\tilde{r}$  and  $r$  are the scalar curvature of the connection  $\nabla$  and  $\tilde{\nabla}$  respectively.

DEFINITION ;The projective curvature tensor is defined as[21]

$$P(X, Y)Z = R(X, Y)Z - \frac{1}{n-1}\{S(Y, Z)X - S(X, Z)Y\} \tag{2.31}$$

DEFINITION ;The concircular curvature tensor is defined as [20]

$$C(X, Y, Z) = R(X, Y, Z) - \frac{r}{n(n-1)}\{g(Y, Z)X - g(X, Z)Y\}$$

DEFINITION ; The M-projective curvature tensor (W) is defined as [20]

$$\begin{aligned} W(X, Y, Z, U) &= R(X, Y, Z, U) - \frac{1}{2(n-1)}[S(Y, Z)g(X, U) - S(X, Z)g(Y, U)] \\ &- \frac{1}{2(n-1)}[g(Y, Z)S(X, U) - g(X, Z)S(Y, U)] \end{aligned}$$

DEFINITION; The projective Ricci curvature tensor is defined as

$$L(X, Y)Z = \frac{2n+1}{2n}S(X, Z)Y - \frac{r}{2n}g(X, Y)Z$$

### 3.EXISTENCE OF SPECIAL SEMI-SYMMETRIC RECURRENT METRIC CONNECTION

Let  $\nabla$  be a linear connection in  $M$ , given by

$$\tilde{\nabla}_X Y = \nabla_X Y + H(X, Y) \tag{3.1}$$

Where  $H$  is a tensor of type (1,2) Now , we determine tensor field  $H$  such that  $\tilde{\nabla}$  satisfies (2.23)and (2.27). from equation(3.1), we have (3.2)

$$T(X, Y) = H(X, Y) - H(Y, X) \tag{3.2}$$

Let (3.3)

$$G(X, Y, Z) = (\tilde{\nabla}_X g)(Y, Z) \tag{3.3}$$

then(3.4)

$$g(H(X,Y),Z)+g(H(X,Z),Y)=-G(X,Y,Z) \tag{3.4}$$

From (3.2), we have

$$g(\tilde{T}(X,Y),Z)+g(\tilde{T}(Z,X),Y)+g(\tilde{T}(Z,Y),X) = g(H(X,Y),Z)-g(H(Y,X),Z) \\ +g(H(Z,X),Y)-g(H(X,Z),Y) \\ +g(H(Z,Y),X)-g(H(Y,Z),X)$$

Now from (3.1) and (3.4) we have

$$g(T(X,Y),Z)+g(T(Z,X),Y)+g(T(Z,Y),X) = 2g(H(X,Y),Z)+2\eta(X)g(Y,Z) \\ -2\eta(Z)g(X,Y)+2\eta(Y)g(X,Z)$$

This gives

$$H(X,Y)=1/2\{\tilde{T}(X,Y)+\tilde{T}(X,Y)+\tilde{T}(Y,X)\}-\eta(X)Y-\eta(Y)X=g(X,Y) \tag{3.5}$$

where

$$g(\tilde{T}(X,Y),Z)=g(\tilde{T}(Z,X),Y) \tag{3.6}$$

From (2.23) and (3.6), we get

$$\tilde{T}(X,Y)=\eta(X)Y-g(X,Y) \tag{3.7}$$

Now from (3.5) and (3.7) we have

$$H(X,Y)=-\eta(X)Y \tag{3.8}$$

Thus from (3.1) we obtain  $\tilde{\nabla}$

$$\tilde{\nabla}_x Y = \nabla_x Y - \eta(X)Y$$

Conversely, the connection  $\tilde{\nabla}$  defined by (2.23), satisfies (2.25) and (2.27) so that the connection  $\tilde{\nabla}$  is a special semi-symmetric recurrent metric connection.

**Theorem 3.1:** Let  $(M, g)$  be an SP-Sasakian manifold with a contact metric structure  $(\phi, \xi, \eta, g)$ , then it admits a special semi-symmetric recurrent metric connection defined as below

$$\tilde{\nabla}_x Y = \nabla_x Y - \eta(X)Y$$

#### **4. CURVATURE TENSOR OF THE MANIFOLD WITH RESPECT TO SPECIAL SEMI-SYMMETRIC RECURRENT METRICS CONNECTION $\tilde{\nabla}$**

The curvature tensor of the manifold  $M^n$  with respect to special semi - symmetric recurrent metrics connection  $\tilde{\nabla}$  is given by

$$\tilde{R}(X, Y, Z) = \tilde{\nabla}_X \tilde{\nabla}_Y Z - \tilde{\nabla}_Y \tilde{\nabla}_X Z - \tilde{\nabla}_{[X, Y]} Z$$

Thus from equations (2.25) and (2.15) the above equation becomes

$$\tilde{R}(X, Y, Z) = R(X, Y, Z) \tag{4.1}$$

This can be stated as follows:

**PROPOSITION 4.1 .** The curvature tensor  $R$  of the special semi-symmetric recurrent metrics connection  $\tilde{\nabla}$  of manifold  $M$  coincides with that of connection  $\nabla$  of Riemannian manifold.

Now taking inner product of (4.1) with W, we obtain

$$\tilde{R}(X, Y, Z, W) = R(X, Y, Z, W) \quad (4.2)$$

Where  $R(X, Y, Z, W) = g(R(X, Y, Z), W)$

Now from (4.2), we have

$$\tilde{R}(X, Y, Z, W) = -R(X, Y, Z, W) \quad (4.3)$$

$$\tilde{R}(X, Y, Z, W) = -R(X, Y, W, Z) \quad (4.4)$$

Thus from (4.3) and (4.4) we get

$$\tilde{R}(X, Y, Z, W) = R(X, Y, W, Z) \quad (4.5)$$

Also, the curvature tensor R satisfies

$$\tilde{R}(X, Y, Z) + \tilde{R}(Y, Z, X) + \tilde{R}(Z, X, Y) = 0 \quad (4.6)$$

which is the Bianchi's first identity for connection  $\tilde{\nabla}$

Again contracting (4.2) over X and W, we get

$$\tilde{S}(Y, Z) = S(Y, Z) \quad (4.7)$$

where  $\tilde{S}$  and S denote the Ricci tensor of the connection  $\tilde{\nabla}$  and  $\nabla$  respectively.

Contracting (4.7) over Y and Z, we have as

$$\tilde{r} = r \quad (4.8)$$

Thus we have the following proposition

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**PROPOSITION 4.2.** From- dimensional SP-Sasakian manifold with special semi-symmetric recurrent metric connection  $\tilde{\nabla}$

(1) The curvature tensor R is given by the equation (4.1)

(2) The Ricci tensor S is given by the equation (4.7)

(3)  $\tilde{r} = r$

**5. CONCURRICULAR CURVATURE TENSOR OF SP-SASAKIAN MANIFOLD WITH RESPECT TO THE SPECIAL SEMI-SYMMETRIC RECURRENT METRIC CONNECTION  $\tilde{\nabla}$**

Define concircular curvature tensor with respect to the special semi-symmetric recurrent metric connection  $\tilde{\nabla}$

$$C(X, Y, Z) = R(X, Y, Z) - \frac{r}{n(n-1)}g(Y, Z)X - g(X, Z)Y \quad (5.1)$$

Using (4.1) in the above equation, we get

$$\tilde{C}(X, Y, Z) = C(X, Y, Z) \quad (5.2)$$

Therefore, we have

**PROPOSITION 5.1 ;**The manifold  $M^n$  coincides with Riemannian manifold.

An n-dimensional SP-Sasakian manifold is called ( $\phi$ )concircularly flat if it satisfied the condition

$$\phi^2C(\phi X, \phi Y)\phi Z=0 \tag{5.3}$$

Let us suppose that M be an n- dimensional  $\phi$ - concircularly flat SP-Sasakian manifold with respect to special semi-symmetric recurrent metric connection. Thus

$$\phi^2C(\phi X, \phi Y)\phi Z=0 \tag{5.4}$$

$$\text{if } g(C(\phi X, \phi Y)\phi Z, \phi W)=0 \tag{5.5}$$

for all  $X, Y, Z, W \in T(M)$

Using (5.1) in (5.5) we have

$$g(R(\phi X, \phi Y)\phi Z, \phi W)=r/n(n-1)\{g(\phi Y, \phi Z)g(\phi X, \phi W)-g(\phi X, \phi Z)g(\phi Y, \phi W)\} \tag{5.6}$$

Let  $\{e_1, e_2, \dots, \xi\}$  be a local orthogonal basis of the vector in  $M^n$ . Thus using the fact that

$\{\phi e_1, \phi e_2, \dots, \xi\}$  is also a orthogonal basis, taking  $X=W=e_i$  in ( 5.6) and summing

with respect to  $i$ , we have

$$S(\phi Y, \phi Z)=\frac{r}{2n}g(\phi Y, \phi Z) \tag{5.7}$$

Putting  $Y=\phi Y, Z=\phi Z$  in (5.7) and using the fact that S is symmetric, we have

$$g(R(\phi X, \phi Y)\phi Z, \phi W)=0$$

This can be stated as:

**THEOREM 5.1;** An n- dimensional  $\phi$  - concircularly flat SP-Sasakian manifold with respect to special semi-symmetric recurrent metric connection coincides with Riemannian manifold.

Also, if  $M^n$  is  $\phi$  - concircularly flat SP-Sasakian manifold with respect to special semi-symmetric recurrent metric connection, then from (5.05) we have

$$g(R(\phi X, \phi Y)\phi Z, \phi W) = \frac{r}{n(n-1)}\{g(\phi Y, \phi Z)g(\phi X, \phi W) -g(\phi X, \phi Z)g(\phi Y, \phi W)\}$$

Contracting over X and W and summing over i, we get

$$S(\phi Y, \phi Z) + g(\phi Y, \phi Z) = \frac{r}{n(n-1)}\{(n-1)g(\phi Y, \phi Z)-g(\phi Y, \phi Z)\}$$

$$S(\phi Y, \phi Z) = \frac{r(n-2)}{n(n-1)-1}g(\phi Y, \phi Z)$$

Using (2.12) and (2.3) in the above equation, we get

$$S(Y, Z) = \frac{r(n-2)}{n(n-1)-1}g(Y, Z) - \frac{r(n-2)}{n(n-1)-n}\eta(Y)\eta(Z) \tag{5.8}$$

Which is of the form.

$$S(Y, Z) = ag(Y, Z) + b\eta(Y)\eta(Z)$$

$$\text{Where } a = \frac{r(n-2)}{n(n-1)-1} \text{ and } b = \frac{r(n-2)}{n(n-1)-n}$$

Thus we have the following theorem:

**THEOREM 5.2 ;**  $\phi$ -concircularly flat SP-Sasakian manifold with respect to special semi-symmetric metric connections an  $\eta$ -Einstein manifold.

**6.PROJECTIVE CURVATURE TENSOR OF SP-SASAKIAN MANIFOLD WITH RESPECT TO SPECIAL SEMI-SYMMETRIC RECURRENT METRIC CONNECTION**

Analogous to the definition of projective curvature tensor in a Riemannian manifold

we define the projective curvature tensor P with respect to special semi-symmetric recurrent metric connection by

$$P(X, Y, Z) = R(X, Y, Z) - \frac{1}{(n-1)}S(Y, Z)X - S(X, Z)Y \} \quad (6.1)$$

Using (4.1) and (4.7) in (6.1) we have

$$\tilde{P}(X, Y, Z) = P(X, Y, Z) \quad (6.2)$$

DEFINITION 6.1 ;An SP-Sasakian manifold  $M^n$  is said to be quasi projectively flat with respect to special semi-symmetric recurrent metric connection if

$$g(P(\phi X, Y)Z, \phi W) = 0 \quad (6.3)$$

Where P is the projective curvature tensor with respect to special semi-symmetric recurrent metric connection .

Using (6.1) in (6.3)

$$g(R(\phi X, Y)Z, \phi W) = \{S(Y, Z)g(\phi X, \phi W) - S(\phi X, Z)g(Y, \phi W)\} \quad (6.4)$$

Now from (4.1) in (4.7) , above equation becomes

$$g(R(\phi X, Y)Z, \phi W) = \frac{1}{(n-1)}\{S(Y, Z)g(\phi X, \phi W) - \frac{1}{(n-1)}\{S(\phi X, Z)g(Y, \phi W)\}$$

Substituting  $X=W=e_i$  and summing over  $i, 1 \leq i \leq n-1$

$$\sum g(R(\phi e_i, Y)Z, \phi e_i) = \frac{1}{(n-1)}\sum [S(Y, Z)g(\phi e_i, \phi e_i) - S(\phi e_i, Z)g(Y, \phi e_i)]$$

This implies

$$S(Y, Z) + g(Y, Z) = \frac{1}{(n-1)}\{(n-1)S(Y, Z) - S(Y, Z)\}$$

This gives

$$S(Y, Z) = -(n-1)g(Y, Z) \quad (6.7)$$

Which is of the form

$$S(Y, Z) = ag(Y, Z)$$

Where  $a = -(n-1)$

So we have following theorem

THEOREM 6.1; A quasi-projectively flat SP- Sasakian manifold with respect to semi-symmetric recurrent metric is an Einstein manifold.

**7.PROJECTIVE RICCI CURVATURE TENSOR OF SP-SASAKIAN MANIFOLD WITH RESPECT TO SPECIAL SEMI-SYMMETRIC RECURRENT METRIC CONNECTION**

we define the projective ricci curvature tensor L with respect to special semi-symmetric recurrent metric connection by

$$L(X,Y)=\frac{n}{(n-1)}S(X,Y)-\frac{r}{(n-1)}g(X, Y) \tag{7.1}$$

Analogous to this definition of projective ricci curvature tensor in a Riemannian manifold, we define the projective ricci curvature tensor L with respect to special semi-symmetric recurrent metric connection by

$$\tilde{L}(X,Y)=\frac{n}{(n-1)}\tilde{S}(X,Y)-\frac{\tilde{r}}{(n-1)}g(X, Y) \tag{7.2}$$

**THEOREM 7.1** ;The projective ricci curvature tensor of SP-sasakian manifold admitting special semi-symmetric recurrent metric connection is identical with the the projective ricci curvature tensor of SP- Sasakian manifold with levi-civita connection.

**PROOF 7.1** ; using (4.7) and (4.8) in equation (7.2)

$$\tilde{L}(X,Y)=\frac{n}{(n-1)}S(X,Y)-\frac{r}{(n-1)}g(X, Y) \tag{7.3}$$

And equating (7.3) and (7.1), we have

$$\tilde{L}(X,Y)=L(X,Y) \tag{7.4}$$

Thus the theorem follows.

**8. M-PROJECTIVE CURVATURE TENSOR OF SP-SASAKIAN MANIFOLD WITH RESPECT TO SPECIAL SEMI-SYMMETRIC RECURRENT METRIC CONNECTION**

we define the M-projective curvature tensor W with respect to special semi-symmetric recurrent metric connection by

**DEFINITION 8.1**; The M-projective curvature tensor (W) is defined as [21]

$$W(X,Y,Z,U)=R(X,Y,Z,U) - \frac{1}{2(n-1)}[S(Y,Z)g(X,U)-S(X,Z)g(Y,U)] - \frac{1}{2(n-1)}[g(Y,Z)S(X,U)-g(X,Z)S(Y,U)] \tag{8.1}$$

Analogous to this definition of M-projective curvature tensor in a Riemannian manifold, we define the M-projective curvature tensor W with respect to special semi-symmetric recurrent metric connection by

$$\tilde{W}(X,Y,Z,U)=\tilde{R}(X,Y,Z,U) - \frac{1}{2(n-1)}[\tilde{S}(Y,Z)g(X,U)-\tilde{S}(X,Z)g(Y,U)] - \frac{1}{2(n-1)}[g(Y,Z)\tilde{S}(X,U)-g(X,Z)\tilde{S}(Y,U)] \tag{8.2}$$

**THEOREM 8.1**; The M-projective curvature tensor of SP-sasakian manifold admitting special semi-symmetric recurrent metric connection is identical with the the M-projective curvature tensor of SP- Sasakian manifold with Levi-civita connection.

**PROOF**; using (4.2), (4.7) and (4.8) in equation (8.2)

$$\tilde{W}(X,Y,Z,U)=R(X,Y,Z,U) - \frac{1}{2(n-1)}[S(Y,Z)g(X,U)-S(X,Z)g(Y,U)] - \frac{1}{2(n-1)}[g(Y,Z)S(X,U)-g(X,Z)S(Y,U)] \tag{8.3}$$



And equating (8.3) and (8.1), we have

$$\tilde{W}(X,Y,Z,U)=W(X,Y,Z,U) \quad (8.4)$$

Thus the theorem follows.

## REFERENCES

- [1] Golab,S.(1975) on semi symmetric and Quarter symmetric linear connections *Tensors*,N.S.29,293-301.Sato,I.(1976) On structure similar to almost contact structure I. *Tensor*,N.S., Vol.30,219-224,
- [2] Adati,T. and Motsumoto,K(1977);On conformally recurrent and conformally symmetric P-Sasakian manifolds.*TRU Math.*, Vol.13,25-32,
- [3] Adati,T. and Miyazama,T.(1977),Some properties of P-Sasakian manifold,*TRU,Math*,Vol,13(1),33-42
- [4] Sinha ,B.B.andKalpana (1980) :Semi-symmetric connection in an SP-Sasakian manifold, *Progress of Mathematics*,Vol. 14,Nos.1&2,77-83.
- [5] Mishra ,R.S. and Pandey S.N.(1980) On Quarter symmetric metric F-connections,*Tensors*,N.S..34.1,-07,
- [6] Yano,K.and Imai, T.(1982) Quarter symmetric connections and their curvature tensor,*Tensors*,N.S..38.13-18;
- [7] Rastogi,S,C,(1987) A note On Quarter symmetric metric connections,*Indian J. Pure App. Math* 18-12 ,1107-112,
- [8] Mukhopadhyay,S, Roy A.K.andBarua,B(1991)Some properties of a Quarter-symmetric metric connection on a riemannian manifold, *Soochow J.Math*.17,205-211,
- [9] Tarafdar,D.andDe,U.C(1993) On a type of P-Sasakian manifold.*Extracta Mathematicae*,Vol.8,31-36
- [10] Biswas, S.C.andDe,U.C.(1997) Quarter symmetric metric connection in SP-Sasakian manifold *commun.fac,Sci.Univ.Ank.series*46,49-56.
- [11] Singh, R.N. , On quarter-symmetric connections, *Vikram Mathematical J.* 17 (1997) 45--54.
- [12] De,U.C.andJ.Sengupta ,(2000)Quarter symmetric metric connection in Sasakian manifold .*commun.fac,Sci.Univ.Ank.seriesA1* Vol49,7-13,
- [13] Jaiswal, V.K,Ojha,R.H.andPrasad,B.(2001).A semi symmetric  $\phi$ -connection in an LP-Sasakian , *J.Nat.Acad . Math*.15,73-78,
- [14] "Ozg"ur C. "Ozg"ur.(2005) : On a class of Para-Sasakian manifolds. *Turk. J. Math.*, Vol.29 , 249-257,
- [15] Singh,R.N.,andPandey, M.K.(2007) On a type of Quarter Symmetric non metric connection in a Kenmotsu manifold, *Bull.Cal. Math. Soc.*, 99(4),433-444,
- [16] De,U.C.andMondal,A.K.(2008) Quarter symmetric connection on sasakian manifold,*Bull Math.Anal.App*.1,99-108.
- [17] Sular,SOzgur, C and De,U.C.(2008)Quarter Symmetric non metric connection in a Kenmotsu manifold, *S.U.T.J.Math*.44,297-308,
- [18] Chaube,S.K.andOjha,R.H.(2010)Quarter symmetric non- metric connection in on an almost Hermitian manifold, *Bull.of math anal.and application*. Vol.2pp77-83,
- [19] Shukla, N.V.C and Saha,R.J.(2014) On semi-symmetric non metric connection in p- sasakian Manifolds *Bull.cal. math soc.*,106(1) 73-7.
- [20] Shukla, N.V.C and Pandey,A.C.(2014) Some properties on semi-symmetric metric T-connection on sasakian Manifold *J.A.M*.1946-1954.
- [21] Shukla, N.V.C and Jaiswal,jyoti (2014)Some curvature properties of LP- sasakian manifold with Quarter-symmetric metric connection *JRAPS* Vol13,pp.193-202.

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