Deriving Equations for Energy Equation by Fem

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Abstract — *In this paper I derived equations for energy equation after applying finite element analysis along with boundary conditions by mathematica 4.1 commands.*

Keywords — ***Energy*** *equation, Finite Element Analysis, Mathematica 4.1.*

I. INTRODUCTION

In the chemical process industries we come across situations where heat is either given out or absorbed, fluids are either heated or cooled and where it is necessary to prevent the loss of heat from a hot vessel or steam pipe. Therefore, in evaporators, dryers, distillation units, furnaces, and reactors, one of the major problems is that of transfer of heat at the desired rate. The control of heat flow in the desired manner forms one of the most important sectors of chemical engineering. Some applications of heat transfer are Heating of water in a gas geyser, heating of a room in winter (using a heat pump), cooling of a room in summer, cooling of water and beverages and preservation of foods (using a refrigerator), cooling of automobile engines and the generation of power/electricity in thermal power plants (heat produced by the combustion of fuels is used to generate high pressure steam which, in turn, used to drive the turbines). [1]

Among different numerical techniques now a days finite element element are using many of them. Compare to other methods in finite element method we descretize the entire domain into subdomains and assembling all these we minimize the error.[2].Stephen Wolform created wonder in Mathematics his team developed mathematics software Mathematica 4 Version. This mathematica software is useful for those who are doing Research. [3].

II. HEAT EQUATION (GOVERNING DIMENSIONLESS EQUATION)

$$\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}+PS\frac{\partial\theta}{\partial y}-\alpha\theta+PEcG(u_y^2+u_z^2)+$$

$$+PEcG\,D^{-1}u^2=N_1PGU \qquad (1)$$

Where

$D=\dfrac{k}{b^2}$ is the Darcy parameter.

$N_1=\dfrac{Ab}{T_1-T_0}$ is the non-dimensional temperature gradient.

$P=\dfrac{\mu C_P}{k_1}$ is the Prandtl number.

$\alpha=\dfrac{QL^2}{k_1}$ is the heat source parameter.

$G=\dfrac{\beta g b^3(T_1-T_0)}{\nu^2}$ is the Grashof number.

$Ec=\dfrac{\beta\, gb}{C_p}$ is the Eckert number.

$S=\dfrac{V_o\, b}{v}$ is the suction Reynolds number.

The corresponding boundary conditions in the non-dimensional form are

$u = 0$ on $y = 1$ $\quad\theta = 1$ $C = 1$ at $x = 0$ on $y = 1$ (2)

$\theta = 1+N_1x$, $C= 1+ N_2x$, $x \neq 0$ on $y = 1$ (3)

$$\frac{\partial u}{\partial y}=0,\ \frac{\partial\theta}{\partial y}=0 \text{ and } \frac{\partial C}{\partial y}=0 \quad \text{on } y=0 \qquad (4)$$

After applying finite element analysis we get

$$\int_{\Omega_i}[\sum_{k=1}^{8}\theta_k^i\{\frac{\partial N_k^i}{\partial y}\frac{\partial N_j^i}{\partial y}+\frac{\partial N_k^i}{\partial z}\frac{\partial N_j^i}{\partial z}+$$

$$+PSN_j^i\frac{\partial N_k^i}{\partial y}-\alpha N_j^iN_k^i\}$$

$$+\int_{\Omega_i}\sum_{k=1}^{8}u_k^i\{PEcG((\frac{\partial N_k^i}{\partial y})^2+(\frac{\partial N_k^i}{\partial z})^2)+$$

$$+D^{-1}(N_k^i)^2)-N_1PGN_k^i\}N_j^i]d\Omega_i=(Q^T)_j^i$$

$$(Q^T)^i_j = \oint_{\Gamma_i} [N^i_j \frac{\partial \theta^i}{\partial y} n_y + N^i_j \frac{\partial \theta^i}{\partial z} n_z] d\Gamma_i,$$

$$, j = 1, 2, \ldots\ldots\ldots, 8. \qquad (5)$$

III.DERIVING EQUATIONS FOR HEAT EQUATION BY MATHEMATICA COMMANDS

Shape functions

$$s_1 := -4*(-1+y)*\left(\frac{y}{2}+\frac{1}{2}\left(-\frac{1}{2}+z\right)\right)*(-1+z)$$

$$s_2 := 4*(-1+y)*(-1+z)*z$$

$$s_3 := -4*(-1+y)*\left(-\frac{y}{2}+\frac{1}{2}\left(-\frac{1}{2}+z\right)\right)*z$$

$$s_4 := -4*(-1+y)*y*z$$

$$s_5 := 4*y*\left(\frac{1}{2}\left(-\frac{1}{2}+y\right)+\frac{1}{2}(-1+z)\right)*z$$

$$s_6 := -4*y*(-1+z)*z$$

$$s_7 := -4*y*\left(\frac{1}{2}(-1+y)+\frac{1}{2}\left(\frac{1}{2}-z\right)\right)*(-1+z)$$

$$s_8 := 4*(-1+y)*y*(-1+z)$$

Writing equation (5) in Mathematica syntax and executing we get the output 8 equations.

$$Do[\mathrm{Print}[t_i = \sum_{j=1}^{8} \theta_j * \int_0^1\int_0^1 \Big(\partial_y s_i * \partial_y s_j + \partial_z s_i * \partial_z s_j + p*s*s_i*\partial_y s_j - \alpha * s_i * s_j\Big) dydz$$
$$+\sum_{j=1}^{8} u_j * \int_0^1\int_0^1 \Big(p*k*G*s_i*\big((\partial_y s_j)\wedge 2 + (\partial_z s_j)\wedge 2\big) + p*k*G*ad*s_i*\big((s_j)\wedge 2\big) - n_1 * p*G*s_i*s_j\Big)$$
$$dydz \,/.\, u_5 -> 0 \,/.\, u_6 -> 0 \,/.\, u_7 -> 0 \,/.\, \theta_5 -> 1 \,/.\, \theta_6 -> 1 \,/.\, \theta_7 -> 1], \{i,1,8\}] \qquad (6)$$

Output for equation (6)

1st Equation

$$\frac{1}{2}-\frac{5ps}{36}+\frac{\alpha}{60}+$$
$$\left(\frac{299G\,K\,P}{1260}+\frac{11\,ad\,G\,K\,P}{2800}-\frac{1}{30}G\,P\,n_1\right)u_1+$$
$$\left(\frac{4G\,K\,P}{63}-\frac{3}{175}ad\;G\,K\,P+\frac{1}{30}G\,P\,n_1\right)u_2$$
$$+\left(-\frac{19}{210}G\,K\,P-\frac{19ad\,G\,K\,P}{5040}-\frac{1}{90}G\,P\,n_1\right)u_3+$$
$$\left(-\frac{64}{315}G\,K\,P-\frac{37ad\,G\,K\,P}{1575}+\frac{2}{45}G\,P\,n_1\right)u_4$$
$$+\left(\frac{4G\,K\,P}{63}-\frac{3}{175}ad\;G\,K\,P+\frac{1}{30}G\,P\,n_1\right)u_8+$$
$$\left(\frac{52}{45}-\frac{P\,S}{15}-\frac{\alpha}{30}\right)\theta_1+\left(-\frac{37}{45}+\frac{7P\,S}{90}+\frac{\alpha}{30}\right)\theta_2$$
$$+\left(\frac{1}{2}+\frac{P\,S}{60}-\frac{\alpha}{90}\right)\theta_3+\left(-\frac{23}{45}+\frac{2\alpha}{45}\right)\theta_4+$$
$$\left(-\frac{37}{45}+\frac{P\,S}{9}+\frac{\alpha}{30}\right)\theta_8$$

2nd Equation

$$-\frac{2}{3}+\frac{ps}{9}+$$
$$\left(\frac{26G\,K\,P}{63}+\frac{1}{84}ad\;G\,K\,P+\frac{1}{30}G\,P\,n_1\right)u_1+$$
$$\left(\frac{16\,G\,K\,P}{21}+\frac{4}{35}ad\;G\,K\,P-\frac{8}{45}G\,P\,n_1\right)u_2$$
$$+\left(\frac{26G\,K\,P}{63}+\frac{1}{84}adG\,K\,P+\frac{1}{30}G\,P\,n_1\right)u_3+$$
$$\left(\frac{32G\,K\,P}{45}+\frac{4}{75}ad\,G\,K\,P-\frac{1}{9}G\,P\,n_1\right)u_4$$
$$+\left(\frac{32G\,K\,P}{45}+\frac{4}{75}ad\;G\,K\,P-\frac{1}{9}G\,P\,n_1\right)u_8+$$
$$\left(-\frac{37}{45}-\frac{13P\,S}{90}+\frac{\alpha}{30}\right)\theta_1+\left(\frac{104}{45}-\frac{4P\,S}{15}-\frac{8\alpha}{45}\right)\theta_2$$
$$+\left(-\frac{37}{45}-\frac{13P\,S}{90}+\frac{\alpha}{30}\right)\theta_3+\left(\frac{2PS}{9}-\frac{\alpha}{9}\right)\theta_4+$$
$$\left(\frac{2P\,S}{9}-\frac{\alpha}{9}\right)\theta_8$$

3rd Equation

$$\frac{1}{2}-\frac{5ps}{36}+\frac{\alpha}{60}+$$
$$\left(-\frac{19}{210}GKP-\frac{19ad\,GKP}{5040}-\frac{1}{90}GP_{n_1}\right)u_1+$$
$$\left(\frac{4\,GKP}{63}-\frac{3}{175}ad\,GKP+\frac{1}{30}GP_{n_1}\right)u_2$$
$$+\left(\frac{299GKP}{1260}+\frac{11adGKP}{2800}-\frac{1}{30}GP_{n_1}\right)u_3+$$
$$\left(\frac{4GKP}{63}-\frac{3}{175}ad\,GKP+\frac{1}{30}GP_{n_1}\right)u_4$$
$$+\left(-\frac{64}{315}GKP-\frac{37ad\,GKP}{1575}+\frac{2}{45}GP_{n_1}\right)u_8+$$
$$\left(\frac{1}{2}+\frac{PS}{60}-\frac{\alpha}{90}\right)\theta_1+\left(-\frac{37}{45}+\frac{7PS}{90}+\frac{\alpha}{30}\right)\theta_2$$
$$+\left(\frac{52}{45}-\frac{PS}{15}-\frac{\alpha}{30}\right)\theta_3+\left(-\frac{37}{45}+\frac{PS}{9}+\frac{\alpha}{30}\right)\theta_4+$$
$$\left(-\frac{23}{45}+\frac{2\alpha}{45}\right)\theta_8$$

4th Equation

$$-\frac{4}{3}+\frac{ps}{3}-\frac{\alpha}{30}+$$
$$\left(\frac{53GKP}{315}+\frac{53ad\,GKP}{6300}+\frac{2}{45}GP_{n_1}\right)u_1+$$
$$\left(\frac{32\,GKP}{45}+\frac{4}{75}ad\,GKP-\frac{1}{9}GP_{n_1}\right)u_2$$
$$+\left(\frac{26GKP}{63}+\frac{1}{84}adGKP+\frac{1}{30}GP_{n_1}\right)u_3+$$
$$\left(\frac{16GKP}{21}+\frac{4}{35}ad\,GKP-\frac{8}{45}GP_{n_1}\right)u_4$$
$$+\left(\frac{128GKP}{315}+\frac{4}{105}ad\,GKP-\frac{4}{45}GP_{n_1}\right)u_8+$$
$$\left(-\frac{23}{45}+\frac{2\alpha}{45}\right)\theta_1+\frac{1}{9}\left(-2PS-\alpha\right)\theta_2$$
$$+\left(-\frac{37}{45}-\frac{PS}{9}+\frac{\alpha}{30}\right)\theta_3+\left(\frac{104}{45}-\frac{8\alpha}{45}\right)\theta_4+$$
$$\left(\frac{16}{45}-\frac{4\alpha}{45}\right)\theta_8$$

5th Equation

$$\frac{5}{6}-\frac{ps}{36}-\frac{\alpha}{90}+$$
$$\left(-\frac{79GKP}{1260}-\frac{31ad\,GKP}{8400}-\frac{1}{60}GP_{n_1}\right)u_1+$$
$$\left(-\frac{64}{315}GKP-\frac{37ad\,GKP}{1575}+\frac{2}{45}GP_{n_1}\right)u_2$$
$$+\left(-\frac{19}{210}GKP-\frac{19adGKP}{5040}-\frac{1}{90}GP_{n_1}\right)u_3+$$
$$\left(\frac{4GKP}{63}-\frac{3}{175}ad\,GKP+\frac{1}{30}GP_{n_1}\right)u_4$$
$$+\left(-\frac{64}{315}GKP-\frac{37ad\,GKP}{1575}+\frac{2}{45}GP_{n_1}\right)u_8+$$
$$\left(\frac{23}{45}+\frac{PS}{60}-\frac{\alpha}{60}\right)\theta_1+\left(-\frac{23}{45}+\frac{7PS}{90}+\frac{2\alpha}{45}\right)\theta_2$$
$$+\left(\frac{1}{2}+\frac{2PS}{45}-\frac{\alpha}{90}\right)\theta_3+\left(-\frac{37}{45}-\frac{PS}{9}+\frac{\alpha}{30}\right)\theta_4+$$
$$\left(-\frac{23}{45}+\frac{2\alpha}{45}\right)\theta_8$$

6th Equation

$$\frac{2}{3}+\frac{5ps}{9}-\frac{\alpha}{9}+$$
$$\left(\frac{53GKP}{315}+\frac{53ad\,GKP}{6300}+\frac{2}{45}GP_{n_1}\right)u_1+$$
$$\left(\frac{128GKP}{315}+\frac{4}{105}ad\,GKP-\frac{4}{45}GP_{n_1}\right)u_2$$
$$+\left(\frac{53GKP}{315}+\frac{53adGKP}{6300}+\frac{2}{45}GP_{n_1}\right)u_3+$$
$$\left(\frac{32GKP}{45}+\frac{4}{75}ad\,GKP-\frac{1}{9}GP_{n_1}\right)u_4$$
$$+\left(\frac{32GKP}{45}+\frac{4}{75}ad\,GKP-\frac{1}{9}GP_{n_1}\right)u_8+$$
$$\left(-\frac{23}{45}+\frac{7PS}{90}+\frac{2\alpha}{45}\right)\theta_1+\left(\frac{16}{45}-\frac{4PS}{15}-\frac{4\alpha}{45}\right)\theta_2$$
$$+\left(-\frac{23}{45}+\frac{7PS}{90}+\frac{2\alpha}{45}\right)\theta_3+\frac{1}{9}\left(-2PS-\alpha\right)\theta_4+$$
$$\left(-\frac{2PS}{9}-\frac{\alpha}{9}\right)\theta_8$$

7th Equation

$$\frac{5}{6}-\frac{ps}{36}-\frac{\alpha}{90}+$$
$$\left(-\frac{19}{210}G\ K\ P-\frac{19ad\ G\ K\ P}{5040}-\frac{1}{90}G\ P\ n_1\right)u_1+$$
$$\left(-\frac{64}{315}G\ K\ P\text{-}\frac{37ad\ G\ K\ P}{1575}+\frac{2}{45}G\ P\ n_1\right)u_2$$
$$+\left(-\frac{79G\ K\ P}{1260}-\frac{31adG\ K\ P}{8400}-\frac{1}{60}G\ P\ n_1\right)u_3+$$
$$\left(-\frac{64}{315}G\ K\ P\text{-}\frac{37ad\ G\ K\ P}{1575}+\frac{2}{45}G\ P\ n_1\right)u_4$$
$$+\left(\frac{4G\ K\ P}{63}-\frac{3}{175}ad\ G\ K\ P+\frac{1}{30}G\ P\ n_1\right)u_8+$$
$$\left(\frac{1}{2}+\frac{2PS}{45}-\frac{\alpha}{90}\right)\theta_1+\left(-\frac{23}{45}+\frac{7PS}{90}+\frac{2\alpha}{45}\right)\theta_2$$
$$+\left(\frac{23}{45}+\frac{P\ S}{60}-\frac{\alpha}{60}\right)\theta_3+\left(-\frac{23}{45}+\frac{2\alpha}{45}\right)\theta_4+$$
$$\left(-\frac{37}{45}-\frac{PS}{9}+\frac{\alpha}{30}\right)\theta_8$$

8th Equation

$$-\frac{4}{3}+\frac{ps}{3}-\frac{\alpha}{30}+$$
$$\left(\frac{26G\ K\ P}{63}+\frac{1}{84}ad\ G\ K\ P+\frac{1}{30}G\ P\ n_1\right)u_1+$$
$$\left(\frac{32G\ K\ P}{45}+\frac{4}{75}ad\ G\ K\ P\text{-}\frac{1}{9}G\ P\ n_1\right)u_2$$
$$+\left(\frac{53G\ K\ P}{315}+\frac{53adG\ K\ P}{6300}+\frac{2}{45}G\ P\ n_1\right)u_3+$$
$$\left(\frac{128G\ K\ P}{315}+\frac{4}{105}ad\ G\ K\ P\text{-}\frac{4}{45}G\ P\ n_1\right)u_4$$
$$+\left(\frac{16G\ K\ P}{21}+\frac{4}{35}ad\ G\ K\ P\text{-}\frac{8}{45}G\ P\ n_1\right)u_8+$$
$$\left(-\frac{37}{45}-\frac{PS}{9}+\frac{\alpha}{30}\right)\theta_1+\left(-\frac{2PS}{9}-\frac{\alpha}{9}\right)\theta_2$$
$$+\left(-\frac{23}{45}+\frac{2\alpha}{45}\right)\theta_3+\left(\frac{16}{45}-\frac{4\alpha}{45}\right)\theta_4+$$
$$\left(\frac{104}{45}-\frac{8\alpha}{45}\right)\theta_8$$

IV. CONCLUSIONS

1. We derived equations for heat equation by finite element analysis along with boundary conditions.

References

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