Product Cordial Labeling Of Some Fusion Graphs

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1. Abstract:

Here we discuss product cordiality of some fusion graphs.These graphs are obtained by fusing a copy of given graph G2at each vertex of G1.It is denoted by G1FG2.If G1 is not isomorphic to G2 then G1FG2 is not same as G2FG1.We obtain product cordial labeling of antenna graph,CnFk$_{1,m}$,(n = 3,4);SnFK$_{1,m}$where Sn is shel on n-cycle.

Key words*: fusion graph, product cordial, labeling, path. cycle,*

Subject Classification: O5C78

2. Introduction:

All graphs considered here are simple,undirected.For terminology and definations we refer Graph theory by F Harary[6] and J.A.Gallian[7].

Sundaram, Ponraj, and Somasundaram [8] introduced the notion of product cordial labelings. A product cordial labeling of a graph G with vertex set V is a function f :V→ {0,1} such that if each edge uv is assigned the label f(u)f(v), the number of vertices labeled with 0i.e$v_f(0)$and the number of vertices labeled with 1 i.e$v_f(1)$differ by at most 1, and the number of edges labeled with 0 i.e. $e_f(0)$ and the number of edges labeled with 1 i.e. $e_f(1)$differ by at most 1. A graph with a product cordial labeling is called as product cordial graph.(pc-graph).

A lot of work in this type of labeling has been done so far.One should refere [7]

3. Preliminaries:

Definition 3.1 : Fusion of vertices :Let v ϵV(G_1), v' ϵ V(G_2) where G_1 and G_2 are twographs. We fuse v and v' by replacing them with a single vertex say w and all edges incident with v in G_1 and that with v' in G_2 are incident with u in the new graph G=G_1FG_2.Deg $_G$u=deg$_{G1}$(v)$_+$ deg$_{G2}$(v') and |V(G)| =|V(G_1)|+|V(G_2)|-1,|E(G)|=|E(G_1)|+|E(G_2)|

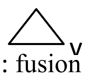
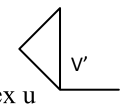
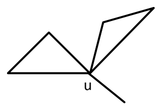
Let G_1=  $G_{2=}$  G= 

Figure3.1: fusion of vertex v and v' at vertex u

The fusion of two vertices in the same graph is described in [5].

Definition3.2: Fusion graph :let G_1 and G_2 be two graphs with |V(G_1)|=P. Take P copies of G_2. Choose a same fixed vertexv' in each copy of G_2.Toeach vertex in G1, fuse v' from one copy each of G_2. The resultant graph is fusion graph of G_1 and G_2 denoted by G_1 F G_2.

Note that |V (G_1 F G_2)|=|V (G_1)| + |V(G_2)|-1.

|E (G_1 F G_2)|=P|E(G_2)| + E(G_1)

Defination3.3 : Antena Graph :Let G be a (p,q) graph. At each vertex of G a path of length m is fused.The resultant graph is antena graph denoted by ant(G,m).We discuss antenna graph of C_3 and C_4.

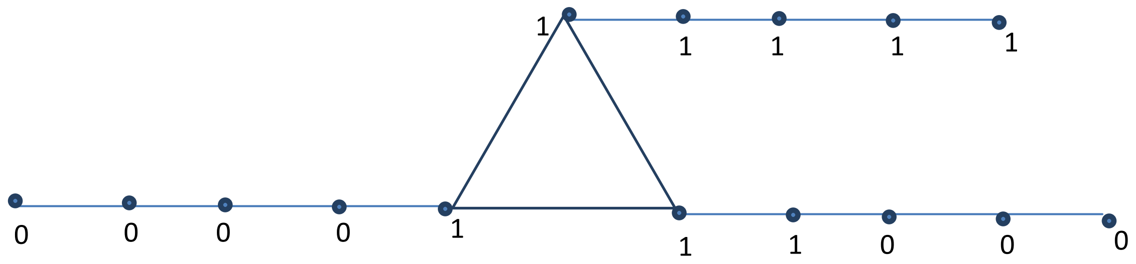
Defination3.4:shel graph: From any one vertex say v of cycle Cn an edge is joined to each vertex on Cnand not adjacent to v.The resultant structure is called as shell S_{n-3}

4. Results Proved:

Theorem4.1 ant(C_3,m) is product cordial.

Let Cn be ($v_1e_1v_2e_2v_3e_{3\ldots}env_{1)}$ The path P_{m+1} fused at vertex i be ($v_ie^i{}_1u^i{}_1e^i{}_2..u^i{}_m$).This will give ant(Cn,m).

Proof: Define a functioin f:V(G)→{0,1} in the following way.



Case m = 2x Fig 4.1 Ant(C_3,4) with product cordial labeling

f(vi) = 1,

$f(u^i{}_j)$ = 1 for i= 1 and j = 1….,m

$f(u^i{}_j)$ = 1 for i= 2 and j = 1,2..,x-1. $f(u^i{}_j)$ = 0 for i = 2 and j = x,…m

$f(u^i{}_j)$ = 0 for i = 3and j = 1…m

Note that $e_f(0)$=3x+2,$e_f(1)$=3x+1 and $v_f(0)$=3x+2,$v_f(1)$=3x+1

Case m = 2x +1

f(vi) = 1,

$f(u^i{}_j)$ = 1 for i = 1 and j = 1….,m

$f(u^i{}_j)$ = 1 for i = 2 and j = 1,2..,x-1. $f(u^i{}_j)$ = 0 for i = 2 and j = x,…m

$f(u^i{}_j)$ = 0 for i = 3and j = 1…m

Note that $e_f(0)$=3x+2,$e_f(1)$=3x+2 and $v_f(0)$=3x+2,$v_f(1)$=3x+2

Theorem 4.3:A fusion graph of C_3 and $K_{1,m}$ is pc graph.

Proof:Let the C_3 be ($v_1e_1v_2e_2v_3e_3v_{1)}$.The copy of $K_{1,m}$ attached at v_i has m pendent edges given by $e^i{}_j=(v_iu^i{}_j)$, i = 1,2,3.;j=1,2,..mWhere$u^i{}_j$ are pendent vertices incident with vertex v_i.

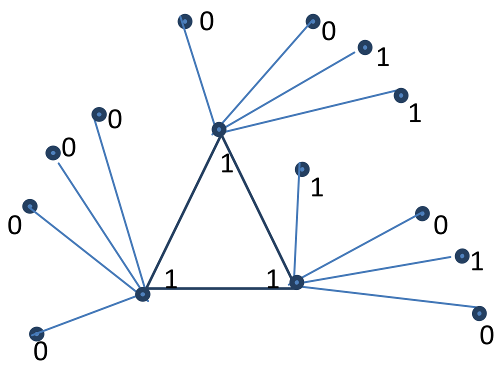


Fig 4.2 $C_3FK_{1,m}$ with product cordial labeling

Define a function f as

f: V→{0,1} given by

f (vi) = 1 for i = 1,2 ,3

$f(u^i_j)$ =0 for all j=1 and i =1,2,3.

$f(u^i_j)$ =1 for all j = 2.. ,x+1 and $f(u^i_j)$ =0 for j=x+2,x+3..m. and i=1,2,3, where m = 2x+1

For m = 2x we have f(vi) = 1 for i = 1,2 ,3

$f(u^i_j)$ =0 for all j=1 and i =1,2,3.

$f(u^i_j)$ =0 for all j=2 and i =1,2 $f(u^i_j)$ =1 for all j=2 and i =,3.

$f(u^i_j)$ =0 for all j=3..m and i =1

.$f(u^i_j)$ =1 for all j=3..m and i = 2.

For i =3 we have,

$f(u^i_j)$ =0 for all j=3,4..,(x+1) and $f(u^i_j)$ =1 for all j=x+2,..m and i =3

Note that when m =2x we have $e_f(0)$=m+x+3 ,$e_f(1)$ = m+x+2 and $v_f(0)$=m+x+3$v_f(1)$=m+x+2 and when m =2x +1 we have $e_f(0)$=m+x+2 ,$e_f(1)$ = m+x+2 and $v_f(0)$=m+x+2,$v_f(1)$=m+x+2 . #

Theorem4.2 Ant(C_4,m) is product cordial.

Fig 4.3 antena(C4,4)

Proof: Define a functioin f:V(G)→{0,1} as

f(vi) = 1,

$f(u^i_j)$ = 1 for i = 1 and j = 1….,m

$f(u^i_j)$ = 1 for i = 2 and j = 1,2..,m-2 $f(u^i_j)$ = 0 for i = 2 and j = m-1,m.

$f(u^i_j)$ = 0 for i = 3,4 and j = 1…m

Note that $e_f(0)$=m+2 =$e_f(1)$ and $v_f(0)$=m+2=$v_f(1)$

m+x+2

Theorem 4.4 $S_nFK_{1,m}$ is product cordial iff (1) n is even or (2) n is odd and m is even

Proof: Let the cycle of shell be defined as Cn= ($v_1e_1v_2e_2v_3e_{3\ldots}env_1$)We draw edges from vertex v_1 to all vertices v_3,v_4..v_{n-1}The pendent edges at vertex v_i be v^i_1,v^i_2,v^i_3…v^i_m

Case n = 2x.

Define f:V(G) →{0,1} as follows:

$f(v_t)$ = 1 for t = 1,2..,x+1

$f(v_t)$ = 0 for t = x+2,x+3,..n

for m = 1,

$f(v^i_j)$ = 1 for j =1,i=1,2

$f(v^i_j) = 0$ for j =1,i=3,4,5

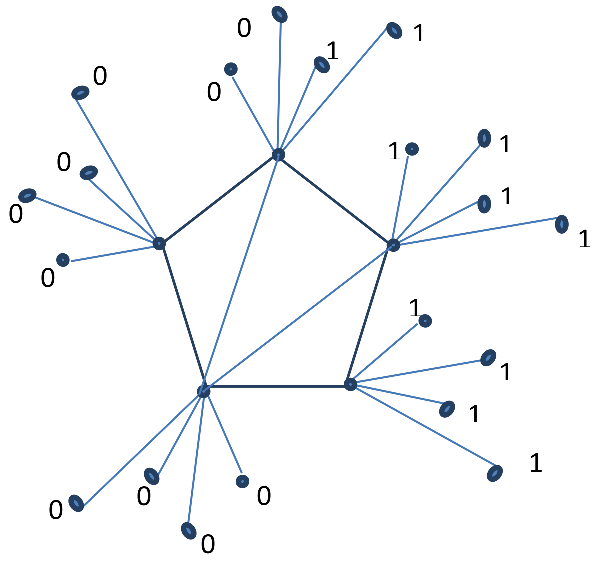


Fig 4.4 : $S_2FK_{1,4}$ Vertex labels are shown.The graph is product cordial.(pc)

At this stage $v_f(0,1) = (4,4)$ and $e_f(0,1) = (5,4)$

For m =2 onwards,

$f(v^i_j) = 1$ for j =2..m and i=1,2

$f(v^i_j) = 0$ for j =2…m,andi=3,4

The resultant label distribution is $v_f(0,1) = (3m+2,3m+2)$ and $e_f(0,1) = (3m+3,3m+2)$

Case n = 2x+1 and m = 2t

Define f:V(G) →{0,1} as follows:

$f(v_t) = 1$ for t = 1,2..,x+1

$f(v_t) = 0$ for t = x+2,x+3,..n

$f(v^i_j) = 1$ for j =1,..m and i=1,2

$f(v^i_j) = 0$ for j =1,….m,andi=4,5

$f(v^i_j) = 0$ for j =1,….t,andi= 3

$f(v^i_j) = 0$ for j =1,….t,andi= 3

The resultant label distribution is $v_f(0,1) = (5(m-1)+2,5(m-1)+3)$ and$e_f(0,1)$ = (5(m-1)+4,5(m-1)+3)

It follows that f is product cordial function.

5. Conclusions:

Concept of fusion has opened high ways to design new families of graphs. In this paper we have tried to develop a few new graphs. We have tried successfully that $Ant(C_n,m)$ is product cordial for n = 3,4.This opens further challenge to check the result for all n for interested researchers.

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