

Research on the Trigonometry as a Main Function of Sine, Secant, Tangent and Formula of Tan and Cot are Inverse

Er. Navin Talkokul

(B.E. Mechanical)

Walchand Institute of Technology, Solapur. India .

Abstract — we all are familiar with the concept of all trigonometry ratios, functions, formulae. In mathematics altogether the trigonometry is mainly concerned with the function of sine, cosine, and tangent. But that is incorrect because the mathematician Aryabhata had introduced the trigonometry. Aryabhata hadn't said anywhere that the trigonometry as main function is sine, cosine, and tangent. Yet, how this function comes into recognition is also a secret. The main purpose of this research paper is to put forward a thesis on trigonometry being a main function of **sine, secant, and tangent** instead of **sine, cosine, and tangent** and also the formula of **tan** and **cot** are inverse.

Keywords— Trigonometry, Aryabhata , sine, secant, and tangent.

INTRODUCTION :-

Trigonometry is the Greek word which means tri – three, Gono – angle and metry means measurement, hence overall it is three angle measurement which is discovered by the great mathematician Aryabhata.

The purpose behind the trigonometry was to find out the largest distance between two objects or to find out the largest height. We don't have a practical scale to find out the distance between earth and sun, but this distance can be found, only with help of triangle because triangle is strongest part in geometry that's why it could have given circle, quadrilateral, hexagon it could give triangle. There are different kind of triangles are available in geometry. So why choose right angle triangle, It is because area of any triangle which is find out by the formula $\frac{1}{2} \times \text{base} \times \text{height}$ and so height is must and most required which is perpendicular to the base and angle between this height and base is 90° . So we must require right angle triangle to find out the length and base meaning sides of triangle.

Trigonometry:-

In astronomy all the planets rotate around the sun in elliptical orbit.

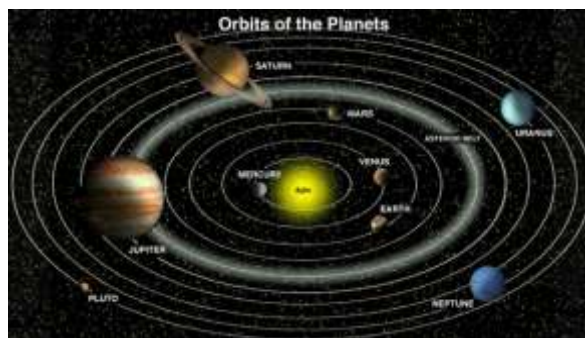


Fig.1

In astronomy sun is the concentric centre of all planets which are moving around the sun. Suppose, if we cut the largest planet which is moving around the sun in elliptical orbit and put it into straight it becomes straight line not arc which is given below.

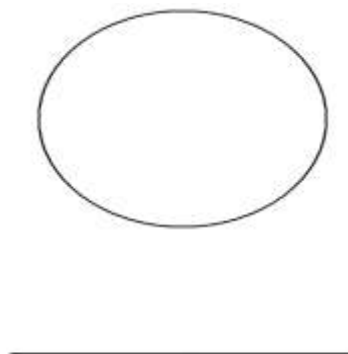


Fig.2

Simultaneously, if we cut all planets which is in elliptical orbit put it into in this manner which given below and join them it becomes triangle.

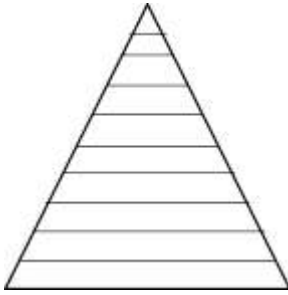


Fig.3

Therefore the triangle is the strongest part in geometry .Then area of any part of geometry which is calculated by triangle like

1) **Square :-**

$$\begin{aligned} \text{Area of square} &= 2 \times \text{area of triangle} \\ &= 2 \times \frac{1}{2} \times a \times a \\ &= a^2 \\ &= \text{side}^2 \end{aligned}$$

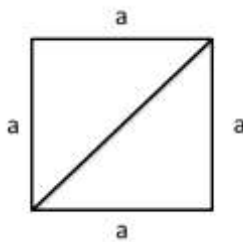


Fig.4

2) **Hexagonal :- (with side 'a') :-**

Area of hexagonal = 6 × area of triangle

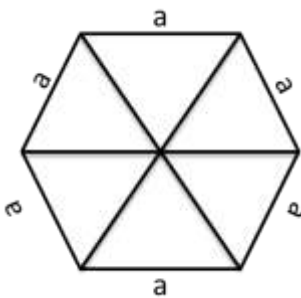


Fig.5

If we increase the number of sides of hexagonal, then it looks like a circle .

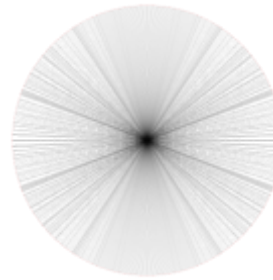


Fig.6

Any part of geometry of its area, height, distance which is calculated by triangle.

In astronomy the purpose of trigonometry is also to find out the distance between sun and earth.

Trigonometry main functions are:-

Sine:-

Consider earth is now at a position 'C' and after some duration at another position 'B' and it is moving around sun in elliptical manner and sun is the center of this ellipse which is 'A', as shown below in the figure .

Join the points A,B and C, then it becomes triangle.

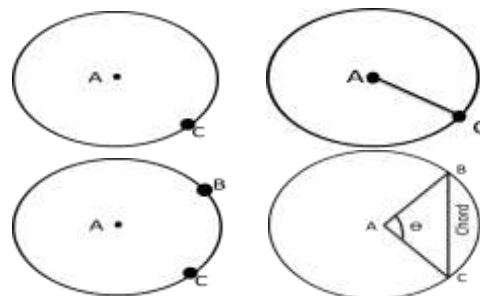


Fig7

Here $AB = AC = x$
 $BC = \text{length of chord}$

The great mathematician Aryabhata named the distance of the chord as jya x which in Sanskrit upheld as half jiva means 'half chord'.

The function sine can be traced to the jyā functions translation from Sanskrit to Arabic and then from Arabic to Latin. The word "sine" comes from Latin mistranslation of the Arabic jiba, which is a transliteration of the Sanskrit word for half the chord.

Draw perpendicular line through point 'A' to the chord which basically bisects the chord at point 'D' and hence $BD = DC = \text{half chord}$, AD is perpendicular to the BC

Hence, $\triangle ADB$ is a right angle triangle.

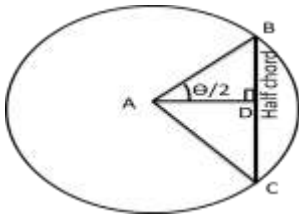


Fig.8

Consider, AD to be the initial arm and AB is the terminal arm and ' $\theta/2$ ' is angle between them. Hence angle $DAB = \theta/2$

Now the definition of sine is

$$\begin{aligned} \text{jya } x &= \text{sine} \times \text{terminal arm} = \text{half chord} \\ \text{sine} \times AB &= BD \\ \text{sine} &= BD/AB \end{aligned}$$

AB is the terminal arm but according to the definition of Pythagoras in right angle triangle the opposite side of 90 is hypotenuse.

AB = terminal arm = hypotenuse
BD = opposite side of angle ($\theta/2$)

The definition of sine :-

It is the ratio of opposite side to the hypotenuse

$$\text{Sin}(\theta/2) = \text{opposite side} / \text{hypotenuse}$$

But, what about AD which is initial arm and also adjacent side of angle ($\theta/2$)

Let start with example of relation,

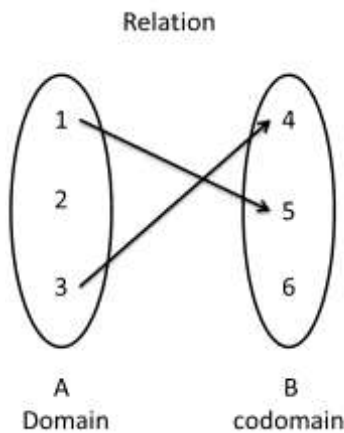


Fig.9

Here 'A' {1, 2, 3} is the domain which is plotted on the x-axis and 'B' {4,5,6} is the codomain which is plotted on y-axis .

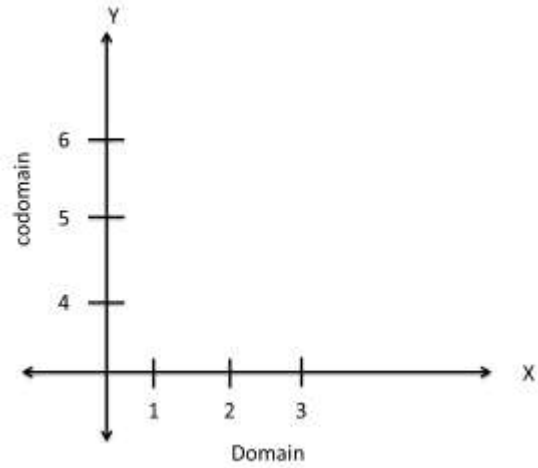


Fig.10

what is y – axis ? and what is codomain?

The y- axis depends on the x – axis i.e. y-axis is related to the x –axis.

Similarly,

Codomain is related to the domain i.e.co-domain means corresponding to the domain

Now, let us take another example to get clearer perspective,

Suppose, A persons interest is mathematics which is on x – axis and B.Sc. M.Sc. and Ph.D. etc. are the field or domain of the same person which is on y-axis. Hence, the domains B.Sc, M.Sc, Ph.d etc. are related to the mathematics which also means that they are corresponding to mathematics i.e. comathematics in this case.

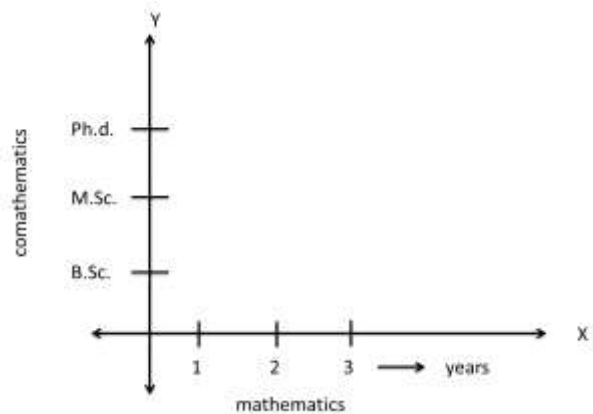


Fig .11

Similarly, in right angle triangle one side is calculated by sine which is opposite to the angle and the other side which is related to sine means corresponding to the sine. i.e. cosine.

The cosine is used to calculate the other side (initial arm) which is adjacent side of a given angle,

Corresponding of sine
 = cosine \times terminal arm = initial arm

$$\text{Cosine} \times AB = AD$$

$$\text{Cosine} = AD/AB$$

Definition of cosine:-

It is the ratio of adjacent side to the hypotenuse

$$\text{Cos}(\theta/2) = \text{adjacent side} / \text{hypotenuse}$$

As we know, the purpose of the trigonometry is to find out the largest distance. Hence, we can find out the same by using the trigonometry functions sine and cosine.

Sine and cosine is only use when triangle is drawn inside ellipse or circle.

Example:-

The compound angles is also found by chord and triangle is drawn inside the circle as shown fig.11.

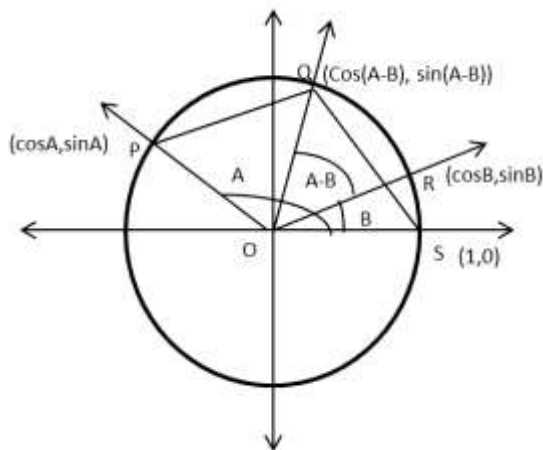


Fig.12

$$\text{Sin}(A+B) = \text{sinAcosB} + \text{cosAsinB}$$

$$\text{Cos}(A-B) = \text{cosAcosB} + \text{sinAsinB}$$

In ellipse,

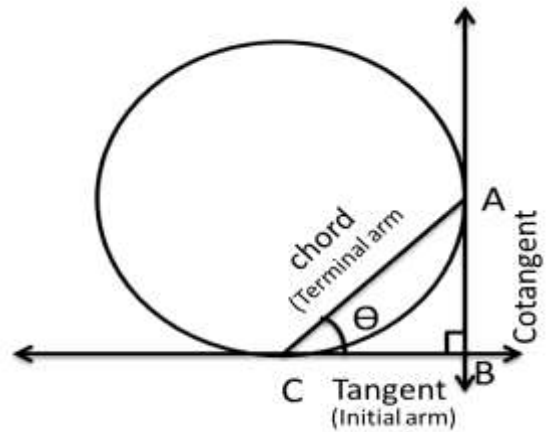


Fig.13

Draw the two tangents to the ellipse which are perpendicular to each other and touching at the point A and point C to the ellipse.

ΔABC is a right angle triangle where AB and BC are the tangents and AC is the chord and angle $ACB = \theta$,

Here, we cannot use the sine and cosine because sine is the jya means half chord.

But, AB and BC are not chords, they are the tangents and AC is chord.

So we can use sine for AC because it is chord and opposite angle of this chord is 90°

$$\text{Sin}90^\circ = \text{opposite} / \text{hypotenuse}$$

But here opposite side and hypotenuse is same i.e. AC

$$\text{Sin}90^\circ = AC/AC = 1$$

$$\text{Hence } \text{sin}90^\circ = 1.$$

BC is the tangent (initial arm) which rotates in anticlockwise direction making an angle θ and AC is chord (terminal arm).

In ΔABC

AB is opposite side, so we cannot declare sine for this side because it is not a chord,

It is a tangent. Also we cannot declare cosine for side BC, because BC also is tangent.

So, how to find out the length of these tangents?

The tangent with adjacent angle ' θ ' is the ratio of first tangent to another tangent.

Tangent with adjacent angle ' θ '

= tangent which is adjacent to the angle ' θ ' / tangent which is opposite to the angle ' θ '

Hence,

$$\tan \theta = \text{adjacent side} / \text{opposite side}$$

$$\tan \theta = BC / AB$$

But, what about AB?

AB is the corresponding of the tangent which is named as ‘Cotangent’.

The cotangent with opposite angle ‘ Θ ’ is the ratio of second tangent to the first tangents

i.e. $\cot \Theta = \text{opposite angle} / \text{adjacent angle}$
 $\cot \Theta = AB / BC$

In above case, AC is the chord and if it passes through the center of ellipse, which is the largest chord which intersects the two tangents at point A and C. Hence, the largest chord becomes secant because it intersecting outside the circle as shown below:

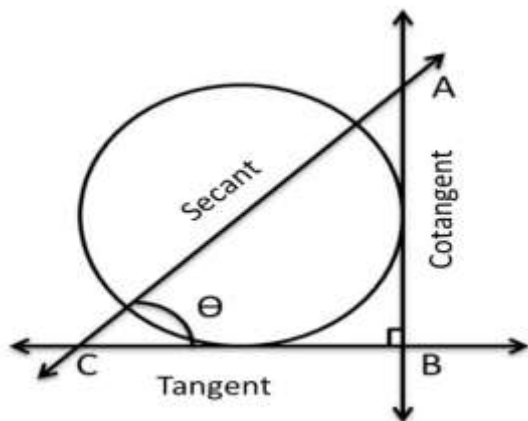


Fig.14

Now in this case AC is not chord, it is secant. Sine is only define for chord. but in above fig.12 AC is not chord, it is secant

We already define for side BC and AB which are tangents to the ellipse

Let’s find out the secant,

The secant is making an angle Θ with initial arm. i.e. tangent.

Secant with an angle Θ is the ratio of terminal arm (secant) to the initial arm (tangent).

Secant with angle $\Theta = \text{terminal arm} / \text{initial arm}$

i. e. $\sec \Theta = \text{hypotenuse side} / \text{adjacent side}$
 $\sec \Theta = AC/BC$

The definition of sec Θ :-

It is ratio of hypotenuse side to the adjacent side of a given triangle.

In above fig.14 AB is the corresponding of the tangent i.e. cotangent which is perpendicular to the tangent BC.

The angle between secant and cotangent is $(90 - \Theta)$ which corresponding angle of Θ , then the secant becomes corresponding to secant i.e. cosecant.

Cosecant with angle $\Theta = \text{secant (hypotenuse)} / \text{Opposite side (cotangent)}$

$\text{cosec } \Theta = AC / AB$

The definition of Cosecant :-

it is the ratio of hypotenuse i.e. secant to the opposite side of given triangle.

The reason for main function sine, secant, tangent :-

Sine is only use for chord which is inside the ellipse as shown in fig.7

Tan is only use when opposite side and adjacent side of triangle are touches to the ellipse or circle which is not inside circle and hypotenuse must a chord as shown in fig.11

Sec is only use when opposite side and adjacent side of triangle are touches to the ellipse or circle which is not inside circle and hypotenuse is not chord as shown fig.12

Remaining **Cos, Cosec, Cot** which are corresponding to the above function, these are the not main function

Also

The previous formula of **tan** and **cot** are also interchanged.

Previous formula of tan and cot :-

$\text{Tan} = \text{opposite side} / \text{adjacent side}$
 $\text{Cot} = \text{adjacent side} / \text{opposite side}$

According to this research paper the formula of tan and cot is :-

$\text{Tan} = \text{adjacent side} / \text{opposite side}$
 $\text{Cot} = \text{opposite side} / \text{adjacent side}$

In fig .13 the primary tangent (initial arm) is adjacent side of given triangle and second tangent is opposite side of given triangle. So we can not say $\tan \Theta$ is opposite side to the adjacent side because primary tangent is adjacent side which is also called as initial arm

So we can say $\tan\theta$ is adjacent side to the opposite side

In fig.14 the primary tangent (initial arm) is adjacent side of given triangle and second tangent is opposite side of given triangle. So we can say $\sec\theta$ is the hypotenuse side to the adjacent side and it is correct because the primary tangent is adjacent side which is also called as initial arm and we can not say $\sec\theta$ is the hypotenuse side to the opposite side because opposite side is not primary tangent.

Sine is use for :-

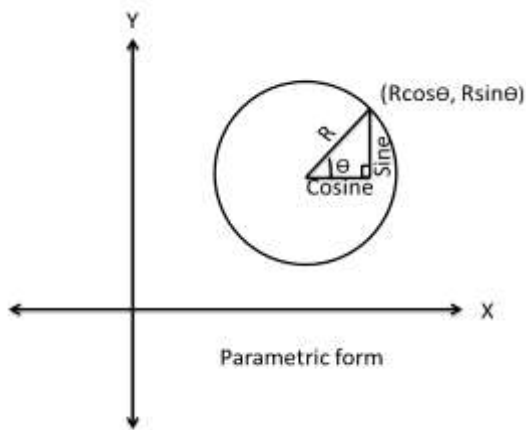


Fig.15

$\sin\theta$ is only use when triangle is drawn inside circle or ellipse.

Tan is use for :-

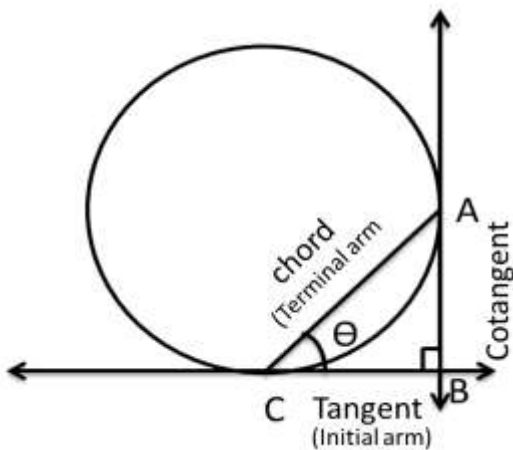


Fig.16

Sec is use for :-

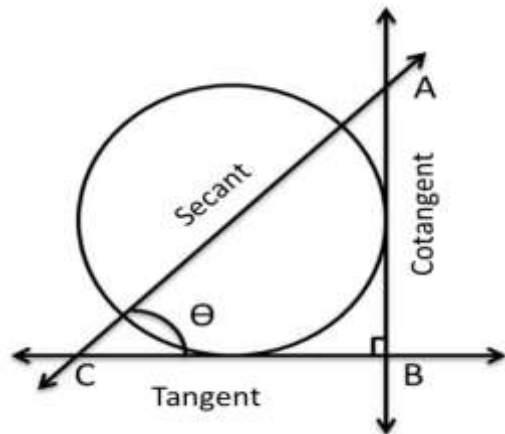


Fig.17

Reference:-

- [1] ww.cso.caltech.edu/outreach/log/NIGHT_DAY/elliptical.htm
- [2] <https://en.wikipedia.org/wiki/Sine>
- [3] 10th textbook geometry, topic name 'circle' edition 2016 Page no.71
- [4] 11th textbook 'mathematics and statistics' part I, topic name 'trigonometric function of compound angles' Edition 2013 ,page no 28
- [5] 11th textbook 'mathematics and statistics' partII, topic name 'Set, Relation and functions' page no. 12 , edition 2011
- [6] https://www.scienceabc.com/wpcontent/uploads/2015/12/shutterstock_208294861.jpg
- [7] 11th textbook 'mathematics and statistics' part I, topic name 'circle and conics ' Edition 2013 ,page no 107
- [8] 10th textbook geometry, topic name 'trigonometry' edition 2016
- [9] <https://image.slidesharecdn.com/aryabhata-150705040835-lva1-app6892/95/aryabhata-4-638.jpg?cb=1436069888>