A Notion of Even Vertex Odd Mean Labeling Graphs

M.Kannan[1], R.Vikrama Prasad[2] , R.Gopi[3]

1.Research Scholar ,Research & Development Centre, Bharathiar University, Coimbatore – 641 046, India
2. PG and Research Department of Mathematics Government Arts College(Autonomous), Salem -636007,India
3.PG and Research Department of Mathematics Srimad Andavan Arts and Science College (Autonomous) Tiruchirappalli – 620005, Tamil Nadu, India

ABSTRACT

A graph with p vertices and q edges is said to have an even vertex odd mean labeling if there exists an injective function f:V(G)→{0, 2, 4, ... 2q-2,2q} such that the induced map f: E(G)→{1, 3, 5, ... 2q-1} defined by* $f^*(uv)=\frac{f(u)+f(v)}{2}$ *is a bijection. A graph that admits an even vertex odd mean labeling is called an even vertex odd mean graph. In this paper we pay our attention to even vertex odd mean labeling of some graphs*

Keywords: *Even vertex odd mean labeling, even vertex odd mean graph*
AMS subject classification (2010): 05C78

1.INTRODUCTION

Throughout this paper we restrict our attention to finite, simple and undirected graphs.The set of vertices and the set of edges of a graph G will be denoted by V(G) and E(G)respectively and let $p = |V(G)|$, $q = |E(G)|$.For general graph theorectic notations we follow F.Harary[7].A graph labeling is a mapping that carries a set of elements(usually vertices and /or edges) into a set of numbers .Many kinds of labeling have been studied an excellent survey of graph labeling can be found in[2].Most of the graph labeling techniques found their origin with graceful labeling which was introduced by Rosa.A(1967) . Let G(V,E) be a graph with p vertices and q edges.

The concept of mean labeling was introduced and studied by Somasundaram and Ponraj [11]. Further some more results on mean graphs are discussed in [4,5]. A graph G is said to be a mean graph if there exists an injective function f:V(G) → {0, 1, 2, …q} such that the induced map f*:E(G)→{1, 2, …q} defined by

$$f^*(uv)=\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$$ is a bijection. Manickam and Marudai [10] have introduced the concept of odd mean labeling of a graph. A graph G is said to be odd mean if there exists an injective map f:V(G)→{0, 1, …2q–1} defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection. The concept of even mean labeling was introduced and studied by Gayathri and Gopi [3]. A graph G is said to be even mean if there exists an injective function f: V(G)→{0,1,…2q} such that the induced map f*:E(G)→{2,4,…2q} defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection.

A graph G is said to have an even vertex odd mean labeling if there exists an injective function f:V(G)→{0, 2, …2q–2, 2q} such that the induced map f*:E(G)→{1,3, … 2q–1} defined by $f^*(uv) = \frac{f(u)+f(v)}{2}$ is a bijection. A graph that admits an even vertex odd mean labeling is called even vertex odd mean graph [1, 13].

In this paper, we inspect the even vertex odd meanness of some graphs

Definition :1.1

Consider a cycle C_n and let $e_k = v_k\, v_{k+1}$ be an edge in it with $e_{k-1} = v_{k-1}\, v_k$ and $e_{k+1} = v_{k+1}\, v_{k+2}$ be its incident edges and $e_k' = v_k'\, v_{k+1}'$ be a new edge.

The duplication of an edge e_k by an edge $e_k^{'}$ produces a new graph G in such a way that $N(v_k) \cap N(v_k^{'}) = \{v_{k-1}\}$ and $N(v_{k+1}) \cap N(v_{k+1}^{'}) = \{v_{k+2}\}$ which is called edge duplication of C_n and denoted by $ED(C_n)$ where $N(v_k)$ denotes the set of vertices adjacent to v_k

Definition :1.2

Le $P_m + \overline{K_n}$ be the graph with the vertex set V (G) $=\{u_i, v_j : 1 \le i \le m, 1 \le j \le n\}$ and the edge set E(G) = $\{u_i u_{i+1}, u_1 v_j, u_m v_j : 1 \le i \le m-1, 1 \le j \le n\}$.

Definition :1.3

$T_{t,n,m}$ is a graph obtained by joining the centers of $K_{1,n}$ and $K_{1,m}$ by a path P_t. It consists of t+n +m vertices and t+n +m -1 edges.

2.MAIN RESULTS

Theorem :2.1

The graph $ED(C_n)$, $n \ge 4$ is an even vertex odd mean graph for n is even

Proof :

Let $\{v_i, 1 \le i \le n, v_1^{'}, v_2^{'}\}$ be the vertices and $\{e_i, 1 \le i \le n, e_1^{'}, e_2^{'}, e_3^{'}\}$ be the edges which are denoted as in figure 1.1

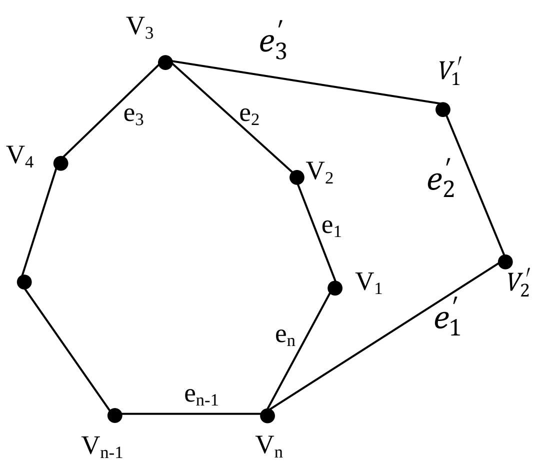

Figure 1.1 Ordinary labeling of $ED(C_n)$, $n \ge 4$

First we label the vertices as follows

Define $f : v \to \{0, 2, \ldots 2q\}$ by

$f(v_1) = 0$, $f(v_2) = 2$, $f(v_1^{'}) = 2n+4$,

$f(v_2^{'}) = 2n+6$

For $3 \le i \le \frac{n}{2}$, $f(v_i) = 2n-2i+6$, i is odd

For $4 \le i \le \frac{n+2}{2}$, $f(v_i) = 2n-2i+10$, i is even

For $\frac{n+4}{2} \le i \le n-1$, $f(v_i) = 2n-2i+2$, i is odd

For $\frac{n+6}{2} \le i \le n$, $f(v_i) = 2n-2i+6$, i is even

Then the induced edge labels are :

$f^*(e_1^{'}) = n+5$, $f^*(e_2^{'}) = 2n+5$, $f^*(e_3^{'}) = 2n+3$

$f^*(e_1) = 1$, $f^*(e_2) = n+1$

For $3 \le i \le \frac{n}{2}$ $f^*(e_i) = 2n-2i+7$ $f^*(e_{\frac{n+2}{2}}) = n+3$

For $\frac{n+4}{2} \le i \le n$ $f^*(e_i) = 2n-2i+3$

Therefore $f^*(E) = \{1, 3, 5, \ldots 2q-1\}$. So, f is a even vertex odd mean labeling and hence the graph $ED(C_n)$, , $n \ge 4$ is an even vertex odd mean graph.Even vertex odd mean labeling of $ED(C_6)$, is shown in Figure 1.2:

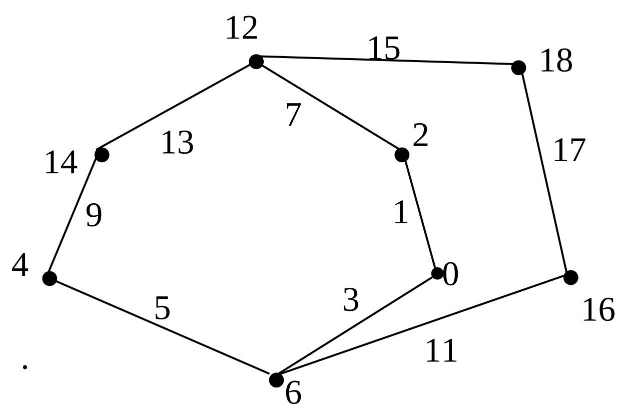

Figure 1.2:Even vertex odd mean labeling of $ED(C_6)$,

Theorem :2.2

The graph $T_{t,n,m}$ $(t,n,m \ge 2)$ an even vertex odd mean graph for any $t \ge m$ when n is even

Proof :

Let $\{u_i\ 1 \le i \le t, v_i, 1 \le i \le n, w_i, 1 \le i \le m\}$ be the vertices and $\{a_i, 1 \le i \le t+n+m-1\}$ be the edges which are denoted as in figure 1.3

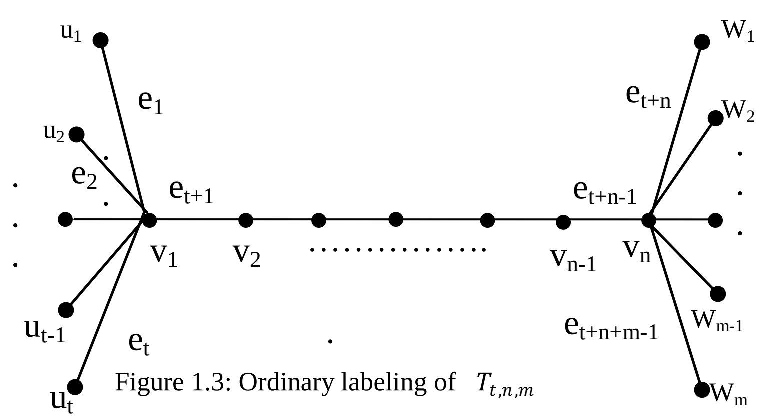

Figure 1.3: Ordinary labeling of $T_{t,n,m}$

First we label the vertices as follows :

Define f : v→{0, 2, … 2q} by

For $1 \le i \le t$ $f(u_i) = 4(i-1)$

For $1 \le i \le n$

$$f(v_i) = \begin{cases} 2i & i \text{ is odd} \\ 4t + 2(i-2) & i \text{ is even} \end{cases}$$

For $1 \le i \le m$, $f(w_i) = 2n+4i-2$

Then the induced edge labels are :

For $1 \le i \le t+n+m-1$ $f^*(e_i) = 2i-1$

Therefore $f^*(E) = \{1, 3, 5, ...2q-1\}$. So, f is a even vertex odd mean labeling and hence the $T_{t,n,m}$(t,n,m,n≥ 2) is an even vertex odd mean graph.Even vertex odd mean labeling of the $T_{6,4,5}$ is shown in Figure 1.4 :

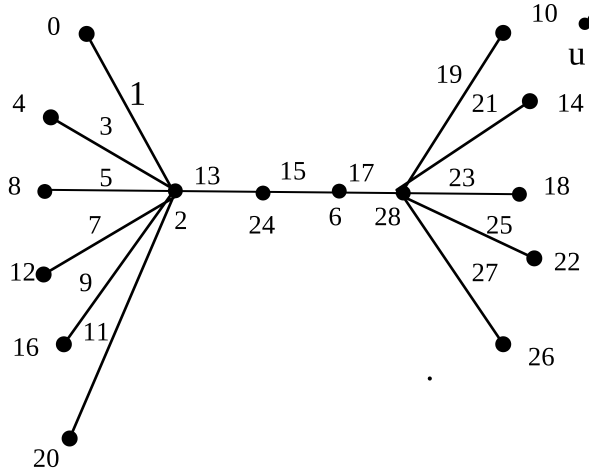

Figure 1.4:Even vertex odd mean labeling of $T_{6,4,5}$

Theorem :2.3

The graph $P_3(+)\overline{k_n}$ is an even vertex odd mean graph

Proof :

Let {u, v,w ,u_i, $1 \le i \le n$ } be the vertices and {a,b, a_i $1 \le i \le 2n$ } be the edges which are denoted as in figure 1.5

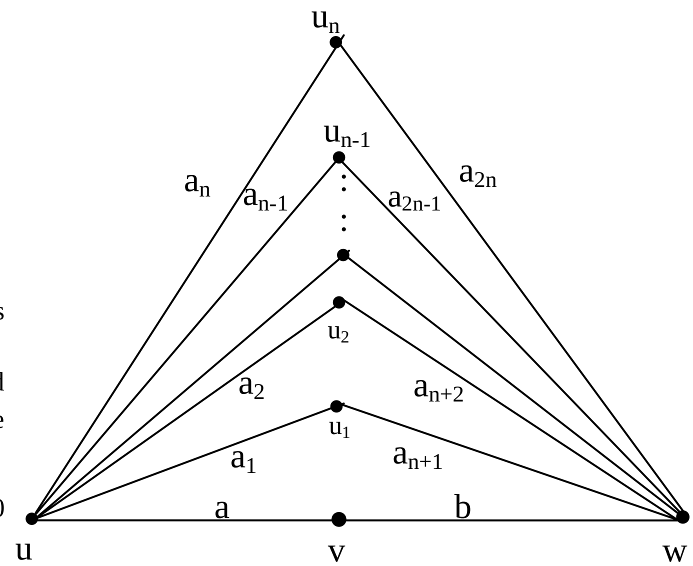

Figure 1.5: Ordinary labeling of $P_3(+)\overline{k_n}$

First we label the vertices as follows :

Define f : v→{0, 2, … 2q} by

f(u)=0 , f(v) =4n+2 ,f(w)= 4(n+1)

For $1 \le i \le n$ $f(u_i) = 4i-2$

Then the induced edge labels are :

$f^*(a) = 2n+1$, $f^*(b) = 4n+3$

For $1 \le i \le n$ $f^*(a_i) = 2i-1$

For $n+1 \le i \le 2n$ $f^*(a_i) = 2i+1$

Therefore $f^*(E) = \{1, 3, 5, ...2q-1\}$. So, f is a even vertex odd mean labeling and hence the $P_3(+)\overline{k_n}$ is an even vertex odd mean graph.Even vertex odd mean labeling of $P_3(+)\overline{k_4}$ is shown in Figure 1.6 :

11

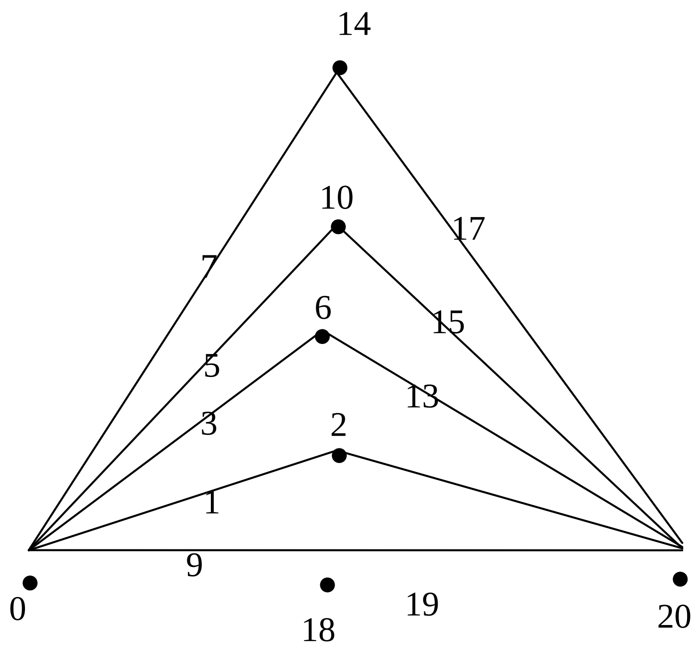

Figure 1.6:Even vertex odd mean labeling of $P_3(+)\overline{k_4}$

REFERENCES

1. S.Arockiaraj and B.S. Mahadevaswamy,Even vertex odd mean labeling of graphs obtained from graph operations, Int . Journal of Advance Research in Edu., Tech. & management, 3(1)(2015), 192.
2. J.A.Gallian, A dynamical survey of graph labeling, The Electronic Journal of Combinatorics, 17(2014), #DS6.
3. B.Gayathri and R.Gopi, K-even mean labeling of $D_{m,n}@C_n$, International Journal of Engineering Sciences, Advanced Computing and Bio-Technology, 1(3)(2010), 137-145.
4. B.Gayathri and R.Gopi, Necessary condition for mean labeling, International Journal of Engineering Sciences, Advance Computing and Bio-Technology, 4(3), July-Sep (2013), 43-52.
5. B.Gayathri and R.Gopi, Cycle related mean graphs, Elixir International Journal of Applied Sciences, 71(2014), 25116-25124.
6. B. Gayathri, M. Duraisamy, Even Edge - Graceful Labeling of Tt,n,m, International Journal of Science and Research,Vol.5(10),(2016),1081-1083
7. F.Harary, Graph Theory, Addision Wesely, Reading Mass, 1972.
8. M.Kannan,R.Vikramaprasad ,R.Gopi, Even Vertex Odd Mean Labeling of Some Graphs,Global journal of pure and applied mathematicsVol.13 No.3(2017),1019-1033
9. M.Kannan,R.Vikramaprasad ,R.Gopi Some Graph Operations Of Even Vertex Odd Mean Labeling Graphs,International Journal Of Applied Engineering Research,Vol.12,N0.18,(2017),7749-7753
10. K.Manickam and M.Marudai, Odd Mean Labeling of Graphs, Bulletin of Pure and Applied Sciences, 25 E(1) (2006), 149-153.
11. R.Ponraj and S.Somasundaram, Mean Labeling of Graphs, National Academy Science Letter, 26 (2003), 210-213.
12. S.K.Vaidya and C.M.Barasara,Harmonic Mean Labeling In The Context Of Duplication Of Graph Element ,Elixir Discrete Mathematics ,48(2012),9482-9485
13. R.Vasuki, A.Nagarajan and S.Arockiaraj, Even Vertex Odd Mean Labeling of Graphs, Sut J.Math, 49(2)(2013), 79-92.