

Soft Multi Generalized Closed Sets in Soft Multi Topological Spaces

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Abstract

Many researchers defined some basic notions on soft multi topological spaces and studied many properties. In this paper, we define soft multi generalized closed and open sets in soft multi topological spaces and studied their some properties. We introduce these concepts which are defined over an initial universe with a fixed set of parameters. We investigate behaviour relative to union, intersection and soft multi subspaces of soft multi generalized closed sets. Also, we investigate many basic properties of these concepts.

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1 Introduction

In 1999, Molodtsov [9] introduced the concept of soft set theory and started to develop the basics of corresponding theory as a new approach for modelling uncertainties. In 2011, Alkhazaleh et al. [1] as a generalization of Molodtsov's soft set, we introduce the definition of a soft multiset, its basic operations such as complement, union and intersection etc.

In 2011, Cagman et al. [3] introduced soft topology. Topological structure of soft set were introduced by Sabir and Naz [12]. They defined the soft topological spaces which are defined over a initial universe with a fixed set of parameters. Tokat and Osmanoglu [14] introduced soft multi topology. In 2013, Anjan Mukherjee et al. [2] introduced topological structure formed by soft multi sets and soft multi compact space. In 2017, Gowri and Sahithyabharathi [6] [7] introduced and studied the concept of separation axioms and higher separation axioms in soft multi topological spaces.

In this paper, we introduce soft multi generalized closed sets in soft multi topological spaces which are defined over an initial universe with a fixed set of parameters. We investigate behaviour relative to soft multi subspaces of soft multi generalized closed set in a soft multi compact space is also soft multi compact. Then, we show that a soft multi compact set in a soft multi regular space is soft multi generalized closed and disjoint soft multi generalized closed sets in a soft multi normal space generally cannot be separated soft multi open sets. Finally, we investigate the properties of soft multi generalized open sets.

2 Preliminaries

Definition 2.1 [9] Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a non-empty subset of E . A pair (F, A) is called a soft set over U , where F is a mapping given by $F : A \rightarrow P(U)$.

In other words, a soft set over U is a parametrized family of subsets of the universe U . For $e \in A$, $F(e)$ may be considered as the set of e - approximate elements of the soft set (F, A) .

Definition 2.2 [1] Let $\{U_i : i \in I\}$ be a collection of universes and such that $\bigcap_{i \in I} U_i = \emptyset$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i \in I} P(U_i)$ where $P(U_i)$ denotes the power set of U_i , $E = \prod_{i \in I} E_{U_i}$ and $A \subseteq E$. A pair (F, A) is called a soft multiset over U (briefly $SMS(U, E)$), where F is a mapping given by $F : A \rightarrow U$.

Definition 2.3 [2] A soft multi set $(F, A) \in SMS(U, E)$ is called a soft multi point in (U, E) , denoted by $e_{(F,A)}$, if for the element $e \in A$, $F(e) \neq \emptyset$ and $\forall e' \in A - \{e\}, F(e') = \emptyset$.

Definition 2.4 [2] A sub family τ of $SMS(U, E)$ is called soft multi topology on (U, E) , if the following axioms are satisfied:

- (O₁). $\emptyset, \tilde{E} \in \tau$,
- (O₂). the union of any number of soft multi sets in τ belongs to τ , ie. for any $\{(F_k, A_k) : k \in K\} \subseteq \tau \implies \tilde{\cup}_{k \in K} (F_k, A_k) \in \tau$,
- (O₃). If $(F, A), (G, B) \in \tau$, then $(F, A) \tilde{\cap} (G, B) \in \tau$.

Example 2.5 Suppose that there are three universes U_1, U_2 and U_3 . Let us consider a soft multi set (F, A) which describes the “attractiveness of houses”, “cars” and “hotels” $U_1 = \{h_1, h_2, h_3, h_4\}, U_2 = \{c_1, c_2, c_3\}$ and $U_3 = \{v_1, v_2\}$. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where $E_{U_1} = \{e_{U_1,1} = \text{expensive}, e_{U_1,2} = \text{cheap}, e_{U_1,3} = \text{wooden}, e_{U_1,4} = \text{in green surroundings}\}$, $E_{U_2} = \{e_{U_2,1} = \text{expensive}, e_{U_2,2} = \text{cheap}, e_{U_2,3} = \text{sporty}\}$, $E_{U_3} = \{e_{U_3,1} = \text{expensive}, e_{U_3,2} = \text{cheap}\}$

Let $U = \prod_{i=1}^3 P(U_i), E = \prod_{i=1}^3 E_{U_i}$, and

$$A_1 = \{e_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), e_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1})\}$$

$$A_2 = \{e_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), e_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1})\}$$

Suppose that

$$(F_1, A_1) = \{(e_1, (\{h_1, h_2\}, \{c_1, c_2\}, \{v_1\})), (e_2, (\{h_3, h_4\}, \{c_1, c_3\}, \{v_2\}))\},$$

$$(F_2, A_2) = \{(e_1, (\{h_1, h_3\}, \{c_2, c_3\}, \{v_1, v_2\})), (e_3, (\{h_2, h_4\}, \{c_1, c_2\}, \{v_2\}))\}$$

$$(F_3, A_3) = (F_1, A_1) \tilde{\cup} (F_2, A_2)$$

$$= \{(e_1, (\{h_1, h_2, h_3\}, \{c_1, c_2, c_3\}, \{v_1, v_2\})), (e_2, (\{h_3, h_4\}, \{c_1, c_3\}, \{v_2\})), (e_3, (\{h_2, h_4\}, \{c_1, c_2\}, \{v_2\}))\}$$

$$(F_4, A_4) = (F_1, A_1) \tilde{\cap} (F_2, A_2)$$

$$= \{(e_1, (\{h_1\}, \{c_2\}, \{v_1\})), (e_2, (\{h_3, h_4\}, \{c_1, c_3\}, \{v_2\})), (e_3, (\{h_2, h_4\}, \{c_1, c_2\}, \{v_2\}))\},$$

where

$$A_3 = A_4 = A_1 \cup A_2 = \{e_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), e_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), e_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1})\}$$

Then we observe that the subfamily

$$\tau_1 = \left\{ \emptyset, \tilde{E}, (F_1, A_1), (F_2, A_2), (F_3, A_3), (F_4, A_4) \right\}$$

of $SMS(U, E)$ is a soft multi topology on (U, E) , since it satisfies the necessary three axioms $(O_1), (O_2)$ and (O_3) and $((U, E), \tau_1)$ is a soft multi topological space.

Definition 2.6 [2] Let $((U, E), \tau)$ be a soft multi topological space on (U, E) and (F, A) be a soft multi set in $SMS(U, E)$. Then the union of all soft multi open sets contained in (F, A) is called the interior of (F, A) and is denoted by $sm-Int(F, A)$ and defined by

$$sm - Int(F, A) = \tilde{\cup} \{ (G, B) : (G, B) \text{ is a soft multi open set contained in } (F, A) \}.$$

Definition 2.7 [2] Let $((U, E), \tau)$ be a soft multi topological space on (U, E) and (F, A) be a soft multi set in $SMS(U, E)$. Then the intersection of all soft multi closed set containing (F, A) is called the closure of (F, A) and is denoted by $sm-Cl(F, A)$ and defined by

$$sm - Cl(F, A) = \tilde{\cap} \{ (G, B) : (G, B) \text{ is a soft multi closed set containing } (F, A) \}$$

Observe that $sm-Cl(F, A)$ is a soft multi closed set, since it is the intersection of soft multi closed sets. Furthermore, $sm-Cl(F, A)$ is the smallest soft multi closed set containing (F, A) .

Definition 2.8 [2] Let $((U, E), \tau)$ be an soft multi topological space on (U, E) and (F, A) be an soft multi set in $SMS(U, E)$. Then the soft multi topology

$$\tau_{(F, A)} = \{ (F, A) \tilde{\cap} (G, B) : (G, B) \in \tau \}$$

is called soft multi subspace topology and $((F, A), \tau_{(F, A)})$ is called soft multi topological subspace of $((U, E), \tau)$.

Definition 2.9 [6] A soft multi topological space $((U, E), \tau)$ is said to be soft multi regular space if for all soft multi closed sets F_A and soft multi point $e_{1(f_1, a_1)}$, (Where $(f_1, a_1) \in (F_1, A_1)$) such that $e_{1(f_1, a_1)} \notin F_A$ there exists $G_A, H_A \in \tau$ such that $e_{1(f_1, a_1)} \in G_A, F_A \tilde{\subseteq} H_A$ and $G_A \tilde{\cap} H_A = \emptyset$.

Definition 2.10 [6] A soft multi topological space $((U, E), \tau)$ is said to be soft multi normal space if for every two non empty disjoint soft multi closed sets $(F_1, A), (F_2, A)$ there exists $G_A, H_A \in \tau$ such that $(F_1, A) \in G_A, (F_2, A) \in H_A$ and $G_A \tilde{\cap} H_A = \emptyset$.

Theorem 2.11 [6] A soft multi topological space $((U, E), \tau)$ is soft multi regular if and only if for every $e_{1(f, a)} \in (U, E)$ and every soft multi open set (F_1, A) of (U, E) , there is a soft multi open set (G_1, A) of (U, E) such that $e_{1(f, a)} \in (G_1, A) \subseteq sm - Cl(G_1, A) \subseteq (F_1, A)$.

3 Soft Multi Generalized Closed Sets

Definition 3.1 Let $((U, E), \tau)$ be a soft multi topological space. A soft multi set (F, A) is called a soft multi generalized closed (briefly soft multi g-closed) in (U, E) if $sm - Cl(F, A) \subseteq (G, A)$ whenever $(F, A) \subseteq (G, A)$ and (G, A) is soft multi open in (U, E) .

Example 3.2 Let us consider there are three universes U_1, U_2 and U_3 . Let $U_1 = \{h_1, h_2, h_3, h_4\}, U_2 = \{c_1, c_2, c_3\}$ and $U_3 = \{v_1, v_2\}$. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \{e_{U_1,1} = \text{expensive}, e_{U_1,2} = \text{cheap}, e_{U_1,3} = \text{wooden}, e_{U_1,4} = \text{in green surroundings}\},$$

$$E_{U_2} = \{e_{U_2,1} = \text{expensive}, e_{U_2,2} = \text{cheap}, e_{U_2,3} = \text{sporty}\},$$

$$E_{U_3} = \{e_{U_3,1} = \text{expensive}, e_{U_3,2} = \text{cheap}\},$$

$$\text{Let } U = \prod_{i=1}^3 P(U_i), E = \prod_{i=1}^3 E_{U_i}, \text{ and}$$

$$A = \{e_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), e_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1})\}$$

and $\tau = \{\tilde{\emptyset}, \tilde{E}, (F_1, A), (F_2, A), (F_3, A), (F_4, A)\}$ Suppose that

$$(F_1, A) = \{(e_1, (\{h_1, h_2\}, \{c_1, c_2\}, \{v_1\})), (e_2, (\{h_1, h_3, h_4\}, \{c_1, c_3\}, \{v_2\}))\},$$

$$(F_2, A) = \{(e_1, (\{h_1, h_3, h_4\}, \{c_1, c_3\}, \{v_1, v_2\})), (e_2, (\{h_2, h_4\}, \{c_1, c_2\}, \{v_1, v_2\}))\},$$

$$(F_3, A) = (F_1, A) \cup (F_2, A)$$

$$= \{(e_1, (\{h_1, h_2, h_3, h_4\}, \{c_1, c_2, c_3\}, \{v_1, v_2\})), (e_2, (\{h_1, h_2, h_3, h_4\}, \{c_1, c_2, c_3\}, \{v_1, v_2\}))\},$$

$$(F_4, A) = (F_1, A) \cap (F_2, A)$$

$$= \{(e_1, (\{h_1\}, \{c_1\}, \{v_1\})), (e_2, (\{h_4\}, \{c_1\}, \{v_2\}))\},$$

Then the subfamily, $\tau = \{\tilde{\emptyset}, \tilde{E}, (F_1, A), (F_2, A), (F_3, A), (F_4, A)\}$ of soft multi set of

(U, E) (briefly $\text{sms}(U, E)$) is soft multi topology on (U, E) .

The soft multi closed sets in $((U, E), \tau)$ are $(\emptyset)^c = \tilde{E}, \tilde{E}^c = \emptyset,$

$$(F_1, A)^c = \{(e_1, (\{h_3, h_4\}, \{c_3\}, \{v_2\})), (e_2, (\{h_2\}, \{c_2\}, \{v_1\}))\}$$

$$(F_2, A)^c = \{(e_1, (\{h_2, h_4\}, \{c_2\}, \{\emptyset\})), (e_2, (\{h_1, h_3\}, \{c_3\}, \{\emptyset\}))\},$$

$$(F_3, A)^c = \{(e_1, (\{\emptyset\}, \{\emptyset\}, \{\emptyset\})), (e_2, (\{\emptyset\}, \{\emptyset\}, \{\emptyset\}))\},$$

$$(F_4, A)^c = \{(e_1, (\{h_2, h_3, h_4\}, \{c_2, c_3\}, \{v_2\})), (e_2, (\{h_1, h_2, h_3\}, \{c_2, c_3\}, \{v_1\}))\}$$

$$\text{Let } (F, A) = \{(e_1, (\{h_3\}, \{c_3\}, \{v_2\})), (e_2, (\{h_2\}, \{c_2\}, \{v_1\}))\}$$

$$\text{sm} - Cl(F, A) = \{(e_1, (\{h_3, h_4\}, \{c_3\}, \{v_2\})), (e_2, (\{h_2\}, \{c_2\}, \{v_1\}))\}$$

A soft multi set (F, A) is called a soft multi generalized closed (briefly soft multi g -closed) in (U, E) if $\text{sm} - Cl(F, A) \subseteq (F_3, A)$ whenever $(F, A) \subseteq (F_3, A)$ and (F_3, A) is soft multi open in (U, E) .

Theorem 3.3 If (F_1, A) is soft multi g -closed in $((U, E), \tau)$ and $(F_1, A) \subseteq (F_2, A) \subseteq \text{sm} - Cl(F_1, A)$, then (F_2, A) is soft multi g -closed.

Proof: Let (F_1, A) is a soft multi g -closed in (U, E) and $(F_1, A) \subseteq (F_2, A) \subseteq \text{sm} - Cl(F_1, A)$. Let $(F_2, A) \subseteq (G, A)$ and (G, A) is soft multi open in (U, E) . Since $(F_1, A) \subseteq (F_2, A)$ and $(F_2, A) \subseteq (G, A)$, we have $(F_1, A) \subseteq (G, A)$. Hence $\text{sm} - Cl(F_1, A) \subseteq (G, A)$ (since (F_1, A) is soft multi g -closed). Since $(F_2, A) \subseteq \text{sm} - Cl(F_1, A)$, we have $\text{sm} - Cl(F_2, A) \subseteq \text{sm} - Cl(F_1, A) \subseteq (G, A)$. Therefore (F_2, A) is soft multi g -closed. \square

Theorem 3.4 If (F_1, A) and (F_2, A) are soft multi g -closed sets then so is $(F_1, A) \cup (F_2, A)$.

Proof: Let (F_1, A) and (F_2, A) are soft multi g -closed sets. Let $(F_1, A) \cup (F_2, A) \subseteq (G, A)$ and (G, A) is soft multi open in (U, E) . Since $(F_1, A) \cup (F_2, A) \subseteq (G, A)$, we have $(F_1, A) \subseteq (G, A)$ and $(F_2, A) \subseteq (G, A)$. Since (G, A) is soft multi open in (U, E) and (F_1, A) and (F_2, A) are soft multi g -closed sets, we have $\text{sm} - Cl(F_1, A) \subseteq (G, A)$ and $(F_2, A) \subseteq (G, A)$.

Therefore $sm - Cl(F_1, A) \tilde{\cup} (F_2, A) = sm - Cl(F_1, A) \tilde{\cup} sm - Cl(F_2, A) \subseteq (G, A)$.
This completes the proof. \square

Theorem 3.5 *If a set (F_1, A) is soft multi g-closed in (U, E) if and only if $sm - Cl(F_1, A) \setminus (F_1, A)$ contains only null soft closed sets.*

Proof: Let (F_1, A) is soft multi g-closed in (U, E) . Let (G, A) be soft multi closed and $(G, A) \subseteq sm - Cl(F_1, A) \setminus (F_1, A)$. Since (G, A) is soft multi closed, we have its relative complement $(G, A)^c$ is soft multi open. Since $(G, A) \subseteq sm - Cl(F_1, A) \setminus (F_1, A)$, we have $(G, A) \subseteq sm - Cl(F, A)$ and $(G, A) \subseteq (F_1, A)^c$. Hence $(F_1, A) \subseteq (G, A)^c$. Consequently $sm - Cl(F_1, A) \subseteq (G, A)^c$ (Since (F_1, A) is soft multi g-closed in (U, E)). Therefore, $(G, A) \subseteq sm - Cl(F_1, A)^c$. Hence $(G, A) = \emptyset$. Hence $sm - Cl(F_1, A) \setminus (F_1, A)$ contains only null soft multi closed sets. \square

Theorem 3.6 *A soft multi g-closed (F_1, A) is soft multi closed if and only if $sm - Cl(F_1, A) \setminus (F_1, A)$ is soft multi closed.*

Proof: If (F_1, A) is soft multi closed, then $sm - Cl(F_1, A) \setminus (F_1, A) = \emptyset$.
Conversely, suppose that $sm - Cl(F_1, A) \setminus (F_1, A)$ is soft multi closed, since (F_1, A) is soft multi g-closed, $sm - Cl(F_1, A) \setminus (F_1, A) = \emptyset$ (by theorem (3.5)). Hence (F_1, A) is soft multi closed. \square

Remark 3.7 *The intersection of two soft multi g-closed sets are generally not a soft multi g-closed set.*

Theorem 3.8 *Let $((U, E), \tau)$ be a soft multi topological space over (U, E) , let $(V, E) \subseteq (U, E)$, if (F_1, A) is soft multi g-closed in (U, E) . Then (F_1, A) is soft multi g-closed relative to $((V, E), \tau_V)$.*

Proof: Let $(F_1, A) \subseteq (V, E) \tilde{\cap} (G, A)$ and suppose that (G, A) is soft multi open in (U, E) . Then $(F_1, A) \subseteq (G, A)$ and hence $sm - Cl(F_1, A) \subseteq (G, A)$. It follows that $(V, E) \tilde{\cap} sm - Cl(F_1, A) \subseteq (V, E) \tilde{\cap} (G, A)$. \square

Theorem 3.9 *In a soft multi topological space $((U, E), \tau)$, $\tilde{\tau} = \tilde{\tau}^c$ if and only if every soft multi set over (U, E) is a soft multi g-closed set.*

Proof: Let $\tilde{\tau} = \tilde{\tau}^c$ and that $(F_1, A) \subseteq (G, A)$ where $(G, A) \in \tau$. Then $sm - Cl(F_1, A) \subseteq sm - Cl(G, A) = (G, A)$ and (F_1, A) is soft multi g-closed.
Conversely, suppose that every soft multi set over (U, E) is soft multi g-closed. Let $(G, A) \in \tau$. Since $(G, A) \subseteq (G, A)$ and (G, A) is soft multi g-closed, we have $sm - Cl(G, A) \subseteq (G, A)$ and $(G, A) \in \tau^c$. Thus $\tau \subseteq \tau^c$. If $(F_2, A) \in \tau^c$, then $(F_2, A)^c \in \tau \subseteq \tau^c$ and hence $(F_2, A) \in \tau$. Therefore $\tau = \tau^c$ \square

Theorem 3.10 *Let $((U, E), \tau)$ be a soft multi compact space. If (F_1, A) is soft multi closed set in (U, E) , then (F_1, A) is soft multi compact.*

Proof: Let $C = (G_i, A); i \in I$ be a soft multi open cover of (F_1, A) , since (F_1, A) is soft multi closed, $sm - Cl(F_1, A) \subseteq \cup_{i \in I} (G_i, A)$. Therefore $sm - Cl(F_1, A)$ is soft multi compact and hence $(F_1, A) \subseteq sm - Cl(F_1, A) \subseteq ((G_1, A) \cup (G_2, A) \cup \dots \cup (G_n, A))$ where $(G_i, A) \in C$ for $i=1,2,\dots,n$. Hence (F_1, A) is soft multi compact. \square

Theorem 3.11 *Let $((U, E), \tau)$ be a soft multi compact space. If (F_1, A) is soft multi g-closed in (U, E) , then (F_1, A) is soft multi compact.*

Proof: Let $C = \{(G_i, A) : i \in I\}$ be a soft multi open cover of (F_1, A) . Since (F_1, A) is a soft multi g-closed. $sm - Cl(F_1, A) \subseteq \cup_{i \in I} (G_i, A)$, by theorem (3.10) $sm - Cl(F_1, A)$ is soft multi compact and it follows that $(F_1, A) \subseteq sm - Cl(F_1, A) \subseteq (G_1, A) \cup (G_2, A) \cup \dots \cup (G_n, A)$ for some $(G_i, A) \in C$ where $i = 1, 2, \dots, n$. Hence the proof. \square

Theorem 3.12 *If $((U, E), \tau)$ is a soft multi regular and (F_1, A) is soft multi compact set in (U, E) then (F_1, A) is soft multi g-closed.*

Proof: Let $((U, E), \tau)$ be soft multi regular, (F_1, A) be soft multi compact and $(F_1, A) \subseteq (H, A)$, where $(H, A) \in \tau$. For every point $e_{1(f,a)} \in (F_1, A)$ there exists a soft multi open set $(G_1, A)_{e_{1(f,a)}}$ of $e_{1(f,a)}$ such that $e_{1(f,a)} \in (G_1, A)_{e_{1(f,a)}} \subseteq sm - Cl(G_1, A)_{e_{1(f,a)}} \subseteq (H, A)$ by theorem (2.9) soft multi compactness of (F_1, A) , there exists a finite open cover $(G_1, A)_{e_{1(f,a)}} 1, (G_1, A)_{e_{1(f,a)}} 2, \dots, (G_1, A)_{e_{1(f,a)}} k$ of (F_1, A) such that $sm - Cl(G_1, A)_{e_{1(f,a)}} i \subseteq (H, E)$ for each i . Then define (G_1, A) as their finite union of $(G_1, A)_{e_{1(f,a)}} 1, (G_1, A)_{e_{1(f,a)}} 2, \dots, (G_1, A)_{e_{1(f,a)}} k$. Then $(F_1, A) \subseteq (G_1, A)$ and $sm - Cl(G_1, A) = \left\{ sm - Cl((G_1, A)_{e_{1(f,a)}} i); i = 1, 2, \dots, k \right\} \subseteq (H, A)$. Then we obtain $(F_1, A) \subseteq (G_1, A) \subseteq sm - Cl(G_1, A) \subseteq (H, A)$ and it follows that $sm - Cl(F_1, A) \subseteq (H, A)$. Hence the proof. \square

Theorem 3.13 *Let $((U, E), \tau)$ be a soft multi normal space, (V, E) be a non empty subset of (U, E) and suppose that (V, E) be a soft multi g-closed set in (U, E) . Then $((V, E), \tau_V)$ is soft multi normal space.*

Proof: Let (F_1, A) and (F_2, A) be two soft multi closed sets in (U, E) and suppose that $((V, E) \cap (F_1, A)) \cap ((V, E) \cap (F_2, A)) = \emptyset$. Then $(V, E) \subseteq [((F_1, A) \cap (F_2, A))]^c \in \tau$ and hence $sm - Cl(V, E) \subseteq [((F_1, A) \cap (F_2, A))]^c \in \tau$. Thus $[sm - Cl(V, E) \cap (F_1, A)] \cap [sm - Cl(V, E) \cap (F_2, A)] = \emptyset$. Since $((U, E), \tau)$ is soft multi normal, there exist disjoint soft multi open sets (G_1, A) and (G_2, A) such that $sm - Cl(V, E) \cap (F_1, A) \subseteq (G_1, A)$ and $sm - Cl(V, E) \cap (F_2, A) \subseteq (G_2, A)$. It follows that $(V, E) \cap (F_1, A) \subseteq (V, E) \cap (G_1, A)$ and $(V, E) \cap (F_2, A) \subseteq (V, E) \cap (G_2, A)$. Hence the proof. \square

Theorem 3.14 *If $((U, E), \tau)$ is soft multi normal and $(F, A) \cap (H, A) = \emptyset$ where (F, A) is soft multi closed and (H, A) is soft multi g-closed, then there exists disjoint soft multi open sets (G_1, A) and (G_2, A) such that $(F, A) \subseteq (G_1, A)$ and $(H, A) \subseteq (G_2, A)$.*

Proof: Let $(H, A) \subseteq (F, A)^c \in \tau$ and hence $sm - Cl(H, A) \subseteq (F, A)^c$. Thus $sm - Cl(H, A) \cap (F, A) = \emptyset$. Since $((U, E), \tau)$ is soft multi normal, then there exists disjoint soft multi open sets (G_1, A) and (G_2, A) such that $(F, A) \subseteq (G_1, A)$ and $sm - Cl(H, A) \subseteq (G_2, A)$. Since $(H, A) \subseteq sm - Cl(H, A)$, we obtain $(F, A) \subseteq (G_1, A)$ and $(H, A) \subseteq (G_2, A)$. Hence the proof. \square

Remark 3.15 *Disjoint soft multi g-closed sets in a soft multi normal space generally cannot be separated by soft multi open sets.*

Example 3.16 In example (3.2) $((U, E), \tau)$ is a soft multi normal space. Let $(F_1, A) = \{(e_1, (\{h_3\}, \{c_3\}, \{v_2\})), (e_2, (\{h_2\}, \{c_2\}, \{v_1\}))\}$ and $(F_2, A) = \{(e_1, (\{h_2\}, \{c_2\}, \{v_1\})), (e_2, (\{h_1\}, \{c_3\}, \{v_2\}))\}$ are soft multi g-closed in $((U, E), \tau)$. Clearly (F_1, A) and (F_2, A) are disjoint soft multi g-closed sets which cannot be separated by soft multi open sets.

4 Soft Multi Generalized Open Sets

Definition 4.1 Let $((U, E), \tau)$ be a soft multi topological space. A soft multi set (G, A) is called a soft multi generalized open (briefly soft multi g-open) in (U, E) if the complement $(G, A)^c$ is soft multi generalized closed in (U, E) .

Equivalently, a soft multi set (G, A) is called a soft multi generalized open (briefly soft multi g-open) in a soft multi topological space $((U, E), \tau)$ if and only if $(F, A) \subseteq sm - Int(G, A)$ whenever $(F, A) \subseteq (G, A)$ and (F, A) is soft multi closed in (U, E) .

Theorem 4.2 If (F, A) is soft multi g-open in (U, E) and $sm - Int(F, A) \subseteq (G, A) \subseteq (F, A)$, then (G, A) is soft multi g-open.

Proof: Let (F, A) is soft multi g-open in (U, E) and $sm - Int(F, A) \subseteq (G, A) \subseteq (F, A)$. Let $(H, A) \subseteq (G, A)$ where (H, A) is soft multi closed in (U, E) . Since $(G, A) \subseteq (F, A)$ and $(H, A) \subseteq (G, A)$, we have $(H, A) \subseteq (F, A)$. Hence $(H, A) \subseteq int(F, A)$ Since (F, A) is soft multi g-open. Since $sm - Int(F, A) \subseteq (G, A)$, we have $(H, A) \subseteq sm - Int(F, A) \subseteq sm - Int(G, A)$. Therefore, (G, A) is soft multi g-open. Hence the proof. \square

Theorem 4.3 If (F_1, A) and (F_2, A) are soft multi g-open sets then so is $(F_1, A) \cap (F_2, A)$.

Proof: Let (F_1, A) and (F_2, A) are soft multi g-open sets. Let $(H, A) \subseteq (F_1, A) \cap (F_2, A)$ and (H, A) is soft multi closed in (U, E) . Since $(H, A) \subseteq (F_1, A) \cap (F_2, A)$, we have $(H, A) \subseteq (F_1, A)$ and $(H, A) \subseteq (F_2, A)$. Since (H, A) is soft multi closed in (U, E) and (F_1, A) and (F_2, A) are soft multi g-open sets, we have $(H, A) \subseteq sm - Int(F_1, A)$ and $(H, A) \subseteq sm - Int(F_2, A)$. Therefore $(H, A) \subseteq sm - Int(F_1, A) \cap sm - Int(F_2, A)$. This completes the proof. \square

Theorem 4.4 If (F_1, A) and (F_2, A) are soft multi disconnected and soft multi g-open sets, then $(F_1, A) \cup (F_2, A)$ is soft multi g-open.

Proof: Let (G, A) be a soft multi closed subset of $(F_1, A) \cup (F_2, A)$. Then $(G, A) \cap sm - Cl(F_1, A) \subseteq (F_1, A)$ and hence by definition (4.1), $(G, A) \cap sm - Cl(F_1, A) \subseteq sm - Int(F_1, A)$. Similarly, $(G, A) \cap sm - Cl(F_2, A) \subseteq sm - Int(F_2, A)$. Now $(G, A) = (G, A) \cap ((F_1, A) \cup (F_2, A)) \subseteq ((G, A) \cap sm - Cl(F_1, A)) \cup ((G, A) \cap sm - Cl(F_2, A)) \subseteq sm - Int(F_1, A) \cup sm - Int(F_2, A) \subseteq sm - Int((F_1, A) \cup (F_2, A))$. Hence $(G, A) \subseteq sm - Int((F_1, A) \cup (F_2, A))$ and by definition (4.1) $(F_1, A) \cup (F_2, A)$ is soft multi g-open. Hence the proof. \square

Theorem 4.5 A soft multi set (F, A) is soft multi g-open in $((U, E), \tau)$ if and only if $(G, A) = \tilde{E}$ whenever (G, A) is soft multi open and $sm - Int(F, A) \cup (F, A)^c \subseteq (G, A)$.

Proof: Let (G, A) is soft multi open and $sm - Int(F, A) \cup (F, A)^c \subseteq (G, A)$. Now $(G, A)^c \subseteq sm - Cl((F, A)^c) \setminus (F, A)^c$. Since $(G, A)^c$ is soft multi closed and $(F, A)^c$ is soft multi g-closed, by theorem (3.5) it follows that $(G, A)^c = \emptyset$ or $\tilde{E} = (G, A)$. Let (H, A) is a soft multi closed set and $(H, A) \subseteq (F, A)$. By definition (4.1) it sufficient to show that $(H, A) \subseteq sm - Int(F, A)$. Now $sm - Int(F, A) \cup (F, A)^c \subseteq sm - Int(F, A) \cup (H, A)^c$ and hence we obtain $sm - Int(F, A) \cup (H, A)^c = \tilde{E}$. Thus $(H, A) \subseteq sm - Int(F, A)$. Hence the proof. \square

Theorem 4.6 *A soft multi set (F, A) is soft multi g-closed if and only if $sm - Cl(F, A) \setminus (F, A)$ is soft multi g-open.*

Proof: Let (F, A) is soft multi g-closed and $(H, A) \subseteq sm - Cl(F, A) \setminus (F, A)$, where (H, A) is soft multi closed. By Theorem (3.5) $(H, A) = \emptyset$ and hence $(H, A) \subseteq sm - Cl(F, A) \setminus sm - Int(F, A)$. By definition (4.1) $sm - Cl(F, A) \setminus (F, A)$ is soft multi g-open. Let $(F, A) \subseteq (G, A)$ where (G, A) is a soft multi open set. Now $sm - Cl(F, A) \cap (G, A)^c \subseteq sm - Cl(F, A) \cap (F, A)^c = sm - Cl(F, A) \setminus (F, A)$ and since $sm - Cl(F, A) \cap (G, A)^c$ is soft closed and $sm - Cl(F, A) \setminus (F, A)$ is soft multi g-open, it follows that $sm - Cl(F, A) \cap (G, A)^c \subseteq sm - Cl(F, A) \setminus sm - Int(F, A) = \emptyset$. Therefore $sm - Cl(F, A) \cap (G, A)^c = \emptyset$ or $sm - Cl(F, A) \subseteq (G, A)$. Thus we get (F, A) is soft multi g-closed. Hence the proof. \square

5 Conclusion

In this paper, we introduced soft multi generalized closed sets in soft multi topological spaces which are defined over an initial universe with a fixed set of parameters. A sufficient condition for a soft multi generalized closed set to be a soft multi closed is also presented. Moreover, the union and intersection of two soft multi generalized closed sets are discussed.

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