On a general class of multiple Eulerian integrals I

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ABSTRACT
Recently, Raina and Srivastava [5] and Srivastava and Hussain [12] have provided closed-form expressions for a number of a general Eulerian integrals involving multivariable H-functions. Motivated by these recent works, we aim at evaluating a general class of multiple eulerian integrals involving a multivariable Aleph-function with general arguments. These integrals will serve as a key formula from which one can deduce numerous useful integrals.

Keywords :Multivariable Aleph-function, multiple Eulerian integral ,class of polynomials, sequence of functions.

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1.Introduction and prerequisites

The well-known Eulerian Beta integral

$$\int_a^b (z-a)^{\alpha-1}(b-t)^{\beta-1}\mathrm{d}t = (b-a)^{\alpha+\beta-1}B(\alpha,\beta)(Re(\alpha)>0, Re(\beta)>0, b>a) \tag{1.1}$$

is a basic result for evaluation of numerous other potentially useful integrals involving various special functions and polynomials. Raina and Srivastava [5], Saigo and Saxena [6], Srivastava and Hussain [12], Srivastava and Garg [11] etc have established a number of Eulerian integrals involving various general class of polynomials, Meijer's G-function and Fox's H-function of one and more variables with general arguments.

For this study, we need the following series formula for the general sequence of functions introduced by Agrawal and Chaubey [1] and was established by Salim [7].

$$R_n^{\alpha,\beta}[x;E,F,g,h;p,q;\gamma;\delta;e^{-\mathfrak{s}x^{\tau}}] = \sum_{w,v,u,t',e,k_1,k_2} \psi(w,v,u,t',e,k_1,k_2)x^R \tag{1.2}$$

where $\psi(w,v,u,t',e,k_1,k_2) = \dfrac{(-)^{t'+w+k_2}(-v)_u(-t')_e(\alpha)_t l^n}{w!v!u!t'!e!l'_n k_1!k_2!}\dfrac{\mathfrak{s}^{w+k_1}F^{\gamma n-t'}}{(1-\alpha-t')_e}(-\alpha-\gamma n)_e(-\beta-\delta n)_v$

$$g^{v+k_2}h^{\delta n-v-k_2}(v-\delta n)_{k_2}E^{t'}\left(\frac{pe+\tau w+\lambda+qu}{l}\right)_n \tag{1.3}$$

and $\displaystyle\sum_{w,v,u,t',e,k_1,k_2} = \sum_{w=0}^{\infty}\sum_{v=0}^{n}\sum_{u=0}^{v}\sum_{t'=0}^{n}\sum_{e=0}^{t'}\sum_{k_1,k_2=0}^{\infty}$

The infinite series in the right hand side of (1.2) is absolutely convergent and $R = ln+qv+pt'+\tau w+\tau k_1+k_2 q$.

We shall note $R_n^{\alpha,\beta}[x;A,B,c,d;p,q;\gamma,\delta;e^{-\mathfrak{s}x^{\tau}}] = R_n^{\alpha,\beta}[x;A,B,c,d;p,q;\gamma,\delta;\mathfrak{s},\tau]$ (1.4)

We recall here the following definition of the general class of polynomials introduced and studied by Srivastava [10]

$$S_V^U(x) = \sum_{\eta=0}^{[V/U]}\frac{(-V)_{U\eta}A_{V,\eta}}{\eta!}x^{\eta} \tag{1.5}$$

where $V=0,1,\cdots$ and U is an arbitrary positive integer. The coefficients $A_{V,\eta}(V,\eta\geqslant 0)$ are arbitrary constants, real or complex.

The multivariable Aleph-function is an extension of the multivariable I-function recently defined by C.K. Sharma and Ahmad [9] , itself is a generalization of the multivariable H-function defined by Srivastava et al [13]. The multivariable Aleph-function is defined by means of the multiple contour integral :

We have : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};\cdots;p_{i(r)},q_{i(r)};\tau_{i(r)};R^{(r)}}\left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{array}\right|$

$$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}] \; ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$$
$$\cdots\cdots\cdots\cdots \; ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$$

$$\left.\begin{array}{l} [(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}] \\ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}] \end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(\xi_1,\cdots,\xi_r)\prod_{k=1}^{r}\theta_k(\xi_k)z_k^{\xi_k}\,\mathrm{d}\xi_1\cdots\mathrm{d}\xi_r \tag{1.6}$$

with $\omega=\sqrt{-1}$

$$\psi(\xi_1,\cdots,\xi_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}\xi_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}\xi_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}\xi_k)]} \tag{1.7}$$

and $$\theta_k(\xi_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}\xi_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}\xi_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}\xi_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}\xi_k)]} \tag{1.8}$$

For more details, see Ayant [2]. The condition for absolute convergence of multiple Mellin-Barnes type contour can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)}=\sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)}-\tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)}-\tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}-\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m_k}\delta_j^{(k)}-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}>0, \text{ with } k=1,\cdots,r,i=1,\cdots,R\;,i^{(k)}=1,\cdots,R^{(k)} \tag{1.9}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.
We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,)\,,max(\,|z_1|,\cdots,|z_r|\,)\rightarrow 0$$

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,)\,,min(\,|z_1|,\cdots,|z_r|\,)\rightarrow\infty$$

where $k=1,\cdots,r:\alpha_k=min[Re(d_j^{(k)}/\delta_j^{(k)})],j=1,\cdots,m_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$$

For convenience, we will use the following notations in this paper.

$$V = m_1, n_1; \cdots ; m_r, n_r \quad (1.10)$$

$$\mathrm{W} = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \cdots, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \quad (1.11)$$

$$A = \{(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)})_{1,\mathfrak{n}}\}, \{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \cdots, \alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}\} : \{(c_j^{(1)}; \gamma_j^{(1)})_{1,n_1}\},$$

$$\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}; \gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}; \cdots ; \{(c_j^{(r)}; \gamma_j^{(r)})_{1,n_r}\}, \tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}} \quad (1.12)$$

$$B = \{\tau_i(b_{ji}; \beta_{ji}^{(1)}, \cdots, \beta_{ji}^{(r)})_{m+1,q_i}\}, \{(d_j^{(1)}; \delta_j^{(1)})_{1,m_1}\}, \tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}; \delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}} ; \cdots ;$$

$$\{(d_j^{(r)}; \delta_j^{(r)})_{1,m_r}\}, \tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}} \quad (1.13)$$

2. Main integral

In this section, we shall establish the following Eulerian multiple integral of multivariable Aleph-function and we shall use the following notations (2.1) and (2.2).

$$X_j = (b_j - a_j) + \rho_j(t_j - a_j) + \sigma_j(b_j - t_j) \quad (2.1)$$

$$Y_j = \frac{(t_j - a_j)^{\gamma_j}(b_j - t_j)^{\delta_j} X_j^{1-\gamma_j-\delta_j}}{\beta_j(b_j - a_j) + (\beta_j\rho_j + \alpha_j - \beta_j)(t_j - a_j) + \beta_j\sigma_j(b_j - t_j)} \quad (2.2)$$

for $j = 1, \cdots, s$

Lemma ([4] p.287)

$$\int_a^b \frac{(t-a)^{\alpha-1}(b-t)^{\beta-1}}{\{b-a+\lambda(t-a)+\mu(b-t)\}^{\alpha+\beta}} \mathrm{d}t = \frac{(1+\lambda)^{-\alpha}(1+\mu)^{-\beta}\Gamma(\alpha)\Gamma(\beta)}{(b-a)\Gamma(\alpha+\beta)} \quad (2.3)$$

with $t \in [a;b]$ $\; a \neq b, Re(\alpha) > 0, Re(\beta) > 0, \;\; \eta + \lambda(t-a) + \mu(b-t) \neq 0$

Theorem

We have the following result

$$\int_{a_1}^{b_1} \cdots \int_{a_s}^{b_s} \prod_{j=1}^s \frac{(t_j - a_j)^{\lambda_j}(b_j - t_j)^{\mu_j}}{X_j^{\lambda_j+\mu_j+2}} S_U^V\left[a \prod_{j=1}^s \frac{(t_j - a_j)^{S_j}(b_j - t_j)^{T_j}}{X_j^{S_j+T_j}}\right]$$

$$R_n^{\alpha,\beta}\left[b \prod_{j=1}^s Y^{\zeta_j}; A, B, c, d; p, q; \gamma, \delta; \mathfrak{s}, \tau\right] \aleph\left(\begin{array}{c|c} \mathrm{z}_1 \prod_{j=1}^s Y_j^{v_j'} & \mathrm{A} \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r \prod_{j=1}^s Y_j^{v_j^{(r)}} & \mathrm{B} \end{array}\right) \mathrm{d}t_1 \cdots \mathrm{d}t_s$$

$$= \left\{ \prod_{j=1}^{s} \{(b_j - a_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1} \right\} \sum_{K=0}^{[V/U]} \sum_{\eta=0}^{\infty} \sum_{w,v,u,t',e,k_1,k_2} \sum_{\tau_1,\cdots,\tau_s=0}^{\infty} \frac{(-V)_{UK}A_{V,K}}{K!\eta!\Gamma(v-\delta n)}$$

$$\psi(w,v,u,t',e,k_1,k_2) \left\{ \prod_{j=1}^{s} \frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-K_jS_j-\gamma_j\zeta_jR-\tau_j}(1+\sigma_j)^{-KT_j-\delta_j\zeta_jR}}{\tau_j!\beta_j^{\tau_j+\zeta_jR}} \right\} a^K b^R s^\eta$$

$$\aleph_{p_i+3s,q_i+2s,\tau_i;R:W;(1,1)}^{0,\mathfrak{n}+3s:V;(1,1)} \left(\begin{array}{c} \mathrm{z}_1 \prod_{j=1}^{s} \left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j'} \\ \cdot \\ \cdot \\ \mathrm{z}_r \prod_{j=1}^{s} \left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j^{(r)}} \\ \frac{cb^q}{d} \prod_{j=1}^{s} \left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-\zeta_j q} \end{array} \middle| \begin{array}{c} \mathbb{A} \\ \cdot \\ \cdot \\ \mathbb{B} \end{array} \right) \tag{2.4}$$

We obtain a Aleph-function of $(r+1)-$variables

where

$$\mathbb{A} = (1-\tau_j-\zeta_jR; v_j', \cdots, v_j^{(r)}, \zeta_jq)_{1,s}, (-\lambda_j - KS_j - \gamma_j\zeta_jR - \tau_j; \gamma_jv_j', \cdots, \gamma_jv_j^{(r)}, \gamma_j\zeta_jq)_{1,s},$$

$$(-\mu_j - KT_j - \delta_j\zeta_jR - \tau_j; \delta_jv_j', \cdots, \delta_jv_j^{(r)}, \delta_j\zeta_jq)_{1,s}, \{(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)}, 0)_{1,\mathfrak{n}}\},$$

$$\{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \cdots, \alpha_{ji}^{(r)}, 0)_{\mathfrak{n}+1,p_i}\} : \{(c_j^{(1)}; \gamma_j^{(1)})_{1,n_1}\}, \tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}; \gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}; \cdots :$$

$$\{(c_j^{(r)}; \gamma_j^{(r)})_{1,n_r}\}, \tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}; (1-v+\delta\eta, 1)$$

$$\mathbb{B} = (1-\zeta_jR; v_j', \cdots, v_j^{(r)}, \zeta_jq)_{1,s}, (-\lambda_j-\mu_j-K(S_j+T_j)-\zeta_j(\gamma_j+\delta_j)R-\tau_j-1; (\gamma_j+\delta_j)v_j', \cdots, (\gamma_j+\delta_j)v_j^{(r)}, (\gamma_j+\delta_j)\zeta_jq)_{1,s}$$

$$,\{\tau_i(b_{ji}; \beta_{ji}^{(1)}, \cdots, \beta_{ji}^{(r)}, 0)_{m+1,q_i}\} : \{(d_j^{(1)}; \delta_j^{(1)})_{1,m_1}\}, \tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}; \delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}; \cdots ;$$

$$\{(d_j^{(r)}; \delta_j^{(r)})_{1,m_r}\}, \tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}; (0,1) \tag{2.5}$$

$\psi(w,v,u,t',e,k_1,k_2)$ is defined by (1.3)

and

$$R = ln + qv + pt' + \tau w + \tau k_1 + k_2 q \tag{2.6}$$

Provided that

(i) $\lambda_j, \mu_j, s_j, t_j, \zeta_j, v_j^{(i)} > 0, \beta_j \neq 0, b_j - a_j \neq 0, \rho_j \neq -1, \sigma_j - 1,$

$(b_j - a_j) + \rho_j(t_j - a_j) + \sigma_j(b_j - t_j) \neq 0, t_j \in [a_j, b_j]$ for $i = 1, \cdots, r, j = 1, \cdots, s$

(ii) $|(\beta_j - \alpha_j)(t_j - a_j)| < |\beta_j\{(b_j - a_j) + \rho_j(t_j - a_j) + \sigma_j(b_j - t_j)\}|; t_j \in [a_j, b_j]$ for $, j = 1, \cdots, s$

(iii)When $\min(S_j,T_j)>0$

(a) $Re(\lambda_j+\gamma_j\zeta_j R)+\sum_{i=1}^{r}\gamma_j v_j^{(i)}\min_{1\leqslant j\leqslant m_i} Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)+1>0$

(b) $Re(\mu_j+\delta_j\zeta_j R)+\sum_{i=1}^{r}\gamma_j v_j^{(i)}\min_{1\leqslant j\leqslant m_i} Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)+1>0$

When $\max(S_j,T_j)<0$

(c) $Re(\lambda_j+S_j[V/U]+\gamma_j\zeta_j R)+\sum_{i=1}^{r}\gamma_j v_j^{(i)}\min_{1\leqslant j\leqslant m_i} Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)+1>0$

(d) $Re(\mu_j+t_j[V/U]+\delta_j\zeta_j R)+\sum_{i=1}^{r}\gamma_j v_j^{(i)}\min_{1\leqslant j\leqslant m_i} Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)+1>0$

When $S_j>0,T_j<0$ inequalities (a) and (d) are satisfied.

When $S_j<0,T_j>0$ inequalities (b) and (c) are satisfied.

(iv) $|arg z_k|<\frac{1}{2}A_i^{(k)}\pi$, $k=1,\cdots,r$ where $A_i^{(k)}$ is given in (1.9).

Proof

To establish the multiple integral formula (2.4), we first use the series representations for the polynomials sets $S_V^U(x)$ and $R_n^{\alpha,\beta}[x;A,B,c,d;p,q;\gamma,\delta;\mathfrak{s},r]$ respectively and the expansion serie of the exponential function in its left hand side. Further, using contour integral representation for the multivariable Aleph-function and then interchanging the order of integration and summation suitably, which is permissible under the conditions stated above, we find that

$$\text{L.H.S} = \sum_{K=0}^{[V/U]}\sum_{\eta=0}^{\infty}\sum_{w,v,u,t',e,k_1,k_2}\frac{(-V)_{UK}A_{V,K}}{K!\eta!}\,a^K b^R s^\eta\psi(w,v,u,t',e,k_1,k_2)\,\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}$$

$$\psi(\xi_1,\cdots,\xi_r)\prod_{i=1}^{r}\theta_i(\xi_i)z_i^{\xi_i}\int_{a_1}^{b_1}\cdots\int_{a_s}^{b_s}\prod_{j=1}^{s}\frac{(t_j-a_j)^{\lambda_j+KS_j}(b_j-t_j)^{\mu_j+KT_j}}{X_j^{\lambda_j+\mu_j+K(S_j+T_j)+2}}\,Y_j^{\zeta_j R+\sum_{i=1}^{r}\xi_i v_j^{(i)}}$$

$$\left(1+\frac{c}{d}b^q\prod_{j=1}^{s}Y_j^{\zeta_j q}\right)^{\delta n-v}\mathrm{d}t_1\cdots\mathrm{d}t_s\,\mathrm{d}\xi_1\cdots\mathrm{d}\xi_r \tag{2.7}$$

Now by writing $\left(1+\frac{c}{d}b^q\prod_{j=1}^{s}Y_j^{\zeta_j q}\right)^{\delta n-v}$ in terms of contour integral and changing the order of integration therein, we obtain

$$\text{L.H.S} = \sum_{K=0}^{[V/U]}\sum_{\eta=0}^{\infty}\sum_{w,v,u,t',e,k_1,k_2}\frac{(-V)_{UK}A_{V,K}}{K!\eta!\Gamma(v-\delta n)}\,a^K b^R s^\eta\,\psi(w,v,u,t',e,k_1,k_2)\,\frac{1}{(2\pi\omega)^{r+1}}\int_{L_1}\cdots\int_{L_r}\int_{L_{r+1}}$$

$$\psi(\xi_1,\cdots,\xi_r)\prod_{i=1}^{r}\theta_i(\xi_i)z_i^{\xi_i}\left(\frac{cb^q}{d}\right)^{\xi_{r+1}}\Gamma(-\xi_{r+1})\Gamma(v-\delta n+\xi_{r+1})\left[\int_{a_1}^{b_1}\cdots\int_{a_s}^{b_s}\right.$$

$$\left.\left\{\prod_{j=1}^{s}\frac{(t_j-a_j)^{\lambda_j+KS_j}(b_j-t_j)^{\mu_j+KT_j}}{X_j^{\lambda_j+\mu_j+K(S_j+T_j)+2}}Y_j^{\zeta_jR+\sum_{i=1}^{r}\xi_iv_j^{(i)}+\zeta_jq\xi_{r+1}}\right\}\mathrm{d}t_1\cdots\mathrm{d}t_s\right]\mathrm{d}\xi_1\cdots\mathrm{d}\xi_r\mathrm{d}\xi_{r+1} \tag{2.8}$$

Substituting the value of Y_j from (2.2) and after simplifications, we get

$$\text{L.H.S} = \sum_{K=0}^{[V/U]}\sum_{\eta=0}^{\infty}\sum_{w,v,u,t',e,k_1,k_2}\frac{(-V)_{UK}A_{V,K}}{K!\eta!\Gamma(v-\delta n)}a^Kb^Rs^\eta\,\psi(w,v,u,t',e,k_1,k_2)\frac{1}{(2\pi\omega)^{r+1}}\int_{L_1}\cdots\int_{L_r}\int_{L_{r+1}}$$

$$\psi(\xi_1,\cdots,\xi_r)\prod_{i=1}^{r}\theta_i(\xi_i)z_i^{\xi_i}\left(\frac{cb^q}{d}\right)^{\xi_{r+1}}\Gamma(-\xi_{r+1})\Gamma(v-\delta n+\xi_{r+1})$$

$$\left[\int_{a_1}^{b_1}\cdots\int_{a_s}^{b_s}\left\{\prod_{j=1}^{s}\frac{(t_j-a_j)^{\lambda_j+KS_j+\gamma_j\sum_{i=1}^{r}\xi_iv_j^{(i)}+\gamma_j\zeta_j(R+q\xi_{r+1})}}{X_j^{\lambda_j+\mu_j+K(S_j+T_j)+2+(\gamma_j+\delta_j)(R\zeta_j+\sum_{i=1}^{r}\xi_iv_j^{(i)}+\zeta_jq\xi_{r+1})}}\right.\right.$$

$$\left.\frac{(b_j-t_j)^{\mu_j+KT_j+\delta_j\sum_{i=1}^{r}\xi_iv_j^{(i)}+\gamma_j\zeta_j(R+q\xi_{r+1})}}{\beta_j^{(R\zeta_j+\sum_{i=1}^{r}\xi_iv_j^{(i)}+\zeta_jq\xi_{r+1})}}\left(1-\frac{(\beta_j-\alpha_j)(t_j-a_j)}{\beta_jX_j}\right)^{-(\zeta_jR+\sum_{i=1}^{r}\xi_iv_j^{(i)}+\zeta_jq\xi_{r+1})}\right\}$$

$$\left.\mathrm{d}t_1\cdots\mathrm{d}t_s\right]\mathrm{d}\xi_1\cdots\mathrm{d}\xi_r\mathrm{d}\xi_{r+1} \tag{2.9}$$

If $\dfrac{(\beta_j-\alpha_j)(t_j-a_j)}{\beta_jX_j}<1, t_j\in[a_j;b_j]$ for $j=1,\cdots,s$

then use the binomial expansion is valid and we thus find that

$$\text{L.H.S} = \sum_{K=0}^{[V/U]}\sum_{\eta=0}^{\infty}\sum_{w,v,u,t',e,k_1,k_2}\sum_{\tau_1,\cdots,\tau_s=0}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!\eta!\Gamma(v-\delta n)}a^Kb^Rs^\eta\,\psi(w,v,u,t',e,k_1,k_2)\prod_{j=1}^{s}\left\{\frac{(\beta_j-\alpha_j)^{\tau_j}}{\beta_j^{\tau_j}\tau_j!}\right\}$$

$$\frac{1}{(2\pi\omega)^{r+1}}\int_{L_1}\cdots\int_{L_r}\int_{L_{r+1}}\psi(\xi_1,\cdots,\xi_r)\prod_{i=1}^{r}\theta_i(\xi_i)z_i^{\xi_i}\left(\frac{cb^q}{d}\right)^{\xi_{r+1}}\Gamma(-\xi_{r+1})\Gamma(v-\delta n+\xi_{r+1})$$

$$\prod_{i=1}^{s}\left\{\frac{\Gamma(\tau_j+R\zeta_j+\sum_{i=1}^{r}\xi_iv_j^{(i)}+\xi q\xi_{r+1})}{\Gamma(R\zeta_j+\sum_{i=1}^{r}\xi_iv_j^{(i)}+\xi q\xi_{r+1})}\beta_j^{-(R\zeta_j+\sum_{i=1}^{r}\xi_iv_j^{(i)}+\zeta_jq\xi_{r+1})}\right\}$$

$$\left[\int_{a_1}^{b_1}\cdots\int_{a_s}^{b_s}\left\{\prod_{j=1}^{s}\frac{(t_j-a_j)^{\lambda_j+KS_j+\gamma_j\sum_{i=1}^{r}\xi_iv_j^{(i)}+\gamma_j\zeta_j(R+q\xi_{r+1})+\tau_j}}{X_j^{\lambda_j+\mu_j+K(S_j+T_j)+2+(\gamma_j+\delta_j)(R\zeta_j+\sum_{i=1}^{r}\xi_iv_j^{(i)}+\zeta_jq\xi_{r+1})+\tau_j}}\right.\right.$$

$$(b_j-x_j)^{\mu_j+KT_j+\delta_j\sum_{i=1}^r\xi_iv_j^{(i)}+\delta_j\zeta_j(R+q\xi_{r+1})}\mathrm{d}t_1\cdots\mathrm{d}t_s\Big]\mathrm{d}\xi_1\cdots\mathrm{d}\xi_r\mathrm{d}\xi_{r+1} \tag{2.10}$$

Now using (2.1) and then evaluating the inner-most integral by using the lemma (2.3), we get

$$\text{L.H.S} = \left\{\prod_{j=1}^s\{(b_j-a_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\right\}\sum_{K=0}^{[V/U]}\sum_{\eta=0}^{\infty}\sum_{w,v,u,t',e,k_1,k_2}\sum_{\tau_1,\cdots,\tau_s=0}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!\eta!\Gamma(v-\delta n)}$$

$$\psi(w,v,u,t',e,k_1,k_2)\left\{\prod_{j=1}^s\frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-K_jS_j-\gamma_j\zeta_jR-\tau_j}(1+\sigma_j)^{-KT_j-\delta_j\zeta_jR}}{\tau_j!\beta_j^{\tau_j+\zeta_jR}}\right\}a^Kb^Rs^\eta$$

$$\frac{1}{(2\pi\omega)^{r+1}}\int_{L_1}\cdots\int_{L_r}\int_{L_{r+1}}\psi(\xi_1,\cdots,\xi_r)\prod_{i=1}^r\theta_i(\xi_i)z_i^{\xi_i}\left(\frac{cb^q}{d}\right)^{\xi_{r+1}}\Gamma(-\xi_{r+1})\Gamma(v-\delta n+\xi_{r+1})$$

$$\prod_{j=1}^s\left\{\frac{\Gamma(\tau_j+\lambda_j+KS_j+\gamma_j\zeta_jR+\gamma_j\sum_{i=1}^r\xi_iv_j^{(i)}+\gamma_j\zeta_jq\xi_{r+1}+1)}{\Gamma(\lambda_j+\mu_j+K(S_j+T_j)+(\gamma_j+\delta_j)(\zeta_jR+\sum_{i=1}^r\xi_iv_j^{(i)}+\xi_jq\xi_{r+1})+\tau_j+2)}\right.$$

$$\left.\Gamma(\mu_j+K_{tj}+\delta_j\zeta_jR+\delta_j\sum_{i=1}^r\xi_iv_j^{(i)}+\delta_j\xi_jq\zeta_{r+1}+1)\right\}\Gamma(-\xi_{r+1})\Gamma(v-\delta n+\xi_{r+1})$$

$$\prod_{j=1}^s\left\{\frac{(1+\rho_j)^{-\Gamma_j}(1+\sigma_j)^{-\delta_j}}{\beta_j}\right\}^{\sum_{i=1}^r\xi_iv_j^{(i)}}\prod_{j=1}^s\left\{\frac{(1+\rho_j)^{-\gamma_j\zeta_jq}(1+\sigma_j)^{-\delta_j\zeta_jq}cb^q}{\beta_j^{\zeta_jq}d}\right\}^{\xi_{r+1}}\mathrm{d}\xi_1\cdots\mathrm{d}\xi_r\mathrm{d}\xi_{r+1} \tag{2.11}$$

Finally, reinterpreting the multiple Mellin-Barnes contour integral in terms of multivariable Aleph-function, we obtain the result (2.4).

3. Particular cases

The multivariable Aleph-function occurring in the main integral can be suitably specialized to a remarkably wide variety of special functions which are expressible in terms of E, G, H and I-function of one and several variables. Again by suitably specializing various parameters and coefficients, the general class of polynomials and the general sequence of functions can be reduced to a large number of orthogonal polynomials and hypergeometric polynomials. Thus using various special cases of these special functions, we can obtain a large number of others integrals involving simpler special functions and polynomials of one and several variables.

On taking $V=0, U=1$ and $A_{0,0}$ in (2.4), the general class of polynomials $S_V^U(x)$ reduces to unity an we get

Corollary 1

$$\int_{a_1}^{b_1}\cdots\int_{a_s}^{b_s}\prod_{j=1}^s\frac{(t_j-a_j)^{\lambda_j}(b_j-t_j)^{\mu_j}}{X_j^{\lambda_j+\mu_j+2}}R_n^{\alpha,\beta}\left[b\prod_{j=1}^sY^{\zeta_j};A,B,c,d;p,q;\gamma,\delta;\mathfrak{s},\tau\right]$$

$$\aleph\left(\begin{array}{c|c} \mathrm{z}_1\prod_{j=1}^sY_j^{v'_j} & \mathrm{A} \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r\prod_{j=1}^sY_j^{v_j^{(r)}} & \mathrm{B}\end{array}\right)\mathrm{d}t_1\cdots\mathrm{d}t_s=\left\{\prod_{j=1}^s\{(b_j-a_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\right\}$$

$$\sum_{\eta=0}^{\infty}\sum_{w,v,u,t',e,k_1,k_2}\sum_{\tau_1,\cdots,\tau_s=0}^{\infty}\frac{\psi(w,v,u,t',e,k_1,k_2)}{\eta!\Gamma(v-\delta n)}\left\{\prod_{j=1}^{s}\frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-\gamma_j\zeta_j R-\tau_j}(1+\sigma_j)^{-\delta_j\zeta_j R}}{\tau_j!\beta_j^{\tau_j+\zeta_j R}}\right\}$$

$$b^R s^\eta \aleph^{0,\mathfrak{n}+3s:V;(1,1)}_{p_i+3s,q_i+2s,\tau_i;R:W;(1,1)}\left(\begin{array}{c|c} \mathrm{z}_1\prod_{j=1}^{s}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j'} & \mathbb{A}_1\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r\prod_{j=1}^{s}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j^{(r)}} & \mathbb{B}_1\\ \frac{cb^q}{d}\prod_{j=1}^{s}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-\zeta_j q} & \end{array}\right) \quad (3.1)$$

where

$$\mathbb{A}_1=(1-\tau_j-\zeta_j R;v_j',\cdots,v_j^{(r)},\zeta_j q)_{1,s},(-\lambda_j-\gamma_j\zeta_j R-\tau_j;\gamma_j v_j',\cdots,\gamma_j v_j^{(r)},\gamma_j\zeta_j q)_{1,s},$$

$$(-\mu_j-\delta_j\zeta_j R-\tau_j;\delta_j v_j',\cdots,\delta_j v_j^{(r)},\delta_j\zeta_j q)_{1,s},\{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)},0)_{1,\mathfrak{n}}\},$$

$$\{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)},0)_{\mathfrak{n}+1,p_i}\}:\{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}};\cdots:$$

$$\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}};(1-v+\delta\eta,1)$$

$$\mathbb{B}_1=(1-\zeta_j R;v_j',\cdots,v_j^{(r)},\zeta_j q)_{1,s},(-\lambda_j-\mu_j-\zeta_j(\gamma_j+\delta_j)R-\tau_j-1;(\gamma_j+\delta_j)v_j',\cdots,(\gamma_j+\delta_j)v_j^{(r)},(\gamma_j+\delta_j)\zeta_j q)_{1,s}$$

$$,\{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)},0)_{m+1,q_i}\}:\{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}};\cdots;$$

$$\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}};(0,1) \quad (3.2)$$

with the same notations and corresponding validity conditions that (2.4).

Putting $s=1$ in (2.4), we arrive at the following integral formula

Corollary 2

$$\int_{a_1}^{b_1}\frac{(t-a_1)^{\lambda}(b_1-t)^{\mu}}{X_j^{\lambda+\mu+2}}S_U^V\left[a\frac{(t-a_1)^{S_j}(b_1-t)^T}{X^{S+T}}\right]R_n^{\alpha,\beta}\left[bY^{\zeta};A,B,c,d;p,q;\gamma,\delta;\mathfrak{s},\tau\right]$$

$$\aleph\left(\begin{array}{c|c}\mathrm{z}_1 Y^{v'} & \mathrm{A}\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r Y^{v^{(r)}} & \mathrm{B}\end{array}\right)\mathrm{d}t=\left\{(b_1-a_1)^{-1}(1+\rho)^{-\lambda-1}(1+\sigma)^{-\mu-1}\right\}\sum_{K=0}^{[V/U]}\sum_{\eta=0}^{\infty}$$

$$\sum_{w,v,u,t',e,k_1,k_2}\sum_{\tau_1=0}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!\eta!\Gamma(v-\delta n)}\psi(w,v,u,t',e,k_1,k_2)a^K b^R s^\eta\left\{\frac{(\beta-\alpha)^{\tau}(1+\rho)^{-KS-\gamma-\tau}(1+\sigma)^{-KT-\delta\zeta R}}{\tau!\beta^{\tau+\zeta R}}\right\}$$

$$\aleph_{p_i+3,q_i+2,\tau_i;R:W;(1,1)}^{0,\mathfrak{n}+3:V;(1,1)}\left(\begin{array}{c|c} \mathrm{z}_1\left\{\beta(1+\rho)^{\gamma}(1+\sigma)^{\delta}\right\}^{-v'} & \mathbb{A}_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r\left\{\beta(1+\rho)^{\gamma}(1+\sigma)^{\delta}\right\}^{-v^{(r)}} & \mathbb{B}_2 \\ \frac{cb^q}{d}\left\{\beta(1+\rho)^{\gamma}(1+\sigma)^{\delta}\right\}^{-\zeta q} & \end{array}\right) \tag{3.3}$$

where

$\mathbb{A}_2=(1-\tau_1-\zeta R;v',\cdots,v^{(r)},\zeta q),(-\lambda-KS-\gamma\zeta R-\tau_1;\gamma v',\cdots,\gamma v^{(r)},\gamma\zeta q),$

$\{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)},0)_{1,\mathfrak{n}}\},(-\mu-KT-\delta\zeta R-\tau_1;\delta v',\cdots,\delta v^{(r)},\delta\zeta q),$

$\{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)},0)_{\mathfrak{n}+1,p_i}\}:\{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}};\cdots:$

$\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}};(1-v+\delta\eta,1)$

$\mathbb{B}_2=(1-\zeta R;v',\cdots,v^{(r)},\zeta q),(-\lambda-\mu-K(S+T)-\zeta(\gamma+\delta)R-\tau_1-1;(\gamma+\delta)v',\cdots,(\gamma+\delta)v^{(r)},(\gamma+\delta)\zeta q)$

$,\{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)},0)_{m+1,q_i}\}:\{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}};\cdots;$

$\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}};(0,1)$ (3.4)

with the same notations and corresponding validity conditions that (2.4).

Putting $t_j=b_j(b_j-a_j)v_j;j=1,\cdots,s$ in (2.4), we obtain the following result.

Corollary 3

$$\int_0^1\cdots\int_0^1\prod_{j=1}^s\frac{(1-v_j)^{\lambda_j}v_j^{\mu_j}}{X_j'^{\lambda_j+\mu_j+2}}S_U^V\left[a\prod_{j=1}^s\frac{(1-v_j)^{S_j}v_j^{T_j}}{X_j'^{S_j+T_j}}\right]$$

$$R_n^{\alpha,\beta}\left[b\prod_{j=1}^s Y_j'^{\zeta_j};A,B,c,d;p,q;\gamma,\delta;\mathfrak{s},r\right]\aleph\left(\begin{array}{c|c}\mathrm{z}_1\prod_{j=1}^s Y_j^{v_j'} & \mathrm{A}\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r\prod_{j=1}^s Y_j^{v_j^{(r)}} & \mathrm{B}\end{array}\right)\mathrm{d}t_1\cdots\mathrm{d}t_s$$

$$=\left\{\prod_{j=1}^s\{(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\right\}\sum_{K=0}^{[V/U]}\sum_{\eta=0}^{\infty}\sum_{w,v,u,t',e,k_1,k_2}\sum_{\tau_1,\cdots,\tau_s=0}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!\eta!\Gamma(v-\delta n)}$$

$$\psi(w,v,u,t',e,k_1,k_2)\left\{\prod_{j=1}^s\frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-K_jS_j-\gamma_j\zeta_jR-\tau_j}(1+\sigma_j)^{-KT_j-\delta_j\zeta_jR}}{\tau_j!\beta_j^{\tau_j+\zeta_jR}}\right\}a^Kb^Rs^\eta$$

$$\aleph^{0,\mathfrak{n}+3s:V;(1,1)}_{p_i+3s,q_i+2s,\tau_i;R:W;(1,1)}\left(\begin{array}{c|c} \mathrm{z}_1\prod_{j=1}^s\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v'_j} & \mathbb{A} \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r\prod_{j=1}^s\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j^{(r)}} & \mathbb{B} \\ \frac{cb^q}{d}\prod_{j=1}^s\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-\zeta_j q} & \end{array}\right) \quad (3.5)$$

where

$$X'_j = v_j(\rho_j-\sigma_j)+\rho_j+1 \quad (3.6)$$

and

$$Y_j = \frac{((1-v_j)^{\lambda_j}v_j^{\delta_j}(X'_j)^{1-\gamma_j-\delta_j}}{(\alpha_j+\beta_j\rho_j)(1-v_j)+(1+\sigma_j)\beta_j v_j} \quad (3.7)$$

for $j=1,\cdots,s$

with the same notations and corresponding validity conditions that (2.4).

Putting $r=1,\mathfrak{n}=p_i=q_i=0(i=1)$ in (2.4), the multivariable Aleph-function in the L.H.S. reduces to Aleph-function of one variable defined by Sudland [14] and we have

Corollary 4

$$\int_{a_1}^{b_1}\cdots\int_{a_s}^{b_s}\prod_{j=1}^s\frac{(t_j-a_j)^{\lambda_j}(b_j-t_j)^{\mu_j}}{X_j^{\lambda_j+\mu_j+2}}S_U^V\left[a\prod_{j=1}^s\frac{(t_j-a_j)^{S_j}(b_j-t_j)^{T_j}}{X_j^{S_j+T_j}}\right]$$

$$R_n^{\alpha,\beta}\left[b\prod_{j=1}^s Y^{\zeta_j};A,B,c,d;p,q;\gamma,\delta;\mathfrak{s},r\right]\aleph^{M_1,N_1}_{P_\mathrm{i},Q_\mathrm{i},\tau_\mathrm{i};r}\left(\mathrm{z}_1\prod_{j=1}^s Y_j^{v'_j}\ \middle|\ \begin{array}{c}(\mathrm{a}_j,A_j)_{1,\mathfrak{n}},[c_\mathrm{i}(a_{j\mathrm{i}},A_{j\mathrm{i}})]_{\mathfrak{n}+1,p_{\mathrm{i}};r}\\ (\mathrm{b}_j,B_j)_{1,m},[c_\mathrm{i}(b_{j\mathrm{i}},B_{j\mathrm{i}})]_{m+1,q_\mathrm{i};r}\end{array}\right)$$

$$\mathrm{d}t_1\cdots\mathrm{d}t_s=\left\{\prod_{j=1}^s\{(b_j-a_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\right\}\sum_{K=0}^{[V/U]}\sum_{\eta=0}^{\infty}\sum_{w,v,u,t',e,k_1,k_2}\sum_{\tau_1=0}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!\eta!\Gamma(v-\delta n)}$$

$$\psi(w,v,u,t',e,k_1,k_2)\left\{\prod_{j=1}^s\frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-K_jS_j-\gamma_j\zeta_jR-\tau_j}(1+\sigma_j)^{-KT_j-\delta_j\zeta_jR}}{\tau_j!\beta_j^{\tau_j+\zeta_jR}}\right\}a^Kb^Rs^\eta$$

$$\aleph^{M_1,N_1+3s:(1,1)}_{P_\mathrm{i}+3s,Q_\mathrm{i}+2s,\tau_\mathrm{i}:(1,1)}\left(\begin{array}{c|c} \mathrm{z}_1\prod_{j=1}^s\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v'_j} & \mathbb{A}_3 \\ \cdot & \cdot \\ \cdot & \cdot \\ \frac{cb^q}{d}\prod_{j=1}^s\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-\zeta_j q} & \mathbb{B}_3 \end{array}\right) \quad (3.8)$$

We obtain a Aleph-function of two variables defined by Sharma [8], where

$$\mathbb{A}_3=(1-\tau_j-\zeta_jR;v'_j,\zeta_jq)_{1,s},(-\lambda_j-KS_j-\gamma_j\zeta_jR-\tau_j;\gamma_jv'_j,\gamma_j\zeta_jq)_{1,s},$$

$$(-\mu - KT - \delta\zeta R - \tau_1; \delta v'_j, \delta_j\zeta_j q)_{1,s}, (\mathrm{a}_j, A_j)_{1,\mathfrak{n}}, [c_i(a_{ji}, A_{ji})]_{\mathfrak{n}+1,p_i;r}; (1 - v + \delta n, 1)$$

$$\mathbb{B}_3 = (1 - \zeta_j R; v'_j, \zeta_j q)_{1,s}, (-\lambda_j - \mu_j - K(S_j + T_j) - \zeta_j(\gamma_j + \delta_j)R - \tau_j - 1; (\gamma_j + \delta_j)v'_j, (\gamma_j + \delta_j)\zeta_j q)_{1,s},$$

$$(\mathrm{b}_j, B_j)_{1,m}, [c_i(b_{ji}, B_{ji})]_{m+1,q_i;r}; (0,1) \quad (3.9)$$

Remark : If the multivariable Aleph-function reduces to multivariable H-function [13] , we obtain the recents results of Bhargava et al [3].

5. Conclusion

Our main integral formula is unified in nature and possesses manifold generality. It acts a key formula and using various special cases of the multivariable Aleph-function, general class of polynomials and a general sequence of functions, one can obtain a large number of other integrals involving simpler special functions and polynomials of one and several variables.

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