Homomorphism on T – Fuzzy Ideal of ℓ- Near-Rings

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Abstract: *In this paper, we made an attempt to study the properties of T-fuzzy ideal of a ℓ-near-ring and we introduce some theorems an onto homomorphic image, an epimorphic pre-image of a T-fuzzy ideal of a ℓ-near-ring.*

Keywords: *Fuzzy subset, T-fuzzy ideal, homomorphism of T - fuzzy ideal, homomorphic image of T - fuzzy ideal, homomorphic pre-image T-fuzzy ideal and product of T- fuzzy ideal.*

I. Introduction

The concept of fuzzy sets was initiated by Zadeh. L.A, [18] in 1965. After the introduction of fuzzy sets several researchers explored on the generalization of the concept of fuzzy sets. In Ayyappan,M., and Natarajan,R., [3] have introduced Lattice ordered near rings. In Dudek. W.A., and Jun. Y.B., [9] introduced Fuzzy subquasigroups over a t-norm. In 1971, Liu. W., [10] studied fuzzy ideals in rings. In Satyanarayana, Bh., and Syam Prasad, K. [12] introduced Gamma near-rings. In Srinivas, T., Nagaiah, T., and Narasimha Swamy, P., [14] studied anti fuzzy ideals of $\Gamma-$near-rings. Dheena. P., and Mohanraaj. G., [8] have studied several properties of $T-$fuzzy ideals of rings and Akram, M., [2] have studied some results of $T-$fuzzy ideals of near-rings. We extended the results of $T-$fuzzy ideals of a $\ell-$near-rings.

In this paper we define, homomorphism and study the $\ell-$near-ring homomorphism. Wang, Z.D., [15] introduced the basic concepts of TL-ideals and Fuzzy invariant subgroups and fuzzy ideals. We introduced homomorphism in $T-$fuzzy ideals of $\ell-$near- ring. We discuss some of its properties. We have shown that homomorphism, homomorphic image of $T-$fuzzy ideal, homomorphic pre-image $T-$fuzzy ideal and product of $T-$fuzzy ideal of $\ell-$ near-ring

II. Definitions and Examples

Definition: 1

A mapping $T:[0,\ 1]\times[0,\ 1]\to[0,\ 1]$ is called a triangular norm [$t-$norm] if and only if it satisfies the following conditions:

(i). $T(x,\ 1)=T(1,x)=x,$ for all $x\in[0,\ 1]$

(ii). if $x\geq x^*,\ y\geq y^*$ then $T(x,y)\geq T\left(x^*,y^*\right)$

(iii). $T(x,\ y)=T(y,\ x)$, for all $x,\mathrm{y}\in[0,\ 1]$.

(iv). $T(x,\ T(\mathrm{y,z}))=T(T(\mathrm{x,y}),\ \mathrm{z})$.

Definition: 2

A fuzzy subset μ of a ring R is called $T-$fuzzy right (resp. left) ideal if

(i) $\mu(x-y)\geq T(\mu(x),\mu(y))=\min\{\mu(x),\ \mu(y)\}$

(ii) $\mu(xy)\geq\{\mu(x)\}$ (resp. left $\mu(xy)\geq\{\mu(y)\}$), for all x,y in R

Definition: 3

A fuzzy subset μ of a $\ell-$near-ring R is called a $T-$ fuzzy ideal, if the following conditions are satisfied,

(i) $\mu(x-y)\geq T(\mu(x),\ \mu(y))$, for all $x,y\in R$

(ii) $\mu(y+x-y)\geq(\mu(x))$, for all x,y in R

(iii) $\mu(xy)\geq\mu(y);\mu(xy)\geq\mu(x)$, for all $x,\ y\in R$

(iv) $\mu((x+z)y-x\ y)\geq(\mu(z))$, for all $x,y,z\in R$

(v) $\mu(x\vee y)\geq T(\mu(x),\ \mu(y))$, for all $x,\ y\in R$

(vi) $\mu(x\wedge y)\geq T(\mu(x),\ \mu(y))$, for all $x,y\in R$

Example: 1

Now $(R=\{a,\ b,\ c\},+,\cdot,\vee,\wedge)$ is a $\ell-$ near-ring. Consider the fuzzy subset μ of the $\ell-$near-ring R

$$\mu(x)=\begin{cases}0.6 & if\ \ x=a\\ 0.5 & if\ \ x=b\\ 0.7 & if\ \ x=c\end{cases}$$

Then μ is a $T-$fuzzy ideal of $\ell-$near-ring R

Definition: 4

Let R_1 and R_2 be two $\ell-$near rings. Then the function $f:R_1\to R_2$ is called a $\ell-$ near ring homomorphism if satisfies the following conditions

(i) $f(x+y)=f(x)+f(y)$

(ii) $f(xy)=f(x)f(y)$

(iii) $f(x\vee y)=f(x)\vee f(y)$

(iv) $f(x\wedge y)=f(x)\wedge f(y)$, for all x,y in R

Example: 2

Let $R=\{m+n\sqrt{2},\ for\ all\ m,n\in Z\}$, R is a $\ell-$ring under usual addition and multiplication. Define $f:R\to R$ by $f(m+n\sqrt{2})=m-n\sqrt{2}$ is $\ell-$ near ring homomorphism, where Z is set of all integer

Definition: 5

A fuzzy set μ of a $\ell-$ near ring R has the **supremum** property if for any subset N of R, there exists a $a_0\in N$ such that $\mu(a_0)=\sup_{a\in N}\mu(a)$

Definition: 6

Let μ,λ be two fuzzy ideals of a $\ell-$near ring R, then the sum $\mu+\lambda$ is a fuzzy set of R defined by

$(\mu+\lambda)(x)=\{\sup(\min(\mu(y),\lambda(z)))$, if $x=y+z$,

0, otherwise, for all $x,\ y,\ z\in R$.

Definition: 7

Let R_1 and R_2 be two $\ell-$near rings. A mapping $f:R_1\to R_2$ is called a $\ell-$near ring isomorphism if

(i) f is one-to-one

(ii) f is onto, for all x,y in R

Definition: 8

Let M and N be any two sets and let $f:M\to N$ be any function. A fuzzy subset μ of a M is called f-invariant if $f(x)=f(y)\Rightarrow\mu(x)=\mu(y)$, for all $x,\ y\in M$

Definition: 9

Let R be a $\ell-$near ring. Let μ be a fuzzy set of a $T-$fuzzy ideals of a $\ell-$near ring R and f be a function defined on R, then the fuzzy set μ^f in $f(R)$ is defined by $\mu^f(y)=\sup_{n\in f^{-1}(y)}\mu(x)$, for all $y\in f(R)$ and is called the image of μ under f

Definition: 10

Let R be a $\ell-$near ring. Let μ be a fuzzy set of a $T-$fuzzy ideals of a $\ell-$near ring R and f be a function defined on R, if v is a fuzzy set in $f(R)$, then $\mu=v\circ f$ in R is defined by $\mu(x)=v(f(x))$, for all $x\in R$ and is called the pre-image of v under f.

III. THEOREMS

Theorem: 1

Every fuzzy ideal of a $\ell-$ near ring R is a $T-$fuzzy ideal of a $\ell-$near ring R.

Theorem: 2

An onto homomorphism image of a $T-$fuzzy ideal of a $\ell-$ near ring R with **sup** property is a $T-$fuzzy ideal of a $\ell-$near ring R.

Proof:

Let R and S be a $\ell-$ near rings

Let $f:R\to S$ be an epimorphism and A be a $S-$ fuzzy ideal of a $\ell-$ near ring R with **sup** property. Let $x,y\in S$

Let $x_0\in f^{-1}(x)$, $y_0\in f^{-1}(y)$ and $z_0\in f^{-1}(z)$ be such that

$\mu(x_0)=\sup_{n\in f^{-1}(x)}\mu(n)$, $\mu(y_0)=\sup_{n\in f^{-1}(y)}\mu(n)$ and

$\mu(z_0)=\sup_{n\in f^{-1}(z)}\in\mu(n)$ respectively, then

(i)
$$
\begin{aligned}
\mu^f(x-y) &= \sup_{z\in f^{-1}(x-y)}\mu(z)\\
&\geq \mu(x_0-y_0)\\
&\geq \min(\mu(x_0),\ \mu(y_0))\\
&= T(\mu(x_0),\ \mu(y_0))\\
&\geq T\left(\sup_{n\in f^{-1}(x)}\mu(n),\ \sup_{n\in f^{-1}(y)}\mu(n)\right)\\
&= T(\mu^f(x),\ \mu^f(y))
\end{aligned}
$$

Therefore $\mu^f(x-y)\geq T(\mu^f(x),\ \mu^f(y))$, for all $x,y\in S$

(ii) Since $\mu(y+x-y)\geq\mu(x)$

$$
\begin{aligned}
\mu^f(y+x-y) &= \sup_{z\in f^{-1}(y+x-y)}\mu(z)\\
&\geq \mu((y_0+x_0-y_0))\\
&\geq \mu(x_0)=\sup_{n\in f^{-1}(x)}\mu(n)
\end{aligned}
$$

$= \mu^f(x)$

Therefore $\mu^f(y+x-y) \geq \mu^f(x)$, for all $x, y \in S$

(iii) Let μ be a $T-$fuzzy ideals of R and let $x, y \in R$

Since $\mu(xy) \geq \mu(x)$ and $\lambda(xy) \geq \lambda(y)$

we have $\mu^f(xy) = \sup_{z \in f^{-1}(xy)} \mu(z)$

$\geq \mu(x_0 y_0)$

$\geq \mu(x_0) \geq \sup_{n \in f^{-1}(x)} \mu(n)$

$= \mu^f(x)$

Therefore $\mu^f(xy) \geq \mu^f(x)$, for all $x, y \in S$

(iv) Since $\mu((x+z)y - x\,y) \geq \mu(z)$

$\mu^f((x+z)y - xy) = \sup_{z \in f^{-1}((x+z)y - x\,y)} \mu(z)$

$\geq \mu((x_0+z_0)y_0 - x_0 y_0)$

$\geq \mu(z_0)$

$= \sup_{n \in f^{-1}(z)} \mu(n)$

$= \mu^f(z)$

Therefore $\mu^f((x+z)y - xy) \geq \mu^f(z)$, for all $x, y \in S$

(v) $\mu^f(x \vee y) = \sup_{z \in f^{-1}(x \vee y)} \mu(z)$

$\geq \mu(x_0 \vee y_0)$

$\geq \min(\mu(x_0), \mu(y_0))$

$= T(\mu(x_0), \mu(y_0))$

$\geq T\left(\sup_{n \in f^{-1}(x)} \mu(n), \sup_{n \in f^{-1}(y)} \mu(n)\right)$

$= T(\mu^f(x), \mu^f(y))$

Therefore $\mu^f(x \vee y) \geq T(\mu^f(x), \mu^f(y))$, for all $x, y \in S$

(vi) $\mu^f(x \wedge y) = \sup_{z \in f^{-1}(x \wedge y)} \mu(z)$

$\geq \mu(x_0 \wedge y_0) \geq \min(\mu(x_0), \mu(y_0))$

$= T(\mu(x_0), \mu(y_0))$

$\geq T\left(\sup_{n \in f^{-1}(x)} \mu(n), \sup_{n \in f^{-1}(y)} \mu(n)\right)$

$= T(\mu^f(x), \mu^f(y))$

Therefore $\mu^f(x \wedge y) \geq T(\mu^f(x), \mu^f(y))$, for all $x, y \in S$

Thus an onto homomorphic image of a $T-$fuzzy ideal of a $\ell-$near ring R with **sup** property is a $T-$fuzzy ideal of a $\ell-$near-ring R .

Theorem: 3

An epimorphic pre-image of a $T-$fuzzy ideal of a $\ell-$near ring is a $T-$fuzzy ideal of a $\ell-$near ring R.

Proof:

Let R and S be a $\ell-$near rings. Let $f: R \to S$ be an epimorphism. Let ν be a $T-$fuzzy ideal of a $\ell-$near ring S and μ be the pre-image of ν under f for any $x, y, z \in R$.

(i) we have $\mu(x-y) = (\nu \circ f)(x-y)$

$= \nu(f(x-y))$

$= \nu(f(x) - f(y))$

$\geq T(\nu(f(x)), \nu(f(y)))$

$\geq T((\nu \circ f)(x), (\nu \circ f)(y))$

$= T(\mu(x), \mu(y))$

Therefore $\mu(x-y) \geq T(\mu(x), \mu(y))$, for all $x, y \in R$

(ii) Since $\mu(y+x-y) \geq \mu(x)$

$\mu(y+x-y) = (\nu \circ f)(y+x-y)$

$= \nu(f(y+x-y))$

$= \nu(f(x))$

$\geq T(\nu(f(x)))$

$\geq T((\nu \circ f)(x))$

$= (\nu \circ f)(x)$

$= \mu(x)$

Therefore $\mu(y+x-y) \geq \mu(x)$, for all $x, y \in S$

(iii) Since $\mu(xy) \geq \mu(x)$

$\mu(xy) = (\nu \circ f)(xy)$

$= \nu(f(xy))$

$= \nu(f(x) f(y))$

$\geq T(\nu(f(x)))$

$\geq T\left(\left(v \circ f\right)(x)\right)$

$= \left(v \circ f\right)(x)$

$= \mu(x)$

Therefore $\mu(xy) \geq \mu(x)$, for all $x, y \in R$

(iv) Since $\mu\left((x+z)y - x\,y\right) \geq \mu(z)$

$\mu\left((x+z)y - x\,y\right) = \left(v \circ f\right)\left((x+z)y - x\,y\right)$

$= v\left(f\left((x+z)y - x\,y\right)\right) = v\left(f\left(yz\right)\right)$

$\geq T\left(v\left(f(y), f(z)\right)\right) \geq v\left(f(z)\right)$

$\geq T\left(\left(v \circ f\right)(z)\right)$

$= \left(v \circ f\right)(z)$

$= \mu(z)$

Therefore $\mu\left((x+z)y - xy\right) \geq \mu(z)$, for all $x, y \in S$

(v) we have $\mu(x \vee y) = \left(v \circ f\right)(x \vee y)$

$= v\left(f\left(x \vee y\right)\right)$

$= v\left(f(x) \vee f(y)\right)$

$\geq T\left(v\left(f(x)\right), v\left(f(y)\right)\right)$

$\geq T\left(\left(v \circ f\right)(x), \left(v \circ f\right)(y)\right)$

$= T\left(\mu(x), \mu(y)\right)$

Therefore $\mu(x \vee y) \geq T\left(\mu(x), \mu(y)\right)$, for all $x, y \in R$

(vi) we have $\mu(x \wedge y) = \left(v \circ f\right)(x \wedge y)$

$= v\left(f\left(x \wedge y\right)\right)$

$= v\left(f(x) \wedge f(y)\right)$

$\geq T\left(v\left(f(x)\right), v\left(f(y)\right)\right)$

$\geq T\left(\left(v \circ f\right)(x), \left(v \circ f\right)(y)\right)$

$= T\left(\mu(x), \mu(y)\right)$

Therefore $\mu(x \wedge y) \geq T\left(\mu(x), \mu(y)\right)$, for all $x, y \in R$

Thus an epimorphic pre-image of a $T-$fuzzy ideal of a $\ell-$near ring is a $T-$fuzzy ideal of a $\ell-$near-ring R.

Proposition: 1

Let R and S be $\ell-$near rings R and let $f : R \to S$ be a homomorphism. Let μ be $f-$invariant fuzzy ideal of a $\ell-$ near-ring R. If $x = f(a)$, then $f(a)(x) = \mu(a)$, for all $a \in R$.

Theorem: 4

Let $f : R \to S$ be an epimorphism of $\ell-$near rings R and S. If μ is $f-$ invariant fuzzy ideal of a $\ell-$near-ring R, then $f(\mu)$ is a $T-$fuzzy ideal of S.

Proof:

Let $a, b, c \in S$ then there exists $x, y, z \in R$ such that $f(x) = a$, $f(y) = b$ and $f(z) = c$

Suppose μ is $f-$ invariant fuzzy ideal of a $\ell-$near-ring R, then by Proposition 1

(i) we have $f(\mu)(a-b) = f(\mu)\left(f(x) - f(y)\right)$

$= f(\mu)\left(f(x-y)\right)$

$= \mu(x-y)$

$\geq T\left(\mu(x), \mu(y)\right)$

$= T\left(f(\mu)(a), f(\mu)(b)\right)$

Therefore $f(\mu)(a-b) \geq T\left(f(\mu)(a), f(\mu)(b)\right)$, for all $a, b \in S$ and $x, y \in R$

(ii) Since $\mu(y + x - y) \geq \mu(x)$

We have

$f(\mu)(b + a - b) = f(\mu)\left(f(y) + f(x) + f(y)\right)$

$= f(\mu)\left(f(y + x - y)\right)$

$= \mu(y + x - y)$

$\geq \mu(x)$

$= f(\mu)(a)$

Therefore $f(\mu)(b + a - b) \geq f(\mu)(a)$, for all $a, b \in S$ and $x, y \in R$

(iii) Since $\mu(xy) \geq \mu(x)$ and $\lambda(xy) \geq \lambda(y)$

We have $f(\mu)(ab) = f(\mu)\left(f(x)\,f(y)\right)$

$= f(\mu)\left(f(xy)\right)$

$= \mu(xy)$

$\geq \mu(x)$

$= f(\mu)(a)$

Therefore $f(\mu)(ab) \geq f(\mu)(a)$, for all $a, b \in S$ and $x, y \in R$

(iv) Since $\mu\left((x+z)y - xy\right) \geq \mu(z)$

$f(\mu)\left((a+c)b - ab\right)$

$= f(\mu)\left(\left(f(x) + f(z)\right) f(y) - f(x)\, f(y)\right)$

$= f(\mu)\left(f\left((x+z)y - xy\right)\right)$

$= \mu((x+z)y - xy)$

$\geq \mu(z) = f(\mu)(c)$

Therefore $f(\mu)((a+c)b-ab) \geq f(\mu)(c)$, for all $a,b \in S$ and $x,y \in R$

(v) we have $f(\mu)(a \vee b) = f(\mu)(f(x) \vee f(y))$

$= f(\mu)(f(x \vee y))$

$= \mu(x \vee y)$

$\geq T(\mu(x), \mu(y))$

$= T(f(\mu)(a), f(\mu)(b))$

Therefore $f(\mu)(a \vee b) \geq T(f(\mu)(a), f(\mu)(b))$, for all $a,b \in S$ and $x,y \in R$

(vi) we have $f(\mu)(a \wedge b) = f(\mu)(f(x) \wedge f(y))$

$= f(\mu)(f(x \wedge y))$

$= \mu(x \wedge y)$

$\geq T(\mu(x), \mu(y))$

$= T(f(\mu)(a), f(\mu)(b))$

Therefore $f(\mu)(a \wedge b) \geq T(f(\mu)(a), f(\mu)(b))$, for all $a,b \in S$ and $x,y \in R$

Hence, $f(\mu)$ is a $T-$fuzzy ideal of a $\ell-$near ring S.

Theorem: 5

Let $f: R_1 \to R_2$ be an onto homomorphism of a $\ell-$near rings. If μ is $T-$fuzzy ideal of R_1, then $f(\mu)$ is a $T-$fuzzy ideal of R_2.

Proof:

Let μ be a $T-$fuzzy ideal of a $\ell-$near ring R_1

Let $\mu_1 = f^{-1}(y_1)$ and $\mu_2 = f^{-1}(y_2)$, where $y_1, y_2 \in R_2$ are non-empty subsets of R_2

Similarly, $\mu_3 = f^{-1}(y_1 - y_2)$

Consider the set $\mu_1 - \mu_2 = \{a_1 - a_2 / a_1 \in \mu_1, a_2 \in \mu\}$

If $x \in \mu_1 - \mu_2$, then $x = x_1 - x_2$, for some $x_1 \in \mu_1$, $x_2 \in \mu_2$ and so,

$f(x) = f(x_1 - x_2) = f(x_1) - f(x_2) = y_1 - y_2$

$\Rightarrow x \in f^{-1}(y_1 - y_2) = \mu_3$

Thus $\mu_1 - \mu_2 \subseteq \mu_3$ that is $\{x / x \in f^{-1}(y_1 - y_2)\}$

$\supseteq \{x_1 - x_2 / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

Let $y_3 \in R_2$, then

(i) we have

$f(\mu)(y_1 - y_2) = \sup\{\mu(x) / x \in f^{-1}(y_1 - y_2)\}$

$\geq \sup\{\mu(x_1 - x_2) / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

$\geq \sup\{\min\{\mu(x_1), \mu(x_2)\} / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

$\geq \sup\{T(\mu(x_1), \mu(x_2)) / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

$= T(\sup\{\mu(x_1)\} / x_1 \in f^{-1}(y_1), \sup\{\mu(x_2)\} / x_2 \in f^{-1}(y_2))$

$= T(f(\mu)(y_1), f(\mu)(y_2))$

Therefore $f(\mu)(y_1 - y_2) \geq T(f(\mu)(y_1), f(\mu)(y_2))$, for all $y_1, y_2 \in R_2$

(ii) Since $\mu(y + x - y) \geq \mu(x)$

We have

$f(\mu)(y_2 + y_1 - y_2) = \sup\{\mu(x) / x \in f^{-1}(y_2 + y_1 - y_2)\}$

$\geq \sup\{\mu(x_2 + x_1 - x_2) / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

$\geq \sup\{\mu(x_1) / x_1 \in f^{-1}(y_1)\}$

$= f(\mu)(y_1)$

Therefore $f(\mu)(y_2 + y_1 - y_2) \geq f(\mu)(y_1)$, for all $y_1, y_2 \in R_2$

(iii) We have $\mu(xy) \geq \mu(x)$ and $\lambda(xy) \geq \lambda(y)$

We have

$f(\mu)(x_1 x_2) = \sup\{\mu(y) / y \in f^{-1}(x_1 x_2)\}$

$\geq \sup\{\mu(x_1 x_2) / y_1 \in f^{-1}(x_1), y_2 \in f^{-1}(x_2)\}$

$\geq \sup\{\mu(x_2) / y_2 \in f^{-1}(x_2)\}$

$= f(\mu)(x_2)$

Therefore $f(\mu)(x_1 x_2) \geq f(\mu)(x_2)$, for all $y_1, y_2 \in R_2$

(iv) Since $\mu((x+z)y - xy) \geq \mu(z)$

We have $f(\mu)((a+c)b - ab)$

$= f(\mu)((f(x) + f(z)) f(y) - f(x) f(y))$

We have $f(\mu)((y_1 + y_3) y_2 - y_1 y_2)$

$= \sup\{\mu(x) / x \in f^{-1}((y_1 + y_3) y_2 - y_1 y_2)\}$

$\geq \sup\{\mu((y_1 + y_3) y_2 - y_1 y_2) /$

$x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2), x_3 \in f^{-1}(y_3)\}$

$\geq \sup\{\mu(x_3) / x_3 \in f^{-1}(y_3)\}$

$= f(\mu)(y_3)$

Therefore $f(\mu)((y_1+y_3)y_2 - y_1y_2) \geq f(\mu)(y_3)$, for all $y_1, y_2, y_3 \in R_2$

(v) $f(\mu)(y_1 \vee y_2) = \sup\{\mu(x) / x \in f^{-1}(y_1 \vee y_2)\}$

$\geq \sup\{\mu(x_1 \vee x_2) / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

$\geq \sup\{\min\{\mu(x_1), \mu(x_2)\} /$
$x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

$\geq \sup\{T(\mu(x_1), \mu(x_2)) /$
$x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

$= T(\sup\{\mu(x_1)\} / x_1 \in f^{-1}(y_1),$
$\sup\{\mu(x_2)\} / x_2 \in f^{-1}(y_2))$

$= T(f(\mu)(y_1), f(\mu)(y_2))$

Therefore $f(\mu)(y_1 \vee y_2) \geq T(f(\mu)(y_1), f(\mu)(y_2))$, for all $y_1, y_2 \in R_2$

(vi) $f(\mu)(y_1 \wedge y_2) = \sup\{\mu(x) / x \in f^{-1}(y_1 \wedge y_2)\}$

$\geq \sup\{\mu(x_1 \wedge x_2) / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

$\geq \sup\{\min\{\mu(x_1), \mu(x_2)\} / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)$

$\geq \sup\{T(\mu(x_1), \mu(x_2)) / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

$= T(\sup\{\mu(x_1)\} / x_1 \in f^{-1}(y_1), \sup\{\mu(x_2)\} / x_2 \in f^{-1}(y_2))$

$= T(f(\mu)(y_1), f(\mu)(y_2))$

Therefore $f(\mu)(y_1 \wedge y_2) \geq T(f(\mu)(y_1), f(\mu)(y_2))$, for all $y_1, y_2 \in R_2$

Hence $f(\mu)$ is a $T-$fuzzy ideal of a $\ell-$near ring R_2.

Theorem: 6

Let μ and μ be $T-$fuzzy ideal of a $\ell-$near ring R. Then $\mu+\lambda$ is the smallest $T-$fuzzy ideal of R containing both μ and λ.

Proof:

Let μ and μ be $T-$fuzzy ideal of a $\ell-$near ring R and Let $x, y, z \in R$

Then $(x - y) = (a+b) - (c - d)$

$= (a+b) - c - d$

$= (b+a-b) - c + (c+b-c) - d$

$= e + f$,

where $e = (b+a-b) - c$ $f = (c+b-c) - d$

(i) $(\mu+\lambda)(x-y) = \bigvee_{x-y=e+f} [\mu(e) \vee \lambda(f)]$

$\geq \bigvee_{x=a+b,\ y=c+d} [T(\mu(b+a-b), \mu(c)) \vee T(\lambda(c+b-c), \lambda(d))]$

$= \bigvee_{x=a+b,\ y=c+d} [T(\mu(a), \mu(c)) \vee T(\lambda(b), \lambda(d))]$

$= \bigvee_{x=a+b,\ y=c+d} [T(\mu(a), \lambda(b)) \vee T(\mu(c), \lambda(d))]$

$= T\left(\bigvee_{x=a+b} (\mu(a), \lambda(b)) \vee \bigvee_{y=c+d} (\mu(c), \lambda(d))\right)$

$= T((\mu+\lambda)(x), (\mu+\lambda)(y))$

Therefore

$(\mu+\lambda)(x-y) \geq T((\mu+\lambda)(x), (\mu+\lambda)(y))$, for all $x, y \in R$

For any $x = a+b$,

We have $y + x - y = y + a + b - y$

$= (y + a - y) + (y + b - y)$, for each

$y + a - y = c + d$

We have $x = -y + c + d + y$

$= (-y + c + y) + (-y + d + y)$

$= (b + a - b) - c + (c + b - c) - d$

(ii) Since

$\mu(y+x-y) \geq \mu(x)$ and $\lambda(y+x-y) \geq \lambda(x)$

We have $(\mu+\lambda)(y+x-y) = \bigvee_{y+z-y=c+d} [\mu(c), \lambda(d)]$

$= \bigvee_{x=a+b} [\mu(y+a-y), \lambda(y+b-y)]$

$\geq \bigvee_{x=a+b} [\mu(a) \vee \lambda(b)] = (\mu+\lambda)(x)$

Therefore $(\mu+\lambda)(y+x-y) \geq (\mu+\lambda)(x)$, for all $x, y \in R$

(iii) Since $\mu(xy) \geq \mu(x)$ and $\lambda(xy) \geq \lambda(y)$

$(\mu+\lambda)(xy) = (\mu+\lambda)(xy_1 + xy_2)$

$= \bigvee [\mu(xy_1) \vee \lambda(xy_2)]$

$\geq \bigvee [\mu(y_1) \vee \lambda(y_2)]$

$\geq \bigvee_{y=y_1+y_2} [\mu(y_1) \vee \lambda(y_2)]$

$= (\mu+\lambda)(y)$

Therefore $(\mu+\lambda)(xy) \geq (\mu+\lambda)(y)$, for all

$x, y \in R$

If η is a fuzzy ideal of R such that $\eta(x) \geq \mu(x)$ and $\eta(x) \geq \lambda(x)$, for all $x \in R$

(iv) Since

$\mu((x+z)y - xy) \geq \mu(z)$ and $\lambda((x+z)y - xy) \geq \lambda(z)$

We have $(\mu+\lambda)(x) = \bigvee_{x=a+b} [\mu(a) \vee \lambda(b)]$

$$\geq \bigvee_{x=a+b} [\eta(a) \vee \eta(b)]$$

$$= \bigvee_{x=a+b} [\eta(a+b)]$$

$$= \eta(z)$$

Therefore $(\mu+\lambda)(x) \geq \eta(z)$, for all $x, y, z \in R$

(v) We have

$$(\mu+\lambda)(x \vee y) = \bigvee_{x-y=e+f} [\mu(e) \vee \lambda(f)]$$

$$\geq \bigvee_{x=a+b,\ y=c+d} [T(\mu(b+a-b), \mu(c)) \vee T(\lambda(c+b-c), \lambda(d))]$$

$$= \bigvee_{x=a+b,\ y=c+d} [T(\mu(a), \mu(c)) \vee T(\lambda(b), \lambda(d))]$$

$$= \bigvee_{x=a+b,\ y=c+d} [T(\mu(a), \lambda(b)) \vee T(\mu(c), \lambda(d))]$$

$$= T\left(\bigvee_{x=a+b} (\mu(a), \lambda(b)) \vee \bigvee_{y=c+d} (\mu(c), \lambda(d)) \right)$$

$$= T((\mu+\lambda)(x), (\mu+\lambda)(y))$$

Therefore

$(\mu+\lambda)(x \vee y) \geq T((\mu+\lambda)(x), (\mu+\lambda)(y))$, for all $x, y \in R$

(vi) We have

$$(\mu+\lambda)(x \wedge y) = \bigvee_{x-y=e+f} [\mu(e) \vee \lambda(f)]$$

$$\geq \bigvee_{x=a+b, y=c+d} [T(\mu(b+a-b), \mu(c)) \vee T(\lambda(c+b-c), \lambda(d))]$$

$$= \bigvee_{x=a+b, y=c+d} [T(\mu(a), \mu(c)) \vee T(\lambda(b), \lambda(d))]$$

$$= \bigvee_{x=a+b, y=c+d} [T(\mu(a), \lambda(b)) \vee T(\mu(c), \lambda(d))]$$

$$= T\left(\bigvee_{x=a+b} (\mu(a), \lambda(b)) \vee \bigvee_{y=c+d} (\mu(c), \lambda(d)) \right)$$

$$= T((\mu+\lambda)(x), (\mu+\lambda)(y))$$

Therefore

$(\mu+\lambda)(x \wedge y) \geq T((\mu+\lambda)(x), (\mu+\lambda)(y))$, for all $x, y \in R$

Thus $\mu+\lambda$ is a $T-$fuzzy ideal of a $\ell-$ near-ring R.

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