Convergence and (S, T) - Stability Almost Surely for Random Jungck-Noor Type Iterative Scheme with Convergence Comparison

Anju Panwar[1], Ravi Prakash Bhokal[2]

Department of Mathematics, M.D. University, Rohtak-124001, Haryana (India)

Abstract *- The aim of this paper is to establish some convergence and (S,T)-stability almost surely results for random Jungck-Noor type iterative scheme. Our results generalize and unify some deterministic results in the literature. Using the MATLAB programming we shall also compare the convergence rate of some random Jungck type iterative schemes.*

Keywords — *random Jungck-Mann type iterative process, random fixed point, almost surely convergence and (S, T)- stability* Introduction.

I. INTRODUCTION

The study of random fixed point theory was initiated by Prague school of probabilistic. This field of study became popular after the publication of survey paper by Bharucha Reid [3]. Random fixed point theorems are generalizations of fixed point theorems and approximation theorems and have large number of applications in probability theory and non-linear functional analysis. Moreover this field of study has applications in statistics, engineering, economics, game theory, integral equations etc.

In 1953, Mann [9] introduced the one step iterative scheme and used it to prove the fixed point results defined for the non-expansive mappings where Picard iteration scheme is not applicable. In 1974, Ishikawa [7] defined two step iterative scheme as a generalization of Mann iterative scheme and established convergence results for Lipschitzian pseudo contractive type operators. Jungck [8] in 1976, introduced the Jungck type iterative scheme and used it to find the common solution of two sequences satisfying contractive type conditions. Singh et al. [15], gave the concept of Jungck- Mann iterative scheme and Olatinwo [13], generalized it by defining Jungck-Ishikawa and Jungck-Noor iterative schemes.In this direction Okeke and Kim [10] gave random fixed point results for Jangck-Mann type random iteration scheme and jungck-Ishikawa type iteration scheme. Rashwan et al. [14] established some random fixed point results for Jungck-Noor type random iterative scheme.

Spacek [16] and Hans [4,5] proved random fixed point results on separable complete metric space and Itoh [6] extended their work to multivalued contraction mappings. Zhung et al. [17] proved almost sure convergence and T-stability results for random iterative schemes. Recently many mathematicians including [8,9,10,13] has proved convergence and almost sure T- stability results for different iterative schemes.

II. PRELIMINARIES

Let (X, ξ) be a separable banach space where ξ denotes the σ algebra of Boral subset of X and let (Ω, ξ, μ) be a complete probability measure space and suppose that Y be a non-empty subset of X. $S,T : \Omega\times Y \to Y$ be two random operators defined on Y, such that S is injective. Let $x_0(\omega) \in Y$ be an arbitrary mapping for $\omega \in \Omega$, and $T(\omega, Y) \subseteq S(\omega, Y)$, the sequence $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ defined by

$$S(\omega, x_{n+1}(\omega)) = (1-\alpha_n)(S(\omega, x_n(\omega)) + \alpha_n T(\omega, z_n(\omega))$$
$$S(\omega, z_n(\omega)) = (1-\beta_n)(S(\omega, x_n(\omega)) + \beta_n T(\omega, y_n(\omega))$$
$$S(\omega, y_n(\omega)) = (1-\gamma_n)(S(\omega, x_n(\omega)) + \gamma_n T(\omega, x_n(\omega)) \quad (2.1)$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$, $\{\gamma_n\}_{n=0}^{\infty}$, are real sequences in (0,1). The iterative sequence defined by (2.1) is called Jungck-Noor type random iterative scheme.

If we take $\gamma_n = 0$ in the iterative scheme (2.1) then we have the Jungck-Ishikawa type random iterative scheme

$$S(\omega, x_{n+1}(\omega)) = (1-\alpha_n)(S(\omega, x_n(\omega)) + \alpha_n T(\omega, z_n(\omega))$$
$$S(\omega, z_n(\omega)) = (1-\beta_n)(S(\omega, x_n(\omega)) + \beta_n T(\omega, x_n(\omega)) \quad (2.2).$$

If we take $\beta_n = 0$ in (2.2) then we obtain the Jungck-Mann type random iterative scheme

$$S(\omega, x_{n+1}(\omega)) = (1-\alpha_n)(S(\omega, x_n(\omega)) + \alpha_n T(\omega, x_n(\omega)) \quad (2.3)$$

Definition 2.1 [17]. Let (Ω, ξ, μ) be a complete probability measure space and suppose that Y be a non-empty subset of a separable Banach space X. Let $S : \Omega \times Y \to Y$ be a random operator. Then $S(T) = \{\xi : \Omega \to Y$ such that $S(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega\}$ is called the random fixed point of the random operator S.

Definition 2.2 [10]. Let (Ω, ξ, μ) be a complete probability measure space and suppose that E be a non-empty subset of a separable Banach space X. Let $S, T : \Omega \times E \to E$ be two random operators. A map $x^*(\omega)$ is called common random fixed point of the pair (S, T) if $x^*(\omega) = S(\omega, x^*(\omega)) = T(\omega, x^*(\omega))$ for each $\omega \in \Omega$ and some $x^* \in E$. If $\rho(\omega) = S(\omega, x(\omega)) = T(\omega, x(\omega))$ for each $\omega \in \Omega$ and some $x \in E$ the random variable $\rho(\omega)$ is called a random point of coincidence of S and T. The pair (S, T) is said to be weakly compatible if S and T commute at their random coincidence points.

Okeke and Kim [11] introduced the following concept which is stochastic generalization of the definition given by Olatinwo [13].

Definition 2.3 [11]. Let (Ω, ξ, μ) be a complete probability measure space and suppose that E, Y be a non-empty subset of a separable space Banach space X. Let $S, T : \Omega \times E \to Y$ be two random operators such that $T(y) \subseteq S(Y)$. The random operators $S, T : \Omega \times E \to Y$ are said to be generalized ϕ- contractive type if there exists a monotone increasing function $\phi : R^+ \to R^+$ such that $\phi(0) = 0$ for all $x, y \in E$, $\theta(\omega) \in (0,1)$ and $\omega \in \Omega$, we have

$$\begin{aligned} &\| T(\omega, X) - T(\omega, Y) \| \\ &\le \phi(\| S(\omega, X) - T(\omega, X) \|) \qquad (2.4) \\ &+ \theta(\omega) \| S(\omega, X) - S(\omega, Y) \| . \end{aligned}$$

Singh et al. [15] defined the following concept of (S, T)-stability:

Definition 2.4 [15]. Let $S, T : Y \to X$ be two operators such that $T(Y) \subseteq S(Y)$ and ρ a point of coincidence of S and T. Let $\{Sx_n\}_{n=0}^{\infty} \subset X$ be the sequence generated by the iterative scheme

$$Sx_{n+1} = f(T, x_n),\ n = 0,1,2,..., \qquad (2.6)$$

where $x_0 \in X$ is the initial approximation and f is some function. Suppose that $\{Sx_n\}_{n=0}^{\infty}$ converges to ρ. Let $\{Sy_n\}_{n=0}^{\infty} \subset X$ be any arbitrary sequence and $\epsilon_n = d(Sy_n, f(T, y_n))$, $n = 0,1,2,....$. Then the iterative scheme (2.5) is said to be (S, T) stable or stable if and only if $\lim_{n\to\infty} \epsilon_n = 0$ implies $\lim_{n\to\infty} Sy_n = \rho$.

Okeke and Kim [10] defined the stochastic verse of definition (2.4).

Definition 2.5 [10]. Let (Ω, ξ, μ) be a complete probability measure space and suppose that E, Y be a non-empty subset of a separable Banach space X. Let $S, T : \Omega \times E \to Y$ be two random operators such that $T(y) \subseteq S(Y)$ and $\rho(\omega)$ a random point of coincidence of S and T. For any random variable $x_0 : \Omega \to E$, consider the random iterative scheme

$$Sx_{n+1}(\omega) = f(T; x_n(\omega)),\ n = ,1, 2,... \qquad (2.6)$$

where f is some function measurable in second variable. Suppose that $\{Sx_n(\omega)\}$ converges to $\rho(\omega)$. Let $\{Sy_n(\omega)\}_{n=0}^{\infty}$ be an arbitrary sequence of random variable. Denote $\epsilon_n(\omega)$ by

$$\epsilon_n(\omega) = Sy_n(\omega) - f(T; y_n(\omega)), \qquad (2.7)$$

The the iterative scheme (2.6) is (S, T)-stable almost surely if and only if $\omega \in \Omega$, $\epsilon_n(\omega) \to 0$ as $n \to \infty$ implies that $y_n(\omega) \to \rho(\omega)$ almost surely.

Definition 2.6 [1]. Suppose $\{a_n\}$ and $\{b_n\}$ be two convergent sequences with limits a and b respectively. Then $\{a_n\}$ is said to converge faster then $\{b_n\}$ if $\lim_{n\to\infty} \left\| \frac{a_n - a}{b_n - b} \right\| = 0$.

Lemma 2.7 [2]. If δ is a real number such that $0 \le \delta < 1$ and $\{\epsilon_n\}_{n=0}^{\infty}$ is a sequence of positive numbers such that $\lim_{n\to\infty} \epsilon_n = 0$, then for any sequence of positive numbers $\{\beta_n\}_{n=0}^{\infty}$ satisfying

$$\beta_{n+1} \le \delta\beta_n + \epsilon_n,\ n = 0,1,2,..., \qquad (2.8)$$

then $\lim_{n\to\infty} = 0$.

III. CONVERGENCE RESULTS

Theorem 3.1. Let X be a separable Banach space and $S, T : \Omega \times Y \to Y$ be generalized ϕ contractive mapping defined by (2.4) and suppose that $T(Y) \subseteq S(Y)$ where $S(Y)$ is a subset of X. Also $S(\omega, r(\omega)) = T(\omega, r(\omega)) = \rho(\omega)$. For $x_0 \in \Omega \times Y$, let $\{Sx_n(\omega)\}_{n=0}^{\infty}$ be random Jungck-Noor type iterative scheme defined by (2.1). Assume that $\sum \alpha_n = \infty$. Then the Jungck-Noor type random iterative scheme $\{Sx_n(\omega)\}_{n=0}^{\infty}$ converges strongly to $\rho(\omega)$ almost surely. Also $\rho(\omega)$ is unique random common fixed point of operators S and T provided that $Y = X$ and S and T are weakly compatible.

Proof. Using (2.1) and (2.4) we have,

$$\begin{aligned}
&\|Sx_{n+1}(\omega) - \rho(\omega)\| \\
&= \|(1-\alpha_n)S(x_n(\omega)) + \alpha_n T(\omega, z_n(\omega)) - (1-\alpha_n+\alpha_n)\rho(\omega)\| \\
&\le (1-\alpha_n)\|Sx_n(\omega) - \rho(\omega)\| + \alpha_n\|T(\omega, z_n(\omega)) - \rho(\omega)\| \\
&= (1-\alpha_n)\|Sx_n(\omega) - \rho(\omega)\| + \alpha_n\|T(\omega, z_n(\omega)) - T(\omega, r(\omega))\| \\
&\le (1-\alpha_n)\|Sx_n(\omega) - \rho(\omega)\| + \alpha_n(\phi(\|S(\omega, r(\omega)) - T(\omega, r(\omega))\|) + \theta(\omega)\|S(\omega, r(\omega)) - Sz_n(\omega)\|) \\
&= (1-\alpha_n)\|Sx_n(\omega) - \rho(\omega)\| + \alpha_n\theta(\omega)\|Sz_n(\omega) - \rho(\omega)\| \qquad (3.1)
\end{aligned}$$

Also

$$\begin{aligned}
&\|Sz_n(\omega) - \rho(\omega)\| \\
&= \|(1-\beta_n)S(x_n(\omega)) + \beta_n T(\omega, y_n(\omega)) - (1-\beta_n+\beta_n)\rho(\omega)\| \\
&\le (1-\beta_n)\|Sx_n(\omega) - \rho(\omega)\| + \beta_n\|T(\omega, y_n(\omega)) - \rho(\omega)\| \\
&= (1-\beta_n)\|Sx_n(\omega) - \rho(\omega)\| + \beta_n\|T(\omega, y_n(\omega)) - T(\omega, r(\omega))\| \\
&\le (1-\beta_n)\|Sx_n(\omega) - \rho(\omega)\| + \beta_n(\phi(\|S(\omega, r(\omega)) - T(\omega, r(\omega))\|) + \theta(\omega)\|S(\omega, r(\omega)) - Sy_n(\omega)\|) \\
&= (1-\beta_n)\|Sx_n(\omega) - \rho(\omega)\| + \beta_n\theta(\omega)\|Sy_n(\omega) - \rho(\omega)\| \qquad (3.2)
\end{aligned}$$

Now

$$\begin{aligned}
&\|Sy_n(\omega) - \rho(\omega)\| \\
&= \|(1-\gamma_n)S(x_n(\omega)) + \gamma_n T(\omega, x_n(\omega)) - (1-\gamma_n+\gamma_n)\rho(\omega)\| \\
&\le (1-\gamma_n)\|Sx_n(\omega) - \rho(\omega)\| + \gamma_n\|T(\omega, x_n(\omega)) - \rho(\omega)\| \\
&= (1-\gamma_n)\|Sx_n(\omega) - \rho(\omega)\| + \gamma_n\|T(\omega, x_n(\omega)) - T(\omega, r(\omega))\| \\
&\le (1-\gamma_n)\|Sx_n(\omega) - \rho(\omega)\| + \gamma_n(\phi(\|S(\omega, r(\omega)) - T(\omega, r(\omega))\|) + \theta(\omega)\|S(\omega, r(\omega)) - Sx_n(\omega)\|) \\
&= (1-\gamma_n)\|Sx_n(\omega) - \rho(\omega)\| + \gamma_n\theta(\omega)\|Sx_n(\omega) - \rho(\omega)\| \qquad (3.3)
\end{aligned}$$

From (3.2) and (3.3) and using the fact that $\gamma_n\theta(\omega) \le \gamma_n$ we have,

$$\begin{aligned}
&\|Sz_n(\omega) - \rho(\omega)\| \\
&\le (1-\beta_n)\|Sx_n(\omega) - \rho(\omega)\| + \beta_n\theta(\omega)[(1-\gamma_n)\|Sx_n(\omega) - \rho(\omega)\| + \gamma_n\theta(\omega)\|Sx_n(\omega) - \rho(\omega)\|] \\
&\le (1-\beta_n)\|Sx_n(\omega) - \rho(\omega)\| + \beta_n\theta(\omega)[(1-\gamma_n)\|Sx_n(\omega) - z\rho(\omega)\| + \gamma_n\|Sx_n(\omega) - \rho(\omega)\|] \\
&= (1-\beta_n)\|Sx_n(\omega) - \rho(\omega)\| + \beta_n\theta(\omega)[\|Sx_n(\omega) - \rho(\omega)\| \qquad (3.4)
\end{aligned}$$

From (3.1) and (3.4) and using the fact that $\beta_n\theta(\omega) \le \beta_n$ we have,

$$\begin{aligned}
&\|Sx_{n+1}(\omega) - \rho(\omega)\| \\
&\le (1-\alpha_n)\|Sx_n(\omega) - \rho(\omega)\| + \alpha_n\theta(\omega)[(1-\beta_n)\|Sx_n(\omega) - \rho(\omega)\| + \beta_n\theta(\omega)\|Sx_n(\omega) - \rho(\omega)\|] \\
&\le (1-\alpha_n)\|Sx_n(\omega) - \rho(\omega)\| + \alpha_n\theta(\omega)[\|Sx_n(\omega) - \rho(\omega)\| \\
&\le (1-\alpha_n + \alpha_n\theta(\omega))\|Sx_n(\omega) - \rho(\omega)\| \\
&\le (1-\alpha_n(1-\theta(\omega)))\|Sx_n(\omega) - \rho(\omega)\| \\
&\le e^{-(1-\theta(\omega))\sum_{i=0}^{n}\alpha_i}\|Sx_0(\omega) - \rho(\omega)\| \qquad (3.5)
\end{aligned}$$

Since $\sum_{i=0}^{n}\alpha_i = \infty$ we have $e^{-(1-\theta(\omega))\sum_{i=0}^{n}\alpha_i} \to 0$ as $n \to \infty$.

Hence $\lim_{n\to\infty}\|Sx_{n+1}(\omega) - \rho(\omega)\| = 0$. Therefore $\{Sx_n(\omega)\}_{n=0}^{\infty}$ converges strongly to $\rho(\omega)$ almost surely. Now we prove the uniqueness of the random fixed point $\rho(\omega)$ for the random operators S and T.

Let if possible there exists another random fixed point $\rho_1(\omega)$. Then there exists $q_1(\omega) \in \Omega \times X$ of S and T such that $S(\omega, q_1(\omega)) = T(\omega, q_1(\omega)) = \rho_1(\omega)$. Now

$$\begin{aligned}
0 &\le \|\rho(\omega) - \rho_1(\omega)\| \\
&= \|T(\omega, q(\omega)) - T(\omega, q_1(\omega))\| \\
&\le \phi\|S(\omega, q(\omega)) - S(\omega, q_1(\omega))\| + \theta(\omega)\|S(\omega, q(\omega)) - S(\omega, q_1(\omega))\| \\
&= \theta(\omega)\|S(\omega, q(\omega)) - S(\omega, q_1(\omega))\| \\
&= \theta(\omega)\|\rho(\omega) - \rho_1(\omega)\|
\end{aligned}$$

Implies that $\rho(\omega) = \rho_1(\omega)$. This proves the uniqueness of the random fixed point $\rho(\omega)$ for the random operators S and T. Since S and T are weakly compatible and $\rho(\omega) = S(\omega, q(\omega)) = T(\omega, q(\omega))$ we have $T(\omega, \rho(\omega)) = TS(\omega, \rho(\omega)) = ST(\omega, \rho(\omega))$ and hence $S(\omega, \rho(\omega)) = T(\omega, \rho(\omega))$. Hence $T(\omega, \rho(\omega))$ random coincidence point is unique hence $\rho(\omega)$ is unique common random fixed of random operators S and T.

Remark 3.2. Theorem 3.1 generalizes the corresponding results of Olatinwo [13], Singh et. al [15], Okeke and Abbas [11] and Okeke and Kim [10].

Example 3.3. Let $X = \Omega = \Re$ (set of real numbers) and $Y = [0,1]$. Consider the mappings $S, T : \Omega\times Y \to Y \to$ defined by $S(\omega, x) = x$, $T(\omega, x)) = x/2$. Clearly $T(Y) \subseteq S(Y)$, also the random operators S and T are continuous in their domain. Let $\{Sx_n\}$ be defined by (2.1) with $\alpha_n = \beta_n = \gamma_n = 1/2$ such that $\Sigma^{\infty}_{n=0}\alpha_n = \infty$. Now S and T are weakly compatible mappings. So all the requirements of the theorem (3.1) are satisfied. Hence the sequence $\{Sx_n\}$ converges strongly to the random fixed point 0 of the operators S and T almost surely. Moreover 0 is the unique common fixed point of S and T.

IV. Stability Results

Theorem 4.1. Let X be a separable Banach space and let $S, T : \Omega\times Y \to Y$ be generalized $ \phi$ contractive mapping defined by (2.4). Also assume that $T(Y) \subseteq S(Y)$ where $S(Y)$ is a subset of X and $S(\omega, r(\omega)) = T(\omega, r(\omega)) = \rho(\omega)$. For $x_0 \in\Omega\times Y$, let $\{Sx_n(\omega)\}^{\infty}_{n=0}$ be random Jungck-Noor type iterative scheme defined by (2.1) which converges to $\rho(\omega)$. Then the random Jungck-Noor type iterative scheme is (S, T) stable almost surely.

Proof. Let $\{Sy_n(\omega)\}^{\infty}_{n=0} \subseteq \Omega\times Y$ be an arbitrary sequence and let

$$\epsilon_n(\omega) = \| Sy_{n+1}(\omega) - (1-\alpha_n)S(\omega, a_n(\omega)) - \alpha_n T(\omega, a_n(\omega)) \|, \quad n = 0,1,2,... \tag{4.1}$$

where

$$S(\omega, a_n(\omega)) = (1-\beta_n)S(\omega, z_n(\omega)) + \beta_n T(\omega, z_n(\omega)) \tag{4.2}$$

and

$$S(\omega, z_n(\omega)) = (1-\gamma_n)S(\omega, y_n(\omega)) + \gamma_n T(\omega, y_n(\omega)) \tag{4.3}$$

and suppose that $\lim_{n\to\infty} \epsilon_n(\omega) = 0.$

Now using the random Jungck-Noor type iterative scheme we have

$$\begin{aligned}
&\| Sy_{n+1}(\omega) - \rho(\omega) \| \\
&\le \| Sy_{n+1}(\omega) - (1-\alpha_n)S(\omega, a_n(\omega)) - \alpha_n T(\omega, a_n(\omega)) \|
\end{aligned}$$

is a random coincidence point of S and T. Since the

$$\begin{aligned}
&+ \| (1-\alpha_n)S(\omega, a_n(\omega)) + \alpha_n T(\omega, a_n(\omega)) - (1-\alpha_n+\alpha_n)\rho(\omega) \| \\
&\le \epsilon_n(\omega) + (1-\alpha_n)\| S(\omega, a_n(\omega)) - \rho(\omega) \| + \alpha_n \| T(\omega, a_n(\omega)) - \rho(\omega) \| \\
&\le \epsilon_n(\omega) + (1-\alpha_n)\| S(\omega, a_n(\omega)) - \rho(\omega) \| + \alpha_n \| T(\omega, a_n(\omega)) - T(\omega, r(\omega)) \| \\
&\le \epsilon_n(\omega) + (1-\alpha_n)\| S(\omega, a_n(\omega)) - \rho(\omega) \| + \alpha_n [\phi \| S(\omega, r(\omega)) - T(\omega, r(\omega)) \| + \theta(\omega) \| S(\omega, r(\omega)) - S(\omega, a_n(\omega)) \|] \\
&\le \epsilon_n(\omega) + (1-\alpha_n)\| S(\omega, a_n(\omega)) - \rho(\omega) \| + \alpha_n \theta(\omega) \| S(\omega, r(\omega)) - S(\omega, a_n(\omega)) \| \\
&\le \epsilon_n(\omega) + (1-\alpha_n)\| S(\omega, a_n(\omega)) - \rho(\omega) \| + \alpha_n \theta(\omega) \| S(\omega, a_n(\omega)) - \rho(\omega) \| \\
&= (1-\alpha_n + \alpha_n\theta(\omega)) \| S(\omega, a_n(\omega)) - \rho(\omega) \| + \epsilon_n(\omega) \\
&= (1-\alpha_n(1-\theta(\omega))) \| S(\omega, a_n(\omega)) - \rho(\omega) \| + \epsilon_n(\omega)
\end{aligned} \tag{4.4}$$

Now we obtain the following estimate

$$\begin{aligned}
&\| S(\omega, a_n(\omega)) - \rho(\omega) \| \\
&= \| (1-\beta_n)S(\omega, z_n(\omega)) + \beta_n T(\omega, z_n(\omega)) - (1-\beta_n+\beta_n)\rho(\omega) \| \\
&\le (1-\beta_n)\| S(\omega, z_n(\omega)) - \rho(\omega) \| + \beta_n \| T(\omega, z_n(\omega)) - \rho(\omega) \| \\
&\le (1-\beta_n)\| S(\omega, z_n(\omega)) - \rho(\omega) \| + \beta_n [\phi \| S(\omega, r(\omega)) - T(\omega, r(\omega)) \| + \theta(\omega) \| S(\omega, r(\omega)) - S(\omega, z_n(\omega)) \|] \\
&\le (1-\beta_n)\| S(\omega, z_n(\omega)) - \rho(\omega) \| + \beta_n\theta(\omega) \| S(\omega, z_n(\omega)) - \rho(\omega) \| \\
&= (1-\beta_n(1-\theta(\omega))) \| S(\omega, z_n(\omega)) - \rho(\omega) \|
\end{aligned} \tag{4.5}$$

Now estimating the following value

$$\begin{aligned}
&\| S(\omega, z_n(\omega)) - \rho(\omega) \| \\
&= \| (1-\gamma_n)S(\omega, y_n(\omega)) + \gamma_n T(\omega, y_n(\omega)) - (1-\gamma_n+\gamma_n)\rho(\omega) \| \\
&\le (1-\gamma_n)\| S(\omega, y_n(\omega)) - \rho(\omega) \| + \gamma_n \| T(\omega, y_n(\omega)) - \rho(\omega) \| \\
&\le (1-\gamma_n)\| S(\omega, y_n(\omega)) - \rho(\omega) \| + \gamma_n \| T(\omega, y_n(\omega)) - T(\omega, r(\omega)) \| \\
&\le (1-\gamma_n)\| S(\omega, y_n(\omega)) - \rho(\omega) \| + \gamma_n [\phi \| S(\omega, r(\omega)) - T(\omega, r(\omega)) \| + \theta(\omega) \| S(\omega, r(\omega)) - S(\omega, y_n(\omega)) \|] \\
&\le (1-\gamma_n)\| S(\omega, y_n(\omega)) - \rho(\omega) \| + \gamma_n\phi \| S(\omega, y_n(\omega)) - \rho(\omega) \|
\end{aligned} \tag{4.6}$$

$=(1-\gamma_n(1-\theta(\omega)))\,\|S(\omega,y_n(\omega))-\rho(\omega)\|$ Using (4.6) and (4.5) we have,

$\|S(\omega,a_n(\omega))-\rho(\omega)\|$

$\leq(1-\beta_n(1-\theta(\omega)))(1-\gamma_n(1-\theta(\omega)))\,\|S(\omega,y_n(\omega))-\rho(\omega)\|$ Using the above estimate in (4.4) we have

$$\|Sy_{n+1}(\omega)-\rho(\omega)\|\leq(1-\alpha_n(1-\theta(\omega)))(1-\beta_n(1-\theta(\omega)))(1-\gamma_n(1-\theta(\omega)))\,\|S(\omega,y_n(\omega))-\rho(\omega)\| \quad (4.7)$$

Using the fact that

$0<\alpha_n<1,\ 0<\beta_n<1,\ 0<\gamma_n<1$

and

$\theta(\omega)\in[0,1)$,

we have

$(1-\alpha_n(1-\theta(\omega)))<1$,

$(1-\beta_n(1-\theta(\omega)))<1$,

$(1-\gamma_n(1-\theta(\omega)))<1$

Using Lemma (2.7) and (4.7) along with the above estimates we obtain that $Sy_{n+1}(\omega)\to\rho(\omega)$ as $n\to\infty$.

Conversely let us assume that $\epsilon_n(\omega)\to 0$ as $n\to\infty$. Using the condition (2.4) and triangle inequality we have,

$$\epsilon_n(\omega)=\|Sy_{n+1}(\omega)-(1-\alpha_n)S(\omega,a_n(\omega))-\alpha_nT(\omega,a_n(\omega))\|$$

$$\leq\|Sy_{n+1}(\omega)-\rho(\omega)\|+\|(1-\alpha_n+\alpha_n)\rho(\omega)-(1-\alpha_n)S(\omega,a_n(\omega))-\alpha_nT(\omega,a_n(\omega))\|$$

$$\leq\|Sy_{n+1}(\omega)-\rho(\omega)\|+(1-\alpha_n)\|\rho(\omega)-S(\omega,a_n(\omega))\|+\alpha_n\|\rho(\omega)-T(\omega,a_n(\omega))\|$$

$$\leq\|Sy_{n+1}(\omega)-\rho(\omega)\|+(1-\alpha_n)\|S(\omega,a_n(\omega))-\rho(\omega)\|+\alpha_n\|T(\omega,r(\omega))-T(\omega,a_n(\omega))\|$$

$$\leq\|Sy_{n+1}(\omega)-\rho(\omega)\|+(1-\alpha_n)\|S(\omega,a_n(\omega))-\rho(\omega)\|+\alpha_n[\phi(\|S(\omega,r(\omega))-T(\omega,r(\omega))\|)+\theta(\omega)\|S(\omega,r(\omega))-S(\omega,a_n(\omega))\|]$$

$$\leq\|Sy_{n+1}(\omega)-\rho(\omega)\|+(1-\alpha_n)\|S(\omega,a_n(\omega))-\rho(\omega)\|+\alpha_n\theta(\omega)\|S(\omega,a_n(\omega))-\rho(\omega)\|$$

$$\leq\|Sy_{n+1}(\omega)-\rho(\omega)\|+(1-\alpha_n(1-\theta(\omega)))\|S(\omega,a_n(\omega))-\rho(\omega)\|$$

$$\leq\|Sy_{n+1}(\omega)-\rho(\omega)\|+(1-\alpha_n(1-\theta(\omega)))(1-\beta_n(1-\theta(\omega)))(1-\gamma_n(1-\theta(\omega)))\|S(\omega,a_n(\omega))-\rho(\omega)\|$$

Hence $\epsilon_n\to 0$ as $n\to\infty$. Therefore the random Jungck-Noor type iterative scheme is (S,T)-stable almost surely. This completes the proof.

Remark 4.2. Theorem 4.1 generalizes and extends several results in the literature including the work of Olatinwo [13], Singh et. al [15] and Okeke and Kim [10].

V. CONVERGENCE COMPARISON

Theorem 5.1. Let X be a separable Banach space and $S, T:\Omega\times Y\to Y$ be the non self operators satisfying contractive conditions (2.4). Also suppose that $T(Y)\subseteq S(Y)$ where $S(Y)$ is a complete subspace of X and $S(\omega,r(\omega))=T(\omega,r(\omega))=\rho(\omega)$. For $x_0\in\Omega\times Y$, let random Jungck-Noor type iterative scheme, random Jungck-Ishikawa iterative scheme and random Jungck-Mann iterative scheme be defined by (2.1), (2.2) and (2.3) respectively. Then the random Jungck-Noor type iterative scheme converges faster random Jungck-Ishikawa type iterative scheme and random Jungck-Mann type iterative scheme.

Proof. For random Jungck-Mann type iterative scheme (2.3), we have

$$\|Sx_{n+1}(\omega)-\rho(\omega)\|=\|(1-\alpha_n)Sx_n(\omega)+\alpha_nT(\omega,x_n(\omega))-\rho(\omega)\|$$

$$\geq(1-\alpha_n)\|Sx_n(\omega)-\rho(\omega)\|-\alpha_n\|T(\omega,x_n(\omega))-\rho(\omega)\|$$

$$\geq(1-\alpha_n)\|Sx_n(\omega)-\rho(\omega)\|-\alpha_n\|T(\omega,x_n(\omega))-T(\omega,r(\omega))\|$$

$$\geq(1-\alpha_n)\|Sx_n(\omega)-\rho(\omega)\|-\alpha_n[\phi(\|S(\omega,x_n(\omega))-T(\omega,x_n(\omega))\|)+\theta(\omega)\|S(\omega,x_n(\omega))-T(\omega,r(\omega))\|]$$

$$\geq(1-\alpha_n)\|Sx_n(\omega)-\rho(\omega)\|-\alpha_n\theta(\omega)\|S(\omega,x_n(\omega))-\rho(\omega)\|$$

$$\geq(1-\alpha_n-\alpha_n\theta(\omega))\|Sx_n(\omega)-\rho(\omega)\|$$

$$\geq(1-\alpha_n(1+\theta(\omega)))\|Sx_n(\omega)-\rho(\omega)\|$$

$$\geq\prod_{i=1}^{n}(1-\alpha_i(1+\theta(\omega)))\|Sx_0(\omega)-\rho(\omega)\| \quad (5.1)$$

Similarly for random Jungck-Ishikawa (2.2) iterative process

$$\|Sx_{n+1}(\omega)-\rho(\omega)\|=\|(1-\alpha_n)Sx_n(\omega)+\alpha_nT(\omega,y_n(\omega))-\rho(\omega)\|$$

$$\geq(1-\alpha_n)\|Sx_n(\omega)-\rho(\omega)\|-\alpha_n\|T(\omega,y_n(\omega))-\rho(\omega)\|$$

$$\geq(1-\alpha_n)\|Sx_n(\omega)-\rho(\omega)\|-\alpha_n\|T(\omega,y_n(\omega))-T(\omega,r(\omega))\|$$

$$\geq(1-\alpha_n)\|Sx_n(\omega)-\rho(\omega)\|-\alpha_n\|T(\omega,r(\omega))-T(\omega,y_n(\omega))\|$$

$$\geq (1-\alpha_n)\|Sx_n(\omega)-\rho(\omega)\|$$
$$-\alpha_n\theta(\omega)\|S(\omega,y_n(\omega))-\rho(\omega)\|$$
$$\geq (1-\alpha_n)\|Sx_n(\omega)-\rho(\omega)\|$$
$$-\alpha_n\theta(\omega)\|(1-\beta_n)Sx_n(\omega)+\beta_n T(\omega,x_n(\omega))-\rho(\omega)\|$$
$$\geq (1-\alpha_n)\|Sx_n(\omega)-\rho(\omega)\|$$
$$-\alpha_n\theta(\omega)[(1-\beta_n)\|Sx_n(\omega)-\rho(\omega)\|$$
$$-\beta_n\|T(\omega,x_n(\omega))-\rho(\omega)\|]$$
$$\geq (1-\alpha_n)\|Sx_n(\omega)-\rho(\omega)\|$$
$$-\alpha_n\theta(\omega)[(1-\beta_n)\|Sx_n(\omega)-\rho(\omega)\|$$
$$-\beta_n\|T(\omega,r(\omega))-T(\omega,x_n(\omega))\|]$$
$$\geq (1-\alpha_n)\|Sx_n(\omega)-\rho(\omega)\|$$
$$-\alpha_n\theta(\omega)[(1-\beta_n)\|Sx_n(\omega)-\rho(\omega)\|$$
$$-\beta_n[\phi(\|S(\omega,r(\omega))-T(\omega,r(\omega))\|)$$
$$+\theta(\omega)\|S(\omega,r(\omega))-S(\omega,x_n(\omega))\|]$$
$$\geq (1-\alpha_n)\|Sx_n(\omega)-\rho(\omega)\|$$
$$-\alpha_n\theta(\omega)[(1-\beta_n)\|Sx_n(\omega)-\rho(\omega)\|$$
$$-\beta_n\theta(\omega)\|Sx_n(\omega)-\rho(\omega)\|]$$
$$\geq (1-\alpha_n)\|Sx_n(\omega)-\rho(\omega)\|$$
$$-\alpha_n\theta(\omega)[(1-\beta_n(1+\theta(\omega))\|Sx_n(\omega)-\rho(\omega)\|]$$
$$\geq (1-\alpha_n-\alpha_n\theta(\omega)(1-\beta_n(1+\theta(\omega)))$$
$$\|Sx_n(\omega)-\rho(\omega)\|$$
$$\geq (1-\alpha_n-\alpha_n\theta(\omega)+\alpha_n\beta_n(1+\theta(\omega))$$
$$+\alpha_n\beta_n(1+\theta(\omega)))\|Sx_n(\omega)-\rho(\omega)\|$$
$$\geq (1-\alpha_n(1+\theta(\omega))\|Sx_n(\omega)-\rho(\omega)\|$$
$$\geq \prod_{i=1}^{n}(1-\alpha_i(1+\theta(\omega))\|Sx_0(\omega)-\rho(\omega)\| \qquad (5.2)$$

Using (3.5) and (5.2) we have

$$\frac{\|Sx_{n+1}(RJN)-\rho(\omega)\|}{\|Sx_{n+1}(RJI)-\rho(\omega)\|}\leq\frac{e^{-(1-\theta(\omega))\sum_{i=0}^{n}\alpha_i}}{\prod_{i=1}^{n}(1-\alpha_i(1+\theta(\omega))}.$$

Let $a_n=\dfrac{e^{-(1-\theta(\omega))\sum_{i=0}^{n}\alpha_i}}{\prod_{i=1}^{n}(1-\alpha_i(1+\theta(\omega))}$.

Then $\dfrac{a_{n+1}}{a_n}=\dfrac{\alpha_{n+1}}{1-\alpha_{n+1}(1+\theta(\omega))}$.

Using $\lim_{n\to\infty}\alpha_n=0$, we obtain $\lim_{n\to\infty}\dfrac{a_{n+1}}{a_n}=0$

which implies that $\lim_{n\to\infty}\dfrac{\|Sx_{n+1}(RJN)-\rho(\omega)\|}{\|Sx_{n+1}(RJI)-\rho(\omega)\|}=0.$

Hence by definition random Jungck-Noor iterative scheme converges faster than random jungck Ishikawa iterative scheme.

By similar arguments we have $\lim_{n\to\infty}\dfrac{a_{n+1}}{a_n}=0$

which implies that $\lim_{n\to\infty}\dfrac{\|Sx_{n+1}(RJN)-\rho(\omega)\|}{\|Sx_{n+1}(RJM)-\rho(\omega)\|}=0.$

Hence by definition random Jungck-Noor iterative scheme converges faster than random jungck-Mann iterative scheme.

Example 5.2. Let $\Omega=X=\Re$ (set of real numbers) and consider the random operators $S,T:\Omega\times X\to X$ by $S(x,\omega)=2(x-1)$ and $T(x,\omega)=(x+2)^{1/2}$. Consider the sequences $\alpha_n=\beta_n=\gamma_n=1/4$ and initial approximation $(x_0,\omega)=3$. Then the convergence pattern of random Jungck-Noor type iteration scheme, random Jungck-Ishikawa type iteration scheme and random Jungck-Mann type iterative scheme is shown in the Table 1.

Number of Steps	Random Jungck-Mann	Random Jungck-Ishikawa	Random Jungck-Noor
0	2.779508	2.773276	2.773097
1	2.607907	2.598141	2.597860
2	2.474256	2.462786	2.462569
3	2.370097	2.358132	2.357785
4	2.288822	2.277187	2.276848
-	-	-	-
25	2.002065	2.001289	2.000128
26	2.001614	2.000998	2.000123
27	2.001260	2.000773	2.000095
28	2.000985	2.000598	2.000074
29	2.000769	2.000166	2.000057
30	2.000601	2.000129	2.000044
-	-	-	-
35	2.000174	2.000100	2.000001
36	2.000136	2.000077	2.000001
37	2.000106	2.000059	2.000000
38	2.000083	2.000046	2.000000
-	-	-	-
45	2.000009	2.000007	2.000000
46	2.000007	2.000006	2.000000
-	-	-	-
52	2.000001	2.000001	2.000000
-	-	-	-
55	2.000001	2.000000	2.000000
56	2.000000	2.000000	2.000000

Convergence pattern of Iterative Schemes

From Table 1 we observe that random Jungck-Noor type iteration scheme converges faster than the random Jungck Ishikawa rype iterative scheme and random Jungck-Mann type iterative scheme.

VI. Competing Interest

The authors declare that there is no competing interest.

VII. Author's Contributions

Both authors have contributed equally to this work. Both authors approve the final manuscript.

VIII. ACKNOWLEDGEMENTS

The authors are thankful to University Grants Commission for financial assistance.

REFERENCES

[1] Berinde, V., Iterative approximation of fied points, Springer, 2007.
[2] Berinde, V., On the convergence of the Ishikawa iteration in the class of quasi-contractive operators, Acta mathematica universitatis comenianae, 73, 119-126, 2004.
[3] Bharucha-Reid, A.T., Random integral equations, Academic press, New-York, 1972.
[4] Hans, O., Reduzierende Zufallige transformationen, Czechoslovak Mathematical Journal, 7(82) 154-158, 1957.
[5] Hans, O., Random operator equations, Proceeding of 4th Bereley symops. Math. Ststist. and Prof. Vol. II, University of California, Part-1, 185-202, 161.
[6] Itoh, S., Random fixed point theorems with an application to random differential equation in Banach spaces, J. Math. Anal.Appl., 67(2), 261-273, 1979.
[7] Ishikawa, S., Fixed point by a new iteration method, Proceedings of American mathematical society, 44, 147-150, 1974.
[8] Jungck, G., Commuting mappings and fixed points, The American Mathematical Monthly, 83, 261-263, 1976.
[9] Mann, W.R., Mean value method in iteration, Proceedings of American mathematical society, 4, 506-510, 1953.
[10] Okeke G.A. and Kim, J.K., Convergence and (S,T)-stability almost surely for random Jungck type iteration process with applications, Cogent Mathematics, 1-15, 201.
[11] Okeke G.A. and Kim, J.K., Convergence and summable almost T stable of random Picard-Mann hybrid iterative process, Journal of inequalities and applications, 290, 2015.
[12] Okeke G.A. and Abbas, M., Convergence and almost sure T- stability of a random iterative sequence generated by a generalized random operator, Journal of inequalities of applications, 146, 2015.
[13] Olatinwo, M.O., Some stability and strong convergence results for Jungck-Ishikawa iteration process, Creative Mathematicas and Informatics, 17, 33-42, 2008.
[14] Rashwan, R.A., Hammad, H.A. and Okeke, G. A., Convergence and almost sure (S,T)- stability for random iterative schemes, international journal of advances in mathematics, 1, 1-16, 2016.
[15] Singh, S.L., Bhatnagar, C. and Mishra, S.N., Stability of Jungck type iterative procedure, International journal of mathematics and mathematical science, 19, 3035-3043, 2005.
[16] Speck, A., Zufallige Gleichungen, Czechoslovak Mathematical Journal, 5(80) 462-466, 1955.
[17] Zhang S.S., Wang, X.R. and Liu, M., Almost sure T-stability and convergence for random iterative algorithms, Applied mathematics and mechanics(English edition), 32, 805-810, 2011.