Large deflection of a circular plate under non uniform load pertaining to multivariable Aleph-functions

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ABSTRACT
In the present paper the large deflection analysis is carried out to determine the deflections and bending stresses for clamped circular plate under non-uniform load following Berger's approximate method. Here the load shape is assumed in the form of an arbitrary function $P(x)$ involving Jacobi polynomial, Fox-Wright function and multivariable Aleph-functions. The small deflection case is treated as a special cases of large deflection. On account of the general nature in the load shape considered here, the solution of the problem yields many useful and interesting results. Some known and new results have been evaluated by taking suitable values of parameters.

Keywords:Multivariable Aleph-function, Fox-Wright function, Bessel function, Aleph-function of two variable, I-function of two variables, Jacobi polynomial.

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1.Introduction and preliminaries.

In the classical theory of plates, small deflection and elastic behaviour of the material are assumed. When the lateral deflections exceeds one half the plate thickness [13], this classical theory generally is not adequate and the second order effects of the vertical displacements on the membrane stresses need to be considered. Two-coupled non-linear partial differential equations considering these effects where given by [5]. Solutions bases of these differential equations have been known a large deflection solutions. Berger [2] proposed an approximate method for investigating the large deflections of initially flat isotropic plates.

Here we determine the large deflection of a clamped cicular plate under non-uniform load following Berger's approximate method. The applied external pressure $P(x)$ is assumed to be expressive in the following form.

$$P(x) = C_0\left(1-\frac{x^2}{\rho^2}\right)P_\beta^{(a,b)}\left(1-\frac{2x^2}{\rho^2}\right){}_{p'}\psi_{q'}\left(A'\left(1-\frac{x^2}{\rho^2}\right)\right)\aleph\begin{pmatrix}\mathrm{A}_1\left(1-\frac{x^2}{\rho^2}\right)\\ \cdot\\ \cdot\\ \mathrm{A}_r\left(1-\frac{x^2}{\rho^2}\right)\end{pmatrix}\aleph\begin{pmatrix}\mathrm{B}_1\left(1-\frac{x^2}{\rho^2}\right)\\ \cdot\\ \cdot\\ \mathrm{B}_s\left(1-\frac{x^2}{\rho^2}\right)\end{pmatrix} \quad (1.1)$$

where C_0, A', A_i and B_j are constants for $i=1,\cdots,r$ and $j=1,\cdots,s$. $\aleph$ is the multivariable Aleph-function, $P_\beta^{(a,b)}(x)$ is the Jacobi polynomial [6] and ${}_{p'}\psi_{q'}(z)$ is the Fox-Wright function [10].

The function Aleph of several variables is an extension of the multivariable I-function recently study by C.K. Sharma and Ahmad [8] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

$$\aleph(z_1,\cdots,z_s) = \aleph^{0,N:M_1,N_1,\cdots,M_s,N_s}_{P_i,Q_i,\iota_i;r':P_{i(1)},Q_{i(1)},\iota_{i(1)};r^{(1)};\cdots;P_{i(s)},Q_{i(s)};\iota_{i(s)};r^{(s)}}\left(\begin{array}{c|}\mathrm{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_s\end{array}\right.$$

$$\begin{array}{l}[(\mathrm{u}_j;\mu_j^{(1)},\cdots,\mu_j^{(r')})_{1,N}] \quad ,[\iota_i(u_{ji};\mu_{ji}^{(1)},\cdots,\mu_{ji}^{(r')})_{N+1,P_i}]:\\ \cdots\cdots\cdots\cdots \quad ,[\iota_i(v_{ji};\upsilon_{ji}^{(1)},\cdots,\upsilon_{ji}^{(r')})_{M+1,Q_i}]:\end{array}$$

$$\left.\begin{matrix}[(\mathrm{a}_j^{(1)});\alpha_j^{(1)})_{1,N_1}],[\iota_{i^{(1)}}(a_{ji^{(1)}}^{(1)};\alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_i^{(1)}}];\cdots;[(\mathrm{a}_j^{(s)});\alpha_j^{(s)})_{1,N_s}],[\iota_{i^{(s)}}(a_{ji^{(s)}}^{(s)};\alpha_{ji^{(s)}}^{(s)})_{N_s+1,P_i^{(s)}}]\\ [(\mathrm{b}_j^{(1)});\beta_j^{(1)})_{1,M_1}],[\iota_{i^{(1)}}(b_{ji^{(1)}}^{(1)};\beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_i^{(1)}}];\cdots;[(\mathrm{b}_j^{(s)});\beta_j^{(s)})_{1,M_s}],[\iota_{i^{(s)}}(b_{ji^{(s)}}^{(s)};\beta_{ji^{(s)}}^{(s)})_{M_s+1,Q_i^{(s)}}]\end{matrix}\right)$$

$$=\frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\zeta(t_1,\cdots,t_s)\prod_{k=1}^{s}\phi_k(t_k)z_k^{t_k}\,\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.2}$$

with $\omega=\sqrt{-1}$

$$\zeta(t_1,\cdots,t_s)=\frac{\prod_{j=1}^{N}\Gamma(1-u_j+\sum_{k=1}^{s}\mu_j^{(k)}t_k)}{\sum_{i=1}^{r'}[\iota_i\prod_{j=N+1}^{P_i}\Gamma(u_{ji}-\sum_{k=1}^{s}\mu_{ji}^{(k)}t_k)\prod_{j=1}^{Q_i}\Gamma(1-v_{ji}+\sum_{k=1}^{s}v_{ji}^{(k)}t_k)]} \tag{1.3}$$

and $$\phi_k(t_k)=\frac{\prod_{j=1}^{M_k}\Gamma(b_j^{(k)}-\beta_j^{(k)}t_k)\prod_{j=1}^{N_k}\Gamma(1-a_j^{(k)}+\alpha_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{r^{(k)}}[\iota_{i^{(k)}}\prod_{j=M_k+1}^{Q_{i^{(k)}}}\Gamma(1-b_{ji^{(k)}}^{(k)}+\beta_{ji^{(k)}}^{(k)}t_k)\prod_{j=N_k+1}^{P_{i^{(k)}}}\Gamma(a_{ji^{(k)}}^{(k)}-\alpha_{ji^{(k)}}^{(k)}s_k)]} \tag{1.4}$$

Suppose , as usual , that the parameters

$u_j, j=1,\cdots,P; v_j, j=1,\cdots,Q;$

$a_j^{(k)}, j=1,\cdots,N_k; a_{ji^{(k)}}^{(k)}, j=n_k+1,\cdots,P_{i^{(k)}};$

$b_{ji^{(k)}}^{(k)}, j=m_k+1,\cdots,Q_{i^{(k)}}; b_j^{(k)}, j=1,\cdots,M_k;$

with $k=1\cdots,s, i=1,\cdots,r'$, $i^{(k)}=1,\cdots,r^{(k)}$

are complex numbers , and the $\alpha's,\beta's,\gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)}=\sum_{j=1}^{N}\mu_j^{(k)}+\iota_i\sum_{j=N+1}^{P_i}\mu_{ji}^{(k)}+\sum_{j=1}^{N_k}\alpha_j^{(k)}+\iota_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\alpha_{ji^{(k)}}^{(k)}-\iota_i\sum_{j=1}^{Q_i}v_{ji}^{(k)}-\sum_{j=1}^{M_k}\beta_j^{(k)}$$

$$-\iota_{i^{(k)}}\sum_{j=M_k+1}^{Q_{i^{(k)}}}\beta_{ji^{(k)}}^{(k)}\leqslant 0 \tag{1.5}$$

The reals numbers τ_i are positives for $i=1,\cdots,r$, $\iota_{i^{(k)}}$ are positives for $i^{(k)}=1\cdots r^{(k)}$

The contour L_k is in the t_k-p lane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(b_j^{(k)}-\beta_j^{(k)}t_k)$ with $j=1$ to M_k are separated from those of $\Gamma(1-u_j+\sum_{i=1}^{s}\mu_j^{(k)}t_k)$with $j=1$ to N and $\Gamma(1-a_j^{(k)}+\alpha_j^{(k)}t_k)$ with $j=1$ to N_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k|<\frac{1}{2}B_i^{(k)}\pi$, where

$$B_i^{(k)}=\sum_{j=1}^{N}\mu_j^{(k)}-\iota_i\sum_{j=N+1}^{P_i}\mu_{ji}^{(k)}-\iota_i\sum_{j=1}^{Q_i}v_{ji}^{(k)}+\sum_{j=1}^{N_k}\alpha_j^{(k)}-\iota_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\alpha_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{M_k}\beta_j^{(k)} - \iota_{i^{(k)}}\sum_{j=M_k+1}^{q_{i^{(k)}}}\beta_{ji^{(k)}}^{(k)} > 0, \quad \text{with } k=1,\cdots,s, i=1,\cdots,r\ , i^{(k)}=1,\cdots,r^{(k)} \tag{1.6}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1,\cdots,z_s)=0(\,|z_1|^{\alpha_1'},\cdots,|z_s|^{\alpha_s'}\,)\,,max(\,|z_1|,\cdots,|z_s|\,)\rightarrow 0$$

$$\aleph(z_1,\cdots,z_s)\ =0(\,|z_1|^{\beta_1'},\cdots,|z_s|^{\beta_s'}\,)\,,min(\,|z_1|,\cdots,|z_s|\,)\rightarrow \infty$$

where, with $k=1,\cdots,z:\alpha_k'=min[Re(b_j^{(k)}/\beta_j^{(k)})], j=1,\cdots,M_k$ and

$$\beta_k'=max[Re((a_j^{(k)}-1)/\alpha_j^{(k)})], j=1,\cdots,N_k$$

The serie representation of the Aleph-function of $r-$variables is given by Ayant [1]

$$\aleph(z_1,\cdots,z_r)=\sum_{G_k=1}^{m_k}\sum_{g_k=0}^{\infty}\phi\frac{\prod_{k=1}^r\phi_k z_k^{\eta_{G_k,g_k}}(-)^{\sum_{k=1}^r g_k}}{\prod_{k=1}^r\delta_{G^{(k)}}^{(k)}\prod_{i=1}^r g_k!} \tag{1.7}$$

where

$$\phi=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^r\alpha_j^{(i)}S_k)}{\sum_{i=1}^R[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^r\alpha_{ji}^{(k)}S_k)\prod_{j=1}^{q_k}\Gamma(1-b_{ji}+\sum_{k=1}^r\beta_{ji}^{(k)}S_k)]} \tag{1.8}$$

$$\phi_k=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}S_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}S_k)}{\sum_{i^{(i)}=1}^{R^{(i)}}[\tau_{i^{(i)}}\prod_{j=m_i+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}S_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}S_k)]} \tag{1.9}$$

and

$$S_k=\eta_{G_k,g_k}=\frac{d_{g_k}^{(k)}+G_k}{\delta_{g_k}^{(k)}}\text{ for } k=1,\cdots,r \tag{1.10}$$

which is valid under the following conditions : $\epsilon_{M_k}^{(k)}[p_j^{(k)}+p_k']\neq\epsilon_j^{(k)}[p_{M_k}+g_k]$

Also, the Fox-Wright function [10] is defined as

$${}_{p'}\psi_{q'}(z)={}_{p'}\psi_{q'}\left[\begin{array}{c|c}(\mathrm{e}_j,E_j)_{1,p'} & \\ . & \mathrm{z} \\ (\mathrm{f}_j,F_j)_{1,q'} & \end{array}\right]=\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p'}\Gamma(e_j+E_jn)}{\prod_{j=1}^{q'}\Gamma(f_j+F_jn)}\frac{z^n}{n!} \tag{1.11}$$

Where $E_j(j=1,\cdots,p')$ and $F_j(j=1,\cdots,q')$ are real and positive numbers and

$$1+\sum_{j=1}^{q'}F_j-\sum_{j=1}^{p'}E_j>0$$

For convenience, we shall note

$$V = M_1, N_1; \cdots ; M_s, N_s \tag{1.12}$$

$$W = P_{i^{(1)}}, Q_{i^{(1)}}, \iota_{i^{(1)}}; r^{(1)}; \cdots ; P_{i^{(r)}}, Q_{i^{(r)}}, \iota_{i^{(s)}}; r^{(s)} \tag{1.13}$$

$$A = \{(u_j; \mu_j^{(1)}, \cdots, \mu_j^{(s)})_{1,N}\}, \{\iota_i(u_{ji}; \mu_{ji}^{(1)}, \cdots, \mu_{ji}^{(s)})_{N+1,P_i}\} : (a_j^{(1)}; \alpha_j^{(1)})_{1,N_1}, \iota_{i^{(1)}}(a_{ji^{(1)}}^{(1)}; \alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_{i^{(1)}}} ; \cdots ;$$

$$(a_j^{(s)}; \alpha_j^{(s)})_{1,N_s}, \iota_{i^{(s)}}(a_{ji^{(s)}}^{(s)}; \alpha_{ji^{(s)}}^{(s)})_{N_s+1,P_{i^{(s)}}} \tag{1.14}$$

$$B = \{\iota_i(v_{ji}; v_{ji}^{(1)}, \cdots, v_{ji}^{(s)})_{M+1,Q_i}\} : (b_j^{(1)}; \beta_j^{(1)})_{1,M_1}, \iota_{i^{(1)}}(b_{ji^{(1)}}^{(1)}; \beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_{i^{(1)}}} ; \cdots ;$$

$$(b_j^{(s)}; \beta_j^{(s)})_{1,M_s}, \iota_{i^{(s)}}(\beta_{ji^{(s)}}^{(s)}; \beta_{ji^{(s)}}^{(s)})_{M_s+1,Q_{i^{(s)}}} \tag{1.15}$$

$$D_n = \frac{\prod_{j=1}^{p'} \Gamma(e_j + E_j n)}{\prod_{j=1}^{q'} \Gamma(f_j + F_j n)} \tag{1.16}$$

2. Main integral

In this section, we evaluate the following integral. We shall use the above notations.

$$\int_0^1 \theta^{\lambda+1}(1-\theta^2)^{\alpha} P_{\beta}^{(a,b)}(1-2\theta^2)_{p'}\psi_{q'}(A'(1-\theta^2)) \aleph\begin{pmatrix} \mathrm{A}_1(1-\theta^2) \\ \cdot \\ \cdot \\ \mathrm{A}_r(1-\theta^2) \end{pmatrix} \aleph\begin{pmatrix} \mathrm{B}_1(1-\theta^2) \\ \cdot \\ \cdot \\ \mathrm{B}_s(1-\theta^2) \end{pmatrix} J_{\mu}(\theta\tau)\mathrm{d}\theta =$$

$$= \sum_{n,n''=0}^{\infty} \sum_{n'=0}^{\beta} \sum_{G_I=1}^{m_I} \sum_{g_I=0}^{\infty} \phi \frac{\prod_{I=1}^r \phi_I A_I^{\eta_{G_I,g_I}} (-)^{\sum_{I=1}^r g_I}}{\prod_{I=1}^r \delta_{G^{(I)}}^{(I)} \prod_{I=1}^r g_I!} \frac{A'^n(-)^{n''}(-\beta)_{n'} \left(\frac{\tau}{2}\right)^{\mu+2n''}}{2n!n'!n''!\beta!} D_n$$

$$\frac{\Gamma(1+a+\beta)(1+a+b+\beta)_{n'}\Gamma\left(\lambda+n'+n''+\frac{\mu}{2}+1\right)}{\Gamma(1+a+n')\Gamma(1+\mu+n'')} \aleph_{P_i+1,Q_i+1,\iota_i;R:W}^{0,N+1:V}\left(\begin{matrix} \mathrm{B}_1 \\ \cdot \\ \cdot \\ \mathrm{B}_s \end{matrix}\right|$$

$$\left.\begin{matrix} (\text{-}\alpha - n - \sum_{I=1}^r \eta_{G_I,g_I}; 1, \cdots, 1), A \\ \cdot \\ \cdot \\ (\text{-}1\text{-}\lambda - n - n' - n'' - \alpha - \sum_{I=1}^r \eta_{G_I,g_I} - \frac{\mu}{2}; 1, \cdots; 1), B \end{matrix}\right) \tag{2.1}$$

Provided that

$$Re(a) > -1, Re(b) > -1, Re(\lambda) > -1, Re(\alpha) > -1, Re(\mu) > -\frac{1}{2}$$

$$Re(\alpha + \sum_{I=1}^r \eta_{G_I,g_I}) + \sum_{j=1}^s \min_{1 \leqslant k \leqslant M_j} Re\left(\frac{b_k^{(j)}}{\beta_k^{(j)}}\right) > 0\,,\, 1 + \sum_{j=1}^{q'} F_j - \sum_{j=1}^{p'} E_j > 0$$

$|argB_k| < \frac{1}{2}B_I^{(k)}\pi$, where $B_I^{(k)}$ is defined by (1.6) and $|argA_k| < \frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)} - \tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)} + \sum_{j=1}^{n_k}\gamma_j^{(k)} - \tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)} + \sum_{j=1}^{m_k}\delta_j^{(k)}$$

$$- \tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)} > 0, \quad \text{with } k = 1\cdots, r, i = 1,\cdots, R \text{ , } i^{(k)} = 1,\cdots, R^{(k)}$$

Proof

To prove (2.1), first using the definition of Bessel's function, we express in series the Aleph-function of r-variables and the Fox-Wright function ${}_p\psi_{q'}(z)$ with the help of (1.7) and (1.11) respectively. We interchange the order of summations and θ-integral (which is permissible under the conditions stated). Expressing the Aleph-functions of s-variables in terms of Mellin-Barnes type contour integral with the help of (1.2) and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process. Now collect the power of $1-\theta^2$ and use [3]. Interpreting the $s-$dimensional Mellin-Barnes integral in multivariable Aleph-function, we obtain the equation (2.1).

3. Statement of the problem.

Let us assume a clamped cicular plate of thickness t, radius ρ and rigidity R. Then by using Berger's method, the approximate equation for a circular plate undergoing large deflections due an externally applied load $P(x)$ may be given as

$$\left(\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx}\right)\left(\frac{d^2w}{dx^2} + \frac{1}{x}\frac{dw}{dx} - k^2w\right) = \frac{P}{R} = \phi(x) \tag{3.1}$$

where k is a normalized constant of integration given by the equation

$$\frac{dy}{dx} + \frac{y}{x} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2 = \frac{k^2t^2}{12} \tag{3.2}$$

where w is the plate deflection, normal to middle plane of the plate and y is the radial displacement. The boundary conditions of the problem are :

(i) $w = 0 = \frac{dw}{dx}$, at $x = \rho$

(ii) $y = 0$, at $x = \rho$

Solution of the problem

Let us consider

$$w = \sum_i G_i'[J_0(xt_i) - J_0(\rho t_i)] \tag{3.3}$$

where t_i is the $i-$th root of $J_1(\rho t_i) = 0$.

It is clear that the boundary conditions are satisfied by the above equation. Now use (3.3) in the equation (3.1) ,we find

$$\sum_i G_i' t_i^2(k^2 + t_i^2)J_0(xt_i) = \phi(x) \tag{3.4}$$

Now expanding $\phi(x)$ in a series of Bessel's function, we obtain on integrations

$$\int_0^\rho G_i' t_i^2 (k^2 + t_i^2) J_0^2(xt_i) x \mathrm{d}x = \int_0^\rho \phi(x) x J_0(xt_i) \mathrm{d}x \tag{3.5}$$

Now by left hand side of (3.5)

$$\int_0^\rho x J_0^2(xt_i) \mathrm{d}x = \frac{\rho^2}{2} J_0^2(\rho t_i) \tag{3.6}$$

(3.6) becomes

$$G_i' t_i^2 (k^2 + t_i^2) J_0^2(xt_i) \frac{\rho^2}{2} = \int_0^\rho \phi(x) x J_0^2(xt_i) \mathrm{d}x \tag{3.7}$$

and

$$G_i' = \frac{\int_0^\rho x\phi(x) J_0(xt_i) \mathrm{d}x}{\rho^2 t_i^2 (k^2 + t_i^2) J_0^2(\rho t_i)} \tag{3.8}$$

Using (2.1) in view of (1.1) and (3.1), we get

$$G_i' = \frac{C_0 \Gamma(1+a+\beta)}{R\beta!(k^2+t_i^2) J_0^2(\rho t_i)} \sum_{n,n''=0}^{\infty} \sum_{n'=0}^{\beta} \sum_{G_I=1}^{m_I} \sum_{g_I=0}^{\infty} \phi \frac{\prod_{I=1}^r \phi_I A_I^{\eta_{G_I,g_I}} (-)^{\sum_{I=1}^r g_I}}{\prod_{I=1}^r \delta_{G^{(I)}}^{(I)} \prod_{I=1}^r g_I!} \frac{A'^n (-)^{n''} (-\beta)_{n'} \left(\frac{\tau}{2}\right)^{\mu+2n''}}{2n!n'!n''!\beta!} D_n$$

$$\frac{\Gamma(1+n+n'')(1+a+b+\beta)_{n'}}{\Gamma(1+a+n')\Gamma(1+n'')} \aleph_{P_i+1,Q_i+1,\iota_i;R:W}^{0,N+1:V} \left(\begin{array}{c|c} \mathrm{B}_1 & (\text{-}\alpha - n - \sum_{I=1}^r \eta_{G_I,g_I}; 1, \cdots, 1), A \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{B}_s & (\text{-1-n-n'-n''-}\alpha - \sum_{I=1}^r \eta_{G_I,g_I}; 1, \cdots; 1), B \end{array} \right) \tag{3.9}$$

under the same notations and conditions that (2.1). Now combining the equation (3.3) and (3.9), we obtain

$$w = L_1 \sum_i \frac{L_2 [J_0(xt_i) - J_0(\rho t_i)]}{k^2 + t_i^2} \tag{3.10}$$

where

$$L_1 = \frac{C_0 \Gamma(1+a+\beta)}{R\beta!} \tag{3.11}$$

and

$$L_2 = \frac{1}{t_i^2 J_0^2(\rho t_i)} \sum_{n,n''=0}^{\infty} \sum_{n'=0}^{\beta} \sum_{G_I=1}^{m_I} \sum_{g_I=0}^{\infty} \phi \frac{\prod_{I=1}^r \phi_I A_I^{\eta_{G_I,g_I}} (-)^{\sum_{I=1}^r g_I}}{\prod_{I=1}^r \delta_{G^{(I)}}^{(I)} \prod_{I=1}^r g_I!} \frac{A'^n (-)^{n''} (-\beta)_{n'} \left(\frac{\tau}{2}\right)^{\mu+2n''}}{2n!n'!n''!\beta!} D_n$$

$$\frac{\Gamma(1+n+n'')(1+a+b+\beta)_{n'}}{\Gamma(1+a+n')\Gamma(1+n'')} \aleph_{P_i+1,Q_i+1,\iota_i;R:W}^{0,N+1:V} \left(\begin{array}{c|c} \mathrm{B}_1 & (\text{-}\alpha - n - \sum_{I=1}^r \eta_{G_I,g_I}; 1, \cdots, 1), A \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{B}_s & (\text{-1-n-n'-n''-}\alpha - \sum_{I=1}^r \eta_{G_I,g_I}; 1, \cdots; 1), B \end{array} \right) \tag{3.12}$$

under the same notations and conditions that (2.1). Now the radial displacement y can be obtained by using equations (3.2) and (3.3) as

$$y = \frac{k^2t^2x}{24} - \frac{1}{2}\sum_{i=1}^{\infty} G_i'^2 t_i^2 \left[\frac{x}{2}\left(J_i'^2(xt_i) + \left(1 - \frac{1}{x^2t_i^2}\right)J_1^2(xt_i)\right)\right]$$

$$-\frac{1}{2}\sum_{i,j=1}^{\infty} G_i'G_j't_jt_j \left[\frac{t_iJ_2(xt_i)J_1(xt_j) - t_jJ_2(xt_j)J_1(xt_i)}{t_i^2 - t_j^2}\right] + C_1, i \neq j \tag{3.13}$$

where C_1 is athe constant of integration.

Applying the boundary condition $y = 0$ at $x = \rho$ and $J_1(\rho t_i) = 0$, we get

$$C_1 = -\frac{k^2t^2\rho}{24} + \frac{1}{4}\sum_{i=1}^{\infty} G_i'^2 t_i^2 J_1'^2(\rho t_i) \tag{3.14}$$

Hence the radial displacement y is established as

$$y = \frac{k^2t^2(x-\rho)}{24} - \frac{1}{2}\sum_{i=1}^{\infty} G_i'^2 t_i^2 \left[\frac{x}{2}\left(J_i'^2(xt_i) + \left(1 - \frac{1}{x^2t_i^2}\right)J_1^2(xt_i)\right)\right]$$

$$-\frac{1}{2}\sum_{i,j=1}^{\infty} G_i'G_j't_jt_j \left[\frac{t_iJ_2(xt_i)J_1(xt_j) - t_jJ_2(xt_j)J_1(xt_i)}{t_i^2 - t_j^2}\right] + \frac{1}{4}\sum_{i=1}^{\infty} G_i'^2 t_i^2 \rho J_0^2(\rho t_i), i \neq j \tag{3.15}$$

4. Applications

(a) The deflection given by equation (3.10) can be to evaluate the boundary stresses at the surface of the plate which for the circular plate, are given by [1] as

$$\sigma_x = \frac{6R}{t^2}\left(\frac{d^2w}{dx^2} + \frac{\upsilon}{x}\frac{dw}{dx}\right) \tag{4.1}$$

and

$$\sigma_\theta = \frac{6R}{t^2}\left(\upsilon\frac{d^2w}{dx^2} + \frac{1}{x}\frac{dw}{dx}\right) \tag{4.2}$$

where υ is the Poisson's ratio. By using (3.10), we get

$$\sigma_x = -\frac{6R}{t^2}L_1\sum_i \frac{L_2\left[J_0''(xt_i) + \frac{\upsilon}{x}J_0'(xt_i)\right]}{k^2 + t_i^2} \tag{4.3}$$

and

$$\sigma_\theta = -\frac{6R}{t^2}L_1\sum_i \frac{L_2\left[\upsilon J_0''(xt_i) + \frac{1}{x}J_0'(xt_i)\right]}{k^2 + t_i^2} \tag{4.4}$$

Now, putting $x = 0$ in (4.3) and (4.4), we get the bending stresses at the centre of the plate as

$$(\sigma_x)_{x=0} = (\sigma_\theta)_{x=0} = -\frac{3R}{t^2}L_1\sum_i \frac{L_2(1+\upsilon)t_i^2}{k^2 + t_i^2} \tag{4.5}$$

Also by putting $x = \rho$, the bending stresses at the edge of the plate are obtained as

$$(\sigma_x)_{x=\rho} = -\frac{6R}{t^2} L_1 \sum_i \frac{L_2 J_0(\rho t_i) t_i^2}{k^2 + t_i^2} \tag{4.6}$$

$$(\sigma_\theta)_{x=\rho} = -\frac{6R}{t^2} L_1 \sum_i \frac{L_2 J_0(\rho t_i) \upsilon t_i^2}{k^2 + t_i^2} \tag{4.7}$$

(b) When $k = 0$, the partial differential equation (3.1) corresponds to that of small deflection equation and the equation (3.10) leads to

$$w = L_1 \sum_i \frac{L_2[J_0(x t_i) - J_0(\rho t_i)]}{t_i^2} \tag{4.8}$$

(c) By using $x = 0$, we obtain the deflection w_0 at the centre of the plate as

$$w_0 = L_1 \sum_i \frac{L_2[1 - J_0(\rho t_i)]}{k^2 + t_i^2} \tag{4.9}$$

whereas the small deflection will be given by

$$w_0 = L_1 \sum_i \frac{L_2[1 - J_0(\rho t_i)]}{t_i^2} \tag{4.10}$$

5. Aleph-function of two variables

In this section, the two multivariable Aleph-function reduce to Aleph-function of two variables defined by K. Sharma [7]. We have $r = s = 2$, and we obtain

$$w = L_1 \sum_i \frac{L_2'[J_0(x t_i) - J_0(\rho t_i)]}{k^2 + t_i^2} \tag{5.1}$$

where

$$L_1 = \frac{C_0 \Gamma(1 + a + \beta)}{R \beta!} \tag{5.2}$$

and

$$L_2' = \frac{1}{t_i^2 J_0^2(\rho t_i)} \sum_{n,n''=0}^{\infty} \sum_{n'=0}^{\beta} \sum_{G_{I=1}}^{m_I} \sum_{g_{I=0}}^{\infty} \phi \frac{\prod_{I=1}^2 \phi_I A_I^{\eta_{G_I,g_I}} (-)^{\sum_{I=1}^2 g_I}}{\prod_{I=1}^2 \delta_{G^{(I)}}^{(I)} \prod_{I=1}^2 g_I!} \frac{A'^n (-)^{n''} (-\beta)_{n'} \left(\frac{\tau}{2}\right)^{\mu+2n''}}{2n!n'!n''!\beta!} D_n$$

$$\frac{\Gamma(1+n+n'')(1+a+b+\beta)_{n'}}{\Gamma(1+a+n')\Gamma(1+n'')} \aleph_{P_i+1,Q_i+1,\iota_i;R:W}^{0,N+1:V} \left(\begin{array}{c|c} \mathrm{B}_1 & (\text{-}\alpha - n - \sum_{I=1}^2 \eta_{G_I,g_I}; 1,1), A' \\ . & . \\ . & . \\ \mathrm{B}_2 & (\text{-1-n-n'-n''-}\alpha - \sum_{I=1}^2 \eta_{G_I,g_I}; 1,1), B' \end{array} \right) \tag{5.3}$$

Under the same notations and conditions that (2.1) with $r = s = 2$.

The quantities A', B' are equal to A, B respectively for $s = 2$.

In the case, we get the small deflection

$$w = L_1 \sum_i \frac{L_2'[J_0(xt_i) - J_0(\rho t_i)]}{t_i^2} \tag{5.4}$$

and the deflection at the centre of the plate is given by

$$w_0 = L_1 \sum_i \frac{L_2'[1 - J_0(\rho t_i)]}{k^2 + t_i^2} \tag{5.5}$$

6. I-function of two variables

In this section, we have $r = s = 2$, $\tau_i, \tau_{i'}, \tau_{i''} \to 1$ and $\iota_i, \iota_{i'}, \iota_{i''} \to 1$, the multivariable Aleph-functions reduce to I-functions of two variables defined by Sharma and Mishra [9]. We have the following results.

$$w = L_1 \sum_i \frac{L_2''[J_0(xt_i) - J_0(\rho t_i)]}{k^2 + t_i^2} \tag{6.1}$$

where

$$L_1 = \frac{C_0 \Gamma(1 + a + \beta)}{R\beta!} \tag{6.2}$$

and

$$L_2'' = \frac{1}{t_i^2 J_0^2(\rho t_i)} \sum_{n,n''=0}^{\infty} \sum_{n'=0}^{\beta} \sum_{G_I=1}^{m_I} \sum_{g_I=0}^{\infty} \phi' \frac{\prod_{I=1}^2 \phi_I' A_I^{\eta_{G_I,g_I}} (-)^{\sum_{I=1}^2 g_I}}{\prod_{I=1}^2 \delta_{G^{(I)}}^{(I)} \prod_{I=1}^2 g_I!} \frac{A'^n (-)^{n''} (-\beta)_{n'} \left(\frac{\tau}{2}\right)^{\mu+2n''}}{2n!n'!n''!\beta!} D_n$$

$$\frac{\Gamma(1+n+n'')(1+a+b+\beta)_{n'}}{\Gamma(1+a+n')\Gamma(1+n'')} \, I_{P_i+1,Q_i+1;R:W}^{0,N+1:V} \left(\begin{array}{c|c} \mathrm{B}_1 & (\text{-}\alpha - n - \sum_{I=1}^2 \eta_{G_I,g_I}; 1, 1), A'' \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{B}_2 & (\text{ -1-n-n'-n''-}\alpha - \sum_{I=1}^2 \eta_{G_I,g_I}; 1, 1), B'' \end{array} \right) \tag{6.3}$$

Under the same notations and conditions that (2.1) with $r = s = 2$, $\tau_i, \tau_{i'}, \tau_{i''} \to 1$ and $\iota_i, \iota_{i'}, \iota_{i''} \to 1$

The quantities A'', B'' are equal to A', B' respectively for $\iota_i, \iota_{i'}, \iota_{i''} \to 1$.

In the case, we get the small deflection

$$w = L_1 \sum_i \frac{L_2''[J_0(xt_i) - J_0(\rho t_i)]}{t_i^2} \tag{6.4}$$

and the deflection at the centre of the plate is given by

$$w_0 = L_1 \sum_i \frac{L_2''[1 - J_0(\rho t_i)]}{k^2 + t_i^2} \tag{6.5}$$

Remarks

If the multivariable Aleph-functions reduce to multivariable I-function defined by Sharma and Ahmad [8] (respectively to multivariable H-functions defined by Srivastava and Panda [11,12]) we obtain the similar formulae with the multivariable I-functions (respectively the multivariable H-functions).

If the multivariable Aleph-functions reduce to Aleph-functions of one variable defined by Sudland [13], we obtain the same results. For more details, see Gill and Modi [4].

7. Conclusion

Specializing the parameters of the multivariable Aleph-functions, we can obtain large number of results involving various special functions of one and several variables useful in Mathematics analysis, Applied Mathematics, Physics and Mechanics, in particular the problem concerning the large deflection of a circular plate under non uniform load.

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