The Mathematical Formula for Determining the Name of the *Pancawara* Day on the Masehi Calendar

Agung Prabowo¹, Mustafa Mamat², Sukono³, Herlina Napitupulu⁴

¹Department of Mathematics, Faculty of Mathematics and Natural Siences, Universitas Jenderal Soedirman

Jl. Dr. Soeparno No. 61 Karangwangkal Purwokerto, Central Java, Indonesia

²Faculty of Informatics and Computing, Universiti Sultan Zainal Abidin

Tembila Campus, 2200 Besut, Terengganu, Malaysia

^{3,4}Department of Mathematics, Faculty of Mathematics and Natural Siences, Universitas Padjadjaran

Jl. Raya Bandung-Sumedang Km 21, Jatinangor 45363, Sumedang, West Java, Indonesia

Abstract - In the Javanese calendar (Anno Javanica) there are two types of day names, namely pancawara day and saptawara day. Pancawara is a five-day cycle, such as: Legi (Manis), Paing, Pon, Wage and Kliwon. While Saptawara is a seven-day cycle, such as Monday, Tuesday, Wednesday, Thursday, Friday, Saturday and Sunday. But in the Masehi calendar does not recognize this pancawara cycle. In this paper intends to determine the day of the pancawara in the Masehi calendar. For the determination of the pacawara day in the Masehi calendar, in this paper was construct mathematical formulation of the name of the day pancawara for a specific date on the Masehi calendar The results of the discussion obtained two alternative formulas to determine the name of the pancawara day in the Masehi calendar. So it can complement two formulas that have been formulated by previous research.

Keywords: Javanese calendar, Masehi calendar, pancawara, saptawara, mathematical formula.

Mathematical Subject Classification 2010: 11A07

I. INTRODUCTION

Rosen [10] has produced a mathematical formula to determine the name of the day in the seven-day cycle (Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday) on any date, month and year in the Masehi calendar. The Masehi (M) calendar used is the Gregorian Calendar, as a result of a revision of Pope Gregory and used since October 15, 1582 M [3; 5]. The mathematical formula is limited to its use only from March 1, 1600 M. The formula is formulated using the ladder and modulo functions. The result is [10]:

 $w = (k + [[2,6 \cdot m - 0,2]] + Y - 2C + [[Y/4]] + [[C/4]]) \mod 7$ (1) Meanwhile, Kurnia, et al. [4], has produced a mathematical formula to determine the name of the day in the five daily cycle on any date, month and year in the Masehi calendar. The result is:

 $w_p = (k + [0.6 \cdot m + 1.8] + 4C + [Y/4] + [C/4] - 3) \mod 5$ (2)

Prabowo et al. [9], has performed a reconstruction of the formula that Kurnia et al. [4], as well as providing a new formula of a kind, namely:

 $w_p \equiv (k + [0,6 \cdot m - 0,2]] + 4C + [Y/4]] + [C/4] - 1) \mod 5$ (3) Formulas (2) and (3) use references $w_p = 0,1,2,3,4$ respectively for Legi (Manis), Paing, Pon, Wage and Kliwon. This article attempts to offer a different mathematical formula with (2) to determine the name of the day in the five-day cycle (pancawara) as Kurnia et al. [4] and Prabowo et al. [9].

Referring to the above background, the formulation of the problem as to whether the mathematical model to determine the name of the day in the five-day cycle (*pancawara*) for each date on the Masehi calendar. As the boundary problem is the formula generated using the base of calculation March 1, 1600 M, so that the model obtained can only be used for the time interval from March 1, 1600 to March 1, 2000.

The use of the Masehi Calendar in Java and Indonesia, in addition to using the seven-day cycle also uses a five-day cycle. In this paper will be construct a formula to determine the name of the day in the five-day cycle that is still used by the people of Java. The resulting mathematical model can be used by all audiences to determine the name of the *pancawara* the desired date of M, such as the Proclamation of Indonesian Independence on August 17, 1945 can be determined to fall on the day *pancawara Legi (Manis)*.

II. RESEARCH METHODS

The research and results outlined in this paper are solved by research methodology in the form of literature review (literature study). Rosen [10], has produced a mathematical formula for determining the name of the day in the seven daily cycle that has been given at (1). Following the steps taken by Rosen [10], a mathematical formula was developed for the naming of days in the five-day cycle.

III. RESULTS AND DISCUSSION

In this section we will discuss the following: the Masehi calendar, *saptawara*, *pancawara*, modulo, function ladder, leap year, and derivation of mathematical formula.

A. The Masehi Calendar

The current Masehi Calendar can be said to be an ancient Egyptian heritage calendar enhanced by Julius Caesar in 46 BC, so called the Roman Calendar Julian, and refined by Pope Gregory in 1582 M so called Gregorian Calendar [5; 7]. The calendar of the Ancient Egyptians was a solar calendar of calendars construct by following the rotation of the earth around the sun (earth revolution). Age one year of the Ancient Egyptian Calendar exactly 365 days, without leap year [7; 11].

At the time of Julius Caesar conquering Egypt, he perfected the calendar of the Ancient Egyptians and the result was called the Julian Masonian Calendar (MJ). On the MJ Calendar, the age of one year is 365.25 days. Thus, once every four years there will be a leap year of 366 days [8]. In the time of Pope Gregory it was known that the age of one year was 365, 2422 days. That is, there is an excess of 0.0078 days compared to MJ Calendar. Pope Gregory repaired the MJ Calendar by setting tomorrow the day after Thursday, October 4, 1582 was Friday, October 15, 1582 [11]. Thus, 5-14 October 1582 never existed. This fix generates a Masehi Gregorian Calendar (MG). In the MG Calendar also set every four years will occur leap year. Furthermore, the MG calendar will be written with CE and abbreviated as

Current modern knowledge states that the age of one year is 365,2425. Thus, every 10,000 years should be added three days or every 3333 years to be added one day [1]. Associated with the change from the MJ calendar to the MG Calendar, not all nations / countries do it on October 5, 1582. The new British established the validity of the MG Calendar in 1752 by adding 11 days. The Japanese, the Soviet Union and Greece were consecutively new in 1873, 1917 and 1923 [10]. Venetian, Spanish, Portuguese, Dutch, German and Polish were the first countries to officially implement this calendar system through Inter Gravissimas in 1582 (https://id.wikipedia.org/ wiki/Calendar_Gregorius). This means that when the Dutch introduced the Masehi calendar in Indonesia, the calendar used is already a Gregoriane [3].

B. Saptawara

Saptawara comes from the word sapta which means seven and wara meaning day. Thus, the saptawara is the seventh day of a seven-day cycle of time.

The use of *saptawara* in Java has existed since the days of ancient Mataram (732 M). The inscriptions issued by the ancient Mataram kings have carved the names of the *saptawara* days. The names of the day are *Radite, Soma, Anggara, Buda, Respati, Sukra*,

and *Tumpak*. The Javanese have replaced the names of the day, while in Bali it is still maintained. In the Masehi calendar, *saptawara* is identical to Sunday (*Radite*), and so on until Saturday (*Tumpak / Saniscara*) [4; 10].

C. Pancawara

The pancawara is the fifth day (panca = five), the five-day cycle of time travel. The pancawara is not known in the Masehi. However, the use of the Masehi calendar in Java combines saptawara with pancawara. Like the saptawara, the pancawara has also been used by Javanese since the days of ancient Mataram. The names of day pancawara are Legi (Manis), Paing, Pon, Wage, and Kliwon [3; 10].

D. Modulo

Let a and b be an integer. If m is a positive integer greater than 1, then a is congruent to b modulo m, and can be written with $a \equiv b \pmod{m}$ if m divides up (a - b) [10; 2]. In other words, $a \equiv b \pmod{m}$ if a and b give the same rest when divided by m [6].

E. Function Ladder

Function of f(x) = [x] is called the largest integer function and is defined by [10]:

$$f(x) = [[x]] = n$$
; for $n \le x < n+1$, with $x \in R$ and $n \in Z$.

As an example for x = 3.67 then:

$$f(x) = f(3.67) = [3.67] = 3$$
.

Because the function graph is a ladder, the largest integer function is also called ladder function [6].

F. Leap Year

The leap year or long year is the year with the age of one year is 366 days. The one year lifespan for the short year is 365 days. The addition of one day in leap year is done in February so the age of February will be 29 days [10].

The rules for determining a year number will be a leap year or not are [1]:

- 1. A year number will be a leap year if the year number is divisible by 4, except when the year number is divisible by 100 (century years).
- 2. The year of the century will be a leap year when the year number is divisible by 400.

Year numbers of 1700, 1800, 1900 and 2100 are not leap years. However, the years 1200, 1600 and 2000 are leap years.

The use of the Masehi Gregorian calendar (hereafter called the Masehi calendar and written M) by the Javanese society carries the naming of the day with two types of cycles. The first cycle is the sevenday cycle (called *saptawara*) and the second cycle is the five-day cycle (called *pancawara*). The names of days on the *saptawara* cycle are similar to commonly used day names ie Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and

Saturday. The names of the days in the *pancawara* cycle are *Legi (Manis)*, *Paing*, *Pon*, *Wage*, and *Kliwon* [4; 9].

Given the two day cycle types, the formula (1) produced by Rosen [10], will be rewritten with:

$$w_S \equiv (k + [[2.6 \cdot m - 0.2]] + Y - 2C + [[Y/4]] + [[C/4]]) \mod 7$$
 (4) Where:

 w_s : name of day on cycle seven daily ($w_s = 0, 1, 2, 3, 4, 5, 6$ respectively, for Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday).

k: date on the Masehi calendar

m: serial number of the month on the Masehi calendar (m = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 respectively for March, April, May,, December, January, and February).

N: the year numbers on the Masehi calendar with N = 100C + Y.

C: century number with $16 \le Y < 20$.

Y: number after a number of centuries with $0 \le Y \le 99$.

Index s at w_s declared saptawara. Using equation (4) it can be seen that March 1, 1949 fell on a saptawara Tuesday:

$$w_s = (k + [[2.6 \cdot m - 0.2]] - 2C + Y + [[Y/4]] + [[C/4]]) \mod 7$$

$$= (1 + [[2.6 \cdot 1 - 0.2]] - 38 + 49 + [[12.25]] + [[4.75]]) \mod 7$$

$$= (1 + 2 - 38 + 49 + 12 + 4) \mod 7$$

$$= (30) \mod 7$$

$$= 2 \text{ (Tuesday)}$$

Furthermore, following the steps taken by Rosen [10], a formula will be developed to determine the name of the *pancawara* day p on the Masehi calendar symbolized by w_p .

G. Derivation of Formula for W_n

With the revision of the MJ Calendar into the MG Calendar, the formula formulation is based on the beginning of the calculation on March 1, 1600 M. The election of March due to the addition of 1 day at the time of leap year is done in February so the age of February increases from 28 to 29. Given this rule, then values k, m, N, C, Y at (3) for February 23, 1971 was k = 23, m = 12, N = 1970, C = 19, Y = 70. For the record, for January and February, the year number N will decrease 1.

On a March 1, 1600 M calculation basis, for example $d_{p,N}$ declared the name of the *pancawara* day on March 1, of the year N. Between March 1, 1600 to March 1, of the year N apply:

1. If N not leap year then age 365 days. Therefore $365 \equiv 0 \mod 5$, then $d_{p,N} \equiv d_{p,N-1} + 0 \mod 5$ or $d_{p,N} \equiv d_{p,N-1}$.

2. If N leap year then the age is 366 days so $d_{p,N} \equiv d_{p,N-1} + 1 \mod 5$.

To determine the name of the day on March 1, year N, since March 1, 1600, must be calculated the number of leap years between 1600 to N (not including the year 1600, but including the year N).

- 1. There is [(N-1600)/4] pieces of leap year divisible by 4.
- 2. There is [(N-1600)/100] pieces of a leap year divisible by 100.
- 3. There is [(N-1600)/400] pieces of a leap year divisible by 400.

The number of leap years between 1600 and N is L, as follow:

$$L = [[(N-1600)/4]] - [[(N-1600)/100]] + [[(N-1600)/400]],$$

$$= [[N/4]] - [[N/100]] + [[N/400]] - 388.$$
From $N = 100C + Y$, obtained:
$$L = [[25C + (Y/4)]] - [[C + (Y/100)]] + [[(C/4) + (Y/400)]] - 388$$

$$= 25C + [[Y/4]] - C + [[C/4]] - 388$$

= 24C + [[C/4]] + [[Y/4]] - 388

Within a year of the Masehi calendar there will be 365 days for the regular year and 366 days for leap year. Thus, in the normal year there will be 73 times the *pancawara* cycles and within a leap year there will be 73 cycles of *pancawara* plus 1 day. This 1 day extra is called the rest of the day. Furthermore, within a century (100 years) there will be 4 days remaining. In 200 years there will be 3 days remaining and within 300 years there are 2 remaining days. Meanwhile, for 400 years the remaining amount of his day is 1, but since the year 400 is a leap year then the rest of the day becomes 2.

The formula for determining the name of the *pancawara* day on March 1, of the year, counted from March 1, 1600 is

In this match 1, 1000 is
$$d_{p,N} = d_{p,1600} + \text{number of leap years}(L)$$
 $d_{p,N} = (d_{p,1600} + 4C + [[C/4]] + [[Y/4]] - 3) \text{mod } 5$ (5) By using the date of March 1, 1949 which falls on a Tuesday-Pon will be used to determine the day of the pancawara on March 1, 1600. In this case it will be assigned Pon = 0, Wage = 1, Kliwon = 2, Legi (Manis) = 3 and Paing = 4.

$$\begin{split} d_{p,\mathrm{N}} = & \, d_{p,1949} = 0 = \left(d_{p,1600} + 4C + \left[\left[C/4 \right] \right] + \left[\left[Y/4 \right] \right] - 3 \right) \mathrm{mod} \, 5 \\ 0 = \left(d_{p,1600} + 4 \cdot 19 + \left[\left[4.75 \right] \right] + \left[\left[12.25 \right] \right] - 3 \right) \mathrm{mod} \, 5 \\ 0 = & \, d_{p,1600} + 89 \, \mathrm{mod} \, 5 \\ 0 = & \, d_{p,1600} + 4 \, \mathrm{mod} \, 5 \end{split}$$

Thus, obtained $d_{p,1600}=1 \mod 5=1$, meaning March 1, 1600 falls on the day of Wage in

pancawara. Therefore $d_{p,1600}=1$ then the formula (5) can be expressed by $d_{p,N}=\left(1+4C+\left[\left[C/4\right]\right]+\left[\left[Y/4\right]\right]-3\right)\operatorname{mod}5$, or

$$d_{p,N} = (4C + [[C/4]] + [[Y/4]] - 2) \mod 5$$
 (6)

Since this formulation begins March 1, 1600, until February 29, 1600 there will be 2 days' remaining, then equation (6) will be:

$$d_{p,N} = (4C + [[C/4]] + [[Y/4]]) \mod 5$$
 (7)

Formula (7) may be used to determine the name of the *pancawara* day on every March 1, of the year N. Next will be determined a formula to determine the name of the *pancawara* day on every 1st of any month of the year with $N \ge 1600$.

Table 1. Increase between months in the Masehi
Calendar

Calendar						
No	Month Period	Number	Mod 5	Increase		
		of days		Results		
1	1 March – 1 April	31	31 mod 5	1		
2	1 April – 1 May	30	30 mod 5	0		
3	1 May – 1 June	31	31 mod 5	1		
4	1 June – 1 July	30	30 mod 5	0		
5	1 July – 1 August	31	31 mod 5	1		
6	1 August – 1 September	31	31 mod 5	1		
7	1 September – 1 October	30	30 mod 5	0		
8	1 October – 1 November	31	31 mod 5	1		
9	1 November -1	30	30 mod 5	0		
	December					
10	1 December – 1 January	31	31 mod 5	1		
11	1 January – 1 February	31	31 mod 5	1		

Suppose m is the serial number of the month with $1 \le m \le 12$. March has serial number m = 1 and so on. January and February the serial number becomes 11 and 12. In the Masehi calendar, the number of days per month is 30 or 31. In modulo 5, $30 \equiv 0 \mod 5 = 0$ and $31 \equiv 1 \mod 5 = 1$. From Table 1, it can be seen the average increment between the months.

From the last column, the average increase is 7/11 = 0.64 = 0.6. By using the inspection described in Table 2, it can be obtained $I = [0.6 \cdot m - 0.2]$. The inspection of the results is given in Table 2.

In Table 2, from the row to m=1 can be determined b=0. Of all value intervals a, the maximum value on the left side is 0 and the minimum value on the right side is 0.2. Value a which meets the entire interval is $0 < a \le 0.2$. Next selected a=0.2 so that $[0.6 \cdot m-a] - b$ become:

$$I = [0.6 \cdot m - 0.2]. \tag{8}$$

(9)

In the same way, the other inspection results are $I = [0.6 \cdot m + 1.8] - 2$.

Serial Number of MonthsFormula $[[0,6\cdot m-a]]-b$ $a \in R, b \in Z^+$		Increase Results (Last Column of Table 1)	Value of a
1	[[0.6-a]]-b=0	1	$-0.4 < a \le 0.6$
2	[[1.2-a]]-b=1	▼ 1	$-0.8 < a \le 0.2$

Table 2. Inspection

3	[[1.8-a]]-b=1	0	$-0.2 < a \le 0.8$
4	[[2.4-a]]-b=2	1	$-0.6 < a \le 0.4$
5	[[3.0-a]]-b=2	0	$0 < a \le 1$
6	[[3.6-a]]-b=3	1	$-0.4 < a \le 0.6$
7	[[4.2-a]]-b=4	1	$-0.8 < a \le 0.2$
8	[[4.8-a]]-b=4	0	$-0.2 < a \le 0.8$
9	[[5.4-a]]-b=5	1	$-0.6 < a \le 0.4$
10	[[6.0-a]]-b=5	0	$0 < a \le 1$
11	[[6.6-a]]-b=6	1	$-0.4 < a \le 0.6$
12	[[7.2-a]]-b=7	1	$-0.8 < a \le 0.2$

By using equation (8), the name of the day of pancawara on the 1st of the month m year N is the smallest positive residue of:

$$d_{p,m,N} = \left(d_{p,N} + \left[\left[0.6 \cdot m - 0.2\right]\right]\right) \mod 5$$

$$d_{p,m,N} = \left(4C + \left[\left[C/4\right]\right] + \left[\left[Y/4\right]\right] + \left[\left[0.6 \cdot m - 0.2\right]\right]\right) \mod 5 \quad (10)$$
Next, the name of the *pancawara* day on the date k , month m , year N , symbolized by w_p obtained by adding $(k+2)$ in equation (10). Added with $(k+2)$ because there are 2 remaining days for the years after 1600 M.

$$w_p = w_{p,k,m,N} = d_{p,m,N} + (k+2)$$

$$w_D = (k + [0.6 \cdot m - 0.2] + 4C + [Y/4] + [C/4] + 2) \mod 5$$
 (11)

Another result that can be obtained is by using equation (9). By using equation (9), the name of the day of *pancawara* on the 1st of the month m year N is the smallest positive residue of:

$$d_{p,m,N} = \left(d_{p,N} + \left[\left[0.6 \cdot m + 1.8\right]\right] - 2\right) \mod 5$$

$$d_{p,m,N} = \left(4C + \left[\left[C/4\right]\right] + \left[\left[Y/4\right]\right] + \left[\left[0.6 \cdot m + 1.8\right]\right] - 2\right) \mod 5 \quad (12)$$
Next, the name of the *pancawara* day on the date k , month m , year N , symbolized by w_p obtained by adding $(k+2)$ on the equation (12).

$$w_p = w_{p,k.m,N} = d_{p,m,N} + (k+2),$$

$$w_p \equiv (k + [[0.6 \cdot m + 1.8]] + 4C + [[Y/4]] + [[C/4]]) \mod 5$$
 (13)

The following is an example of determining the day of the *pancawara* for some selected dates: July 8, 1633 (The Birth of the Java Calendar). June 1, 1945 (Birthday of Pancasila), and August 17, 1945 (Indonesia Independence) and other dates. Lines 1 and 2 use $w_p = 0,1,2,3,4$ for *Pon, Wage, Kliwon, Legi (Manis), Paing*, calculated by equations (11) and (13). Meanwhile, line 3 uses w = 0,1,2,3,4,5,6 for Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday, calculated by equation (4). The results of the examples of determining the *pancawara* days are given in Table 3.

Table 3. Name of *pancawara* day based on the four formulas obtained

No	1 March 1600	8 July 1633	1 June 1945	17 August 1945	1 March 1949	1 March 1982
1	71 = 1	88 = 3	96 = 1	113 = 3	95 = 0	103 = 3
	Wage	Legi	Wage	Legi	Pon	Legi
2	71 = 1	88 = 3	96 = 1	113 = 3	95 = 0	103 = 3
	Wage	Legi	Wage	Legi	Pon	Legi
3	-25 = 3	33 = 5	33 = 5	54 = 5	30 = 2	71 = 1
	Wednesday	Friday	Friday	Friday	Tuesday	Monday

IV. CONCLUSION

In this paper has been determined day pancawara in the Masehi calendar. Based on the discussion, two alternative formulas are given to determine the name of the day of the pancawara, completing the two formulas that have been obtained by Kurnia et al, [4] and Prabowo et al. [9]. Of course there are many more similar formulas that can be raised. For the record, the formulas obtained can only be used for the determination of the names of days on or after March 1, 1600 to March 1, 2000. Therefore, it is advisable to establish similar models for both saptawara and pancawara dates from March 1, 2000 until March 1, 2400 or before March 1, 1600. Other suggestions that can be advanced is to construct a model of determining the day saptawara and pancawara day referring to the Hijri calendar and the Java calendar.

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