Some EP-cordial Graphs

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1. Abstract: *In EP cordial labeling first time the number -1 was also considered in images of labelling function. We show that Tail graph of C_3, $FL(C_3)^{(k)}$, $FL(C_4)^{(k)}$, <K1:n:K1>,kite(3,m),One point union of k-copies of kite(3,2) are EP cordial.*

Key words : *EP cordial, kite, tailgraph, Flag.*

Subject Classification: 05C78

2. Introduction: The graphs considered are finite,simple and undirected**.**We refer Dynamic survey of graph labeling by Gallian[] and Harary[] for definitions and terminology.Sundaram, Ponraj, and Somasundaram [] introduced the notion of EP-cordial labeling (extended product cordial) labeling of a graph G as a function f from the verticies of a graph to {0,1,-1} such that if each edge uv is assigned the label f(u)f(v),then $|v_f(i)-v_f(j)|\leq 1$ and $e_f(i)-e_f(j)|\leq 1$ where i,j϶{0,1,-1} and $v_f(k)$ and ef(k) denote the number of vertices and edges respectively labeled with k. An EP-cordial graph is one that admits an EP-cordial labeling.To know the work done sofar in this labeling one should refer J.Gallian[]

We use new terminology as $v_f(0,1,-1) = (a,b,c)$.That mean number vertices with label 0 are a in number ,with label 1 are b in number and that with label -1 are c in number. Similarly for $e_f(0,1,-1)=(a,b,c)$ means number edges with label 0 are a in number ,with label 1 are b in number and that with label -1 are c in number.

3. Definitions:

3.1defination : tail(G,m,t).It is a graph with a path of length m attached to some vertex of Gand end vertex of degree t.(t>2).If G is (p,q) graph then p=q and Tail(G,m,t) has p+m+t-1vertices and p+m+t-1 edges.

3.2Definition:kite (n, t) is a cycle of length n witha t-edge path attached to one vertex.

Theorem 4.1 Tail graph tail(C_3,m,3)of C_3 is EP cordial.

Proof: Legraph G= tail(C_3,m,3). It has m+ 5 vertices and m+5 edges.For m=1,2 the figures below gives EP-labelingscheme.We give ordinary labeling to G as follows .The two pendent vertices be u_1,u_2 and the consecutive vertices on path P_{m+1} be the 3-degree vertex be u_3 then consecutively $u_4,u_5,..u_{m+2}$,and three vertices on cycle are u_{m+3},u_{m+4},u_{m+5}

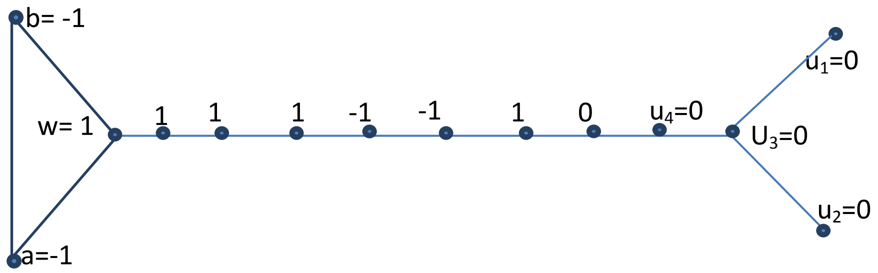


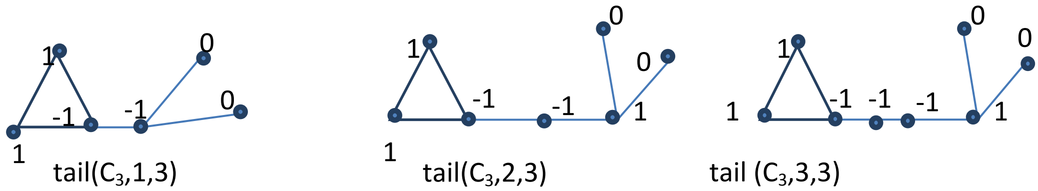
Fig 3.1 :Tail(C_3,9,3):with EP cordial labeling.$v_f(0,1,-1) = (5,5,4)=e_f(0,1,-1)$

Define f:V(G)→{0,1,-1} as

Case 1 p≡0(mod 3).Take p = 3x we must get $v_f(0,1,-1) = (x,x,x)=e_f(0,1,-1)$ subcase x is even number $f(u_i)=0$ for i = 1,2,..x.$f(u_{m+4})=f(u_{m+5})=-1$.$f(u_{x+i})=1$ for i ≡1(mod 3) and i ≤3y , (2y = x-2). $f(u_{x+i})=-1$ for i ≡2,0(mod 3) and i ≤3y.$f(u_{x+i})=1$ for 3y<i ≤(q-3-x-3y).This gives the desired result $v_f(0,1,-1) = (x,x,x)=e_f(0,1,-1)$ subcase x is oddnumber $f(u_i)=0$ for i = 1,2,..x.$f(u_{m+4})=f(u_{m+5})=-1$$f(u_{x+i})=-1$ for i ≡1,0(mod 3) and i ≤3y+1.(2y = x-3) $f(u_{x+i})=1$ for i ≡2(mod 3) and i ≤3y.$f(u_{x+i})=1$ for 3y+1<i ≤(q-3-x-3y).This gives the desired result $v_f(0,1,-1) = (x,x,x)=e_f(0,1,-1)$ case 2 p≡1(mod3) subcase x is even number $f(u_i)=0$ for i = 1, 2,.., x, x+1 $f(u_{m+4})=f(u_{m+5})=-1$.$f(u_{x+1+i})=1$ for i ≡1(mod 3) and i ≤3y , (2y = x-2). $f(u_{x+1+i})=-1$ for i ≡2,0(mod 3) and i ≤3y.$f(u_{x+i})=1$ for 3y<i ≤(q-3-x-3y).This

gives the desired result $v_f(0,1,-1) = (x+1,x,x)=e_f(0,1,-1)$ subcase x is odd number $f(u_i)=0$ for i = 1, 2, .. x, x+1.$f(u_{m+4})=f(u_{m+5})=-1$.$f(u_{x+2})=-1$,$f(u_{x+2+i})=1$ for $i \equiv 1 \pmod 3$ and $i \le 3y$, (2y = x-3). $f(u_{x+2+i})=-1$ for $i \equiv 2,0 \pmod 3$ and $i \le 3y$.$f(u_{x+2+i})=1$ for $3y<i \le (q-3-x-3y)$.This gives the desired result $v_f(0,1,-1) = (x+1,x,x)=e_f(0,1,-1)$ case 3 $p \equiv 2 \pmod 3$

subcase x is even number$f(u_i)=0$ for i = 1,2,..,x,x+1..$f(u_{m+4})=f(u_{m+5})=-1$.$f(u_{x+1+i})=1$ for $i \equiv 1 \pmod 3$ and $i \le 3y$, (2y = x-2). $f(u_{x+1+i})=-1$ for $i \equiv 2,0 \pmod 3$ and $i \le 3y$.$f(u_{x+i})=1$ for $3y<i \le (q-3-x-3y)$.This gives the desired result $v_f(0,1,-1) = (x+1,x+1,x)=e_f(0,1,-1)$ subcase x is odd number $f(u_i)=0$ for i = 1,2,..x,x+1.$f(u_{m+4})=f(u_{m+5})=-1$.$f(u_{x+2})=-1$,$f(u_{x+2+i})=1$ for $i \equiv 1 \pmod 3$ and $i \le 3y$, (2y = x-3). $f(u_{x+2+i})=-1$ for $i \equiv 2,0 \pmod 3$ and $i \le 3y$.$f(u_{x+2+i})=1$ for $3y<i \le (q-4-x-3y)$.This gives the desired result $v_f(0,1,-1) = (x+1,x+1,x)=e_f(0,1,-1)$



Theorem 4.2: One point union of k- copies of C_3 flag i.e.$FL(C_3)^{(k)}$ is EP cordial.**Proof.**For k= 1,K=2 and k=3 we draw separate figures. By suitable combination on these

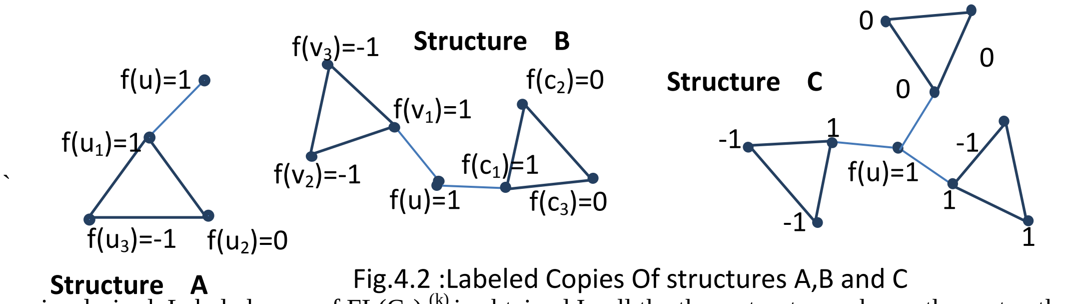


Fig.4.2 :Labeled Copies Of structures A,B and C

copies desired Labeled copy of $FL(C_3)^{(k)}$ is obtained.In all the three structures above the vertex that will be common to all copies of $FL(C_3)$ is u and has label 1 under $f : V(G) \to \{1,0\}$.The table below gives scheme of obtaining $FL(C_3)^{(k)}$ for different values of k.

k	structure to use	v(0,1,-1)	e(0,1,-1)
1	A	V(1,2,1)	e(2,1,1)
2	B	v(2,3,2)	e(3,3,2)
3	c	v(3,4,3)	e(4,4,4)
3x	x times C	v(3x,3x+1,3x)	e(4x,4x,4x)
3x+1	x times C + A	v(3x+1,3x+2,3x+1)	e(4x+2,4x+1,4x+1)
3x+2	x times C + B	v(3x+2,3x+3,3x+2)	e(4x+3,4x+3,4x+2)
v(a,b,c) indicates number vertices with label 0 are a in number ,with label 1 are b in number and that with label -1 are c in number.Similar understanding for e(a,b,c) on edges.			

Fig.4.3

Theorem 4.3 One point union of k- copies of C_4 flag i.e.$FL(C_4)^{(k)}$ is EP cordial. **Proof:** For k= 1,K=2 and k=3 we draw separet figures. By suitable combination on these copies desired Labeled copy of $FL(C_4)^{(k)}$ is obtained.

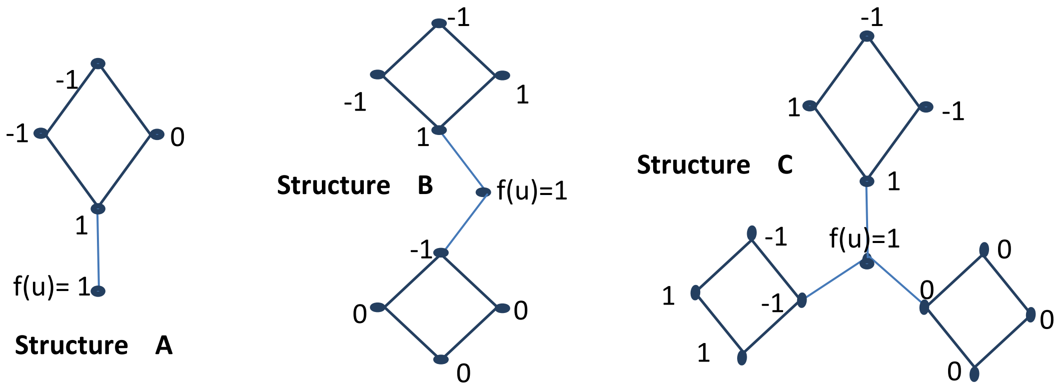


fig 4.5 : EP labeling of $FL(C_4)^{(k)}$ when k =1,2,3.These 3 structures are building blocks of $FL(C_4)^{(k)}$

In all the three structures above the vertex that will be common to all copies of $FL(C_3)$ is u with f(u)=1 under f :V(G) →{1,0}.The table below gives scheme of obtaining $FL(C_4)^{(k)}$ for different values of k.

k	structure to use	v(0,1,-1)	e(0,1,-1)
1	A	V(1,2,2)	e(2,2,1)
2	B	v(3,3,3)	e(4,3,3)
3	C	v(4,5,4)	e(5,5,5)
3x	x times C	v(4x,4x+1,4x)	e(5x,5x,5x)
3x+1	x times c + A	v(4x+1,4x+2,4x+2)	e(4x+2,4x+24x+1)
3x+2	x times C + B	v(4x+3,4x+3,4x+3)	e(4x+4,4x+3,4x+3)
v(a,b,c) indicates number vertices with label 0 are a in number ,with label 1 are b in number and that with label -1 are c in number.Similar understanding for e(a,b,c) on edges.			
Fig.4.4			

Theorem 4.4 G=<K1:n:K1:n>is EP cordial.

Proof. Of the three verticesu,v and w on G , v and w are central vertices of $K_{1,n}$.These two are joined to vertex vby a leaf each of $K_{1,n}$.The vertices incident to u be given by $u_1,u_2,..,u_{n-1}$. And that incident to w be $w_1,w_2,w_2,w_3...,w_{n-1}$.Note that |V(G)| = 2n+1 and |E(G)| =2n .Define a function f:V(G)→{1,0} as
f(u)=f(v)=f(w)=1 **case 1:**2n-2 is of type 3x
f(ui) =0 for i = 1,..x+1,f(ui) = -1 for i= x+2,,,n-1 f(vi) = 1 for i = 1,2,..,x-2,f(vi) = -1 for i =x-1, x,x+1,..,n-1 This results in number distribution on vertices as $v_f(0,1,-1)=(x+1,x+1,x+1)$ and numbers on edges are $e_f(0,1,-1)=(x+1,x,x+1)$

case 2 When 2n-2 is of the type 3x+1.

f(ui) =0 for i = 1,..x+1,f(ui) = -1 for i= x+2,,,n-1.

f(vi) = 1 for i = 1,2,..,x-1,f(vi) = -1 for i = x,x+1,..,n-1 This results in number distribution on vertices as $v_f(0,1,-1)=(x+1,x+2,x+1)$ and numbers on edges are $e_f(0,1,-1)=(x+1,x+2,x+1)$ **case 3** When 2n-2 is of the type 3x+2.
f(ui) =0 for i = 1,..x+2,f(ui) = -1 for i= x+3,,,n-1, f(vi) = 1 for i = 1,2,..,x-1,f(vi) = -1 for i = x,x+1,..,n-1 This results in number distribution on vertices as vf(0,1,-1)=(x+2,x+2,x+1) and numbers on edges aree$_f$(0,1,-1)=(x+2,x+1,x+1) #

Theorem 4.5.Let G' be the kite-(3,2).The one point union of k copies of G'i.e.$G=(G')^{(k)}$ is EP cordial graph.

Proof:G' be a C_3 attached with a 2-edge path at one vertex.We design three structures A,B and C.These are used suitably to obtain G for any value of k.The common point is the vertex u.

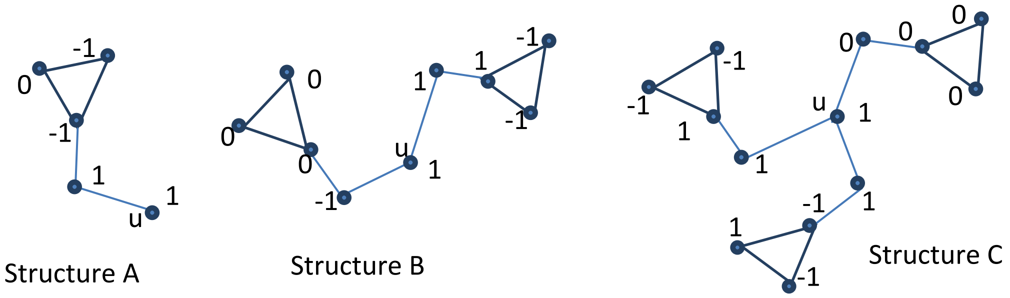


Fig 5.2 : Structure A,B,C with EP Cordial Labels

k	structure to use	v(0,1,-1)	e(0,1,-1)
1	A	V(1,2,2)	e(2,2,1)
2	B	v(3,3,3)	e(4,3,3)
3	C	v(4,5,4)	e(5,5,5)
3x	x times C	v(4x,4x+1,4x)	e(5x,5x,5x)
3x+1	x times c + A	v(4x+1,4x+2,4x+2)	e(4x+2,4x+24x+1)
3x+2	x times C + B	v(4x+3,4x+3,4x+3)	e(4x+4,4x+3,4x+3)
v(a,b,c) indicates number vertices with label 0 are a in number ,with label 1 are b in number and that with label -1 are c in number.Similar understanding for e(a,b,c) on edges.			
Fig.4.4			

The tabel above eleborates how to obtain a EP-cordial copy of $(G')^{(k)}$where G' is the kite-(3,2).

Theorem 4.6kite(3,m) is EP-cordial.

Proof: kite(3,m) has a cycle C3 attached at one point with a path of length m.It has m+3 vertices and m+3 edges. For m>2 Let the consecutive vertices on path from pendent vertex be $v_1,v_2,..v_m,v_{m+1}$.Note that v_{m+1} is degree 3 vertex common to C_3 and p_{m+1}.The two degree vertices on C_3 are u and v respectively.

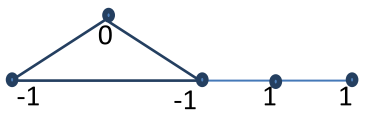


Fig 1.2.$v_f(0,1,-1)=(1,2,2),e_f(0,1,-1)=(2,2,1)$

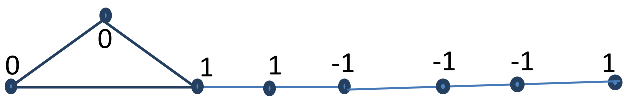


Fig 1.3.$v_f(0,1,-1)=(2,3,3),e_f(0,1,-1)=(3,3,2)$

case 1 m = 3x.**subcase x is even**.Take x-2 =2y
f(u)=f(v)=-1 $f(v_1)$=1,f(vi)=0 for i = 2,3,..,x+1,$f(v_{x+i+2})$ = -1 for i ≡2,3(mod3) and i≤3y.$f(v_{x+2+i})$ = 1 for i≡0 (mod 3) and i≤3y. The numbers on vertices and edges are $v_f(0,1,-1)=(x,x,x),e_f(0,1,-1)=(x+1,x,x)$

subcase 2 x is odd f(u)=f(v)=-1 $f(v_1)$=1,f(vi)=0 for i = 2,3,..,x+1,$f(v_{x+1+i})$= -1 for i≡0,2(mod 3)and i≤3y,where 2y = x-1,$f(v_{x+1+i})$=1for ≡1(mod 3).and i≤3y,f(v1+x+i)=1for i = 3y+1,3y+2,..,m+1

The numbers on vertices and edges are $v_f(0,1,-1)=(x,x+1,x+1)$, $e_f(0,1,-1)=(x+1,x+1,x+1)$ **case 2** m = 3x+1

f(u)=f(v)=-1 $f(v_1)$=1 **subcase x** is even take x-2 = 2y.

$f(v_{1+i})$=0 for i = 1,2,..x.$f(v_{1+x+i})$=-1 for i≡0,2(mod 3)and i ≤3y $f(v_{i+x+1})$ = 1for i≡1(mod 3)and i ≤ 3y.Further $f(v_{1+x+i})$=1 for i= 3y+1,3y+2,..,m+1. The numbers on vertices and edges are $v_f(0,1,-1)=(x,x+1,x)$, $e_f(0,1,-1)=(x+1,x+1,x)$ subcase x is odd.take 2y = x-1, f(u)=f(v)=-1 $f(v_1)$=1,$f(v_{1+i})$=0 for i=1,2,3,..,x. $f(v_{x+1+i})$ =-1 for i≡0,2(mod 3) and i≤3y. f(vi+x+1) =1 for i≤3y.f(vx+i)=1 for i = 3y+1,3y=2,...,m+1. The numbers on vertices and edges are $v_f(0,1,-1)=(x,x,x+1)$, $e_f(0,1,-1)=(x+1,x,x)$.Thus fis EP cordial function.

case 3 m = 2+3x.We give diagrams for x = 1,2..Define F : V(G)→{0,1} as follows. f(u)=f(v)=-1
$f(v_1)$=1 **Subcase**1:x is even.take$\frac{x}{2}$ =y.
f(vi)=0 for fori = 2,3,...,x $f(v_{x+i+2})$=-1 for i ≡o,1(mod3)and i<3yand $f(v_{x+2+i})$=-1for i = 3y+2,3y+3,3y+4

$f(v_{x+2+i}) = 1$ for $i \equiv 2 (mod3)$ and $i \leq 3y+1$.$f(vi) = 1$ for $i > 3y=4$ and $i \leq m+1$ $v_f(0,1,-1)=(x,x+1,x+1)$, $e_f(0,1,-1)=(x+1,x+1,x)$ **subcase** 2:x is odd.Take x-1 =2y. $f(vi) = 0$ for $i = 2,3,..,x+1$. $f(v_{x+2+i})=-1$ for $i \equiv 0,1(mod 3)$.and $i \leq 3y$.$f(v_{x+2+i})=1$for $i=3y+1,3y+2,..m+1$.The numbers on vertices and edges are $v_f(0,1,-1)=(x,x+1,x+1)$, $e_f(0,1,-1)=(x+1,x,x+1)$

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