Fuzzy Inventory Model for Deteriorating Items with Exponentially Decreasing Demand under Fuzzified Cost and Partial Backlogging

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Abstract

Due to technological advancement and business competition, the demand of electronic commodity and software products decreases with time. So in the present paper, we develop an inventory model for deteriorating items with constant deterioration and exponentially decreasing demand. Shortages are allowed in the model and are partially backlogged. This model also considers fuzzy based cost components (holding cost, shortage cost etc.) and deterioration. All related costs are assumed to be trapezoidal fuzzy numbers. Here signed distance and gradient mean integration method is used for defuzzification. We provide simple analytical tractable procedure for optimal inventory replenishment policy of the model and give numerical examples to illustrate the result. Sensitivity analysis of the major parameters with respect to the optimal solution is also carried out. This paper provides an interesting topic for further study, such that the joint influence from some of these parameters may be investigated to show the effects.

Keywords: *Graded Mean Integration, Signed Distance Method, Partial Backlogging.*

1. INTRODUCTION

In inventory management, many researchers have studied inventory models for deteriorating items such as medicines, electronic components, software products and fashion goods. In formulating inventory models, there are two facts of problem, one being the deterioration of items, the other being the variation in the demand rate. In 1915, the first inventory model was developed by F. Harris .Dave and Patel [1981] who derived a lot size model for constant deterioration of items with time proportional demand. Sachan [1984] allowed shortages in Dave and Patel [1981]'s model.

In certain situations uncertainties are due to fuzziness, and such cases are dilated in the fuzzy set theory which was demonstrated by Zadehin[1965]. Kaufmann and Gupta[1991] provided an introduction to fuzzy arithmetic operation and Zimmermann[1985] discussed the concept of the fuzzy set theory and its applications. Park [1987] applied the fuzzy set concepts to EOQ formula by representing the inventory carrying cost with a fuzzy number and solved the economic order quantity model using fuzzy number operations based on the extension principle. Vujosevic et al.[1996] used trapezoidal fuzzy number to fuzzify the order cost in the total cost of the inventory model without backorder, and got fuzzy total cost. Yao and Lee [1996] introduced a backorder inventory model with fuzzy order quantity as triangular and trapezoidal fuzzy numbers and shortage cost as a crisp parameter. Gen et al.[1997] expressed their input data as fuzzy numbers, and then the interval mean value concept was introduced to solve the inventory problem.In 2007, J. K. Syed and L. A. Aziz [2005] applied signed distance method to Fuzzy inventory model without shortages. P. K. Tripathy et al.[2008] developed an entropic order quantity model with fuzzy holding cost and fuzzy disposal cost for perishable items. In 2011, P. K. De and A. Rawat proposed a fuzzy inventory model without shortages using triangular fuzzy number. In 2012, C. K. Jaggi, S. Pareek, A. Sharma and Nidhi presented a fuzzy inventory model for deteriorating items with time-varying demand and shortages. In 2012, SumanaSaha and TriptiChakrabarti proposed a fuzzy EOQ model for time dependent deteriorating items and time dependent demand with shortages. Very recently, D. Dutta and Pavan Kumar published several papers in the area of fuzzy inventory with or with shortages. In 2012, D. Datt et al. presented a fuzzy inventory model without shortage using trapezoidal fuzzy number with sensitivity analysis. Narendra et al.[2016] proposed fuzzy inventory model with exponential demand and time-varying deterioration.

In this paper, we first consider a crisp inventory model with constant deteriorating items with negative exponential demand where shortages are allowed with fully backlogged condition. Thereafter we develop the corresponding fuzzy inventory model for fuzzy deteriorating items with fuzzy demand rate under fully backlogging. The average total inventory cost in fuzzy sense is derived. All inventory parameters including deterioration rate are fuzzified as the trapezoidal fuzzy numbers. The fuzzy model is defuzzified by using the signed distance and graded mean integration method. The solution for minimizing the fuzzy cost function has been derived.

2. Basic concept

A fuzzy set $\tilde{A}$ on the given universal set X is a set of order pairs $\tilde{A} = \{(x,\ \mu_A(x)) : x \in X\}$

Where $\mu_{\tilde{A}} : X \to [0,\ 1]$ is called membership function.

The α -cut of $\tilde{A}$ is defined by

$A_\alpha = \{x : \mu_A(x) = \alpha, \alpha \geq 0\}$

If R is a real line, then a fuzzy number is a fuzzy set $\tilde{A}$ with membership function $\mu_{\tilde{A}} : X \to [0,\ 1]$, having the following properties

(i) $\tilde{A}$ is normal i.e. there exist $x \in R$ such that $\mu_A(x) = 1$

(ii) $\tilde{A}$ is piecewise continuous.

(iii) $\sup p(\tilde{A}) = cl\{x \in R : \mu_A(x) > 0\}$, where cl represents the closure of a set.

(iv) $\tilde{A}$ is a convex fuzzy set.

3. Assumptions and Notation

This inventory model is developed on the basis of the following assumptions and notation:

a) $\theta(t) = \theta$ is the constant rate of deterioration where $0 < \theta < 1$.

b) $D(t) = ae^{-bt}$ where $a,\ b > 0$ is exponentially decreasing demand.

c) Shortage is allowed and partially backlogged.

d) l is the shortage cost per unit per unit time.

e) β is the backlogging rate; $0 \leq \beta \leq 1$.

f) Replenishment is instantaneous; lead time is zero.

g) T is the length of the cycle.

h) A is the order quantity.

i) h is the holding cost per unit time.

j) C is the unit cost of an item.

k) S is the lost sale cost per unit.

l) $TC(t_1, T)$ is the total inventory cost per unit time.

m) $\tilde{D}$ is the fuzzy demand.

n) $\tilde{C}$ is the fuzzy unit cost of an item.

o) $\tilde{\theta}$ is the fuzzy deterioration rate.

p) $\tilde{h}$ is the fuzzy holding cost per unit per unit time.

q) $\tilde{l}$ is the fuzzy shortage cost per unit time.

r) $\tilde{S}$ is the fuzzy lost sale cost per unit.

s) $\tilde{TC}(t_1, T)$ is the total fuzzy inventory cost per unit time.

t) $TC_G(t_1, T)$ is the defuzzify value of $\tilde{TC}(t_1, T)$ by applying Graded Mean Integration.

u) $TC_S(t_1, T)$ is the defuzzify value of $\tilde{TC}(t_1, T)$ by applying Signed Distance Method.

4. Mathematical Model

4.1 Crisp Model

Under above assumption, the behaviour of inventory system at any instant of time is exhibited in Figure 1.

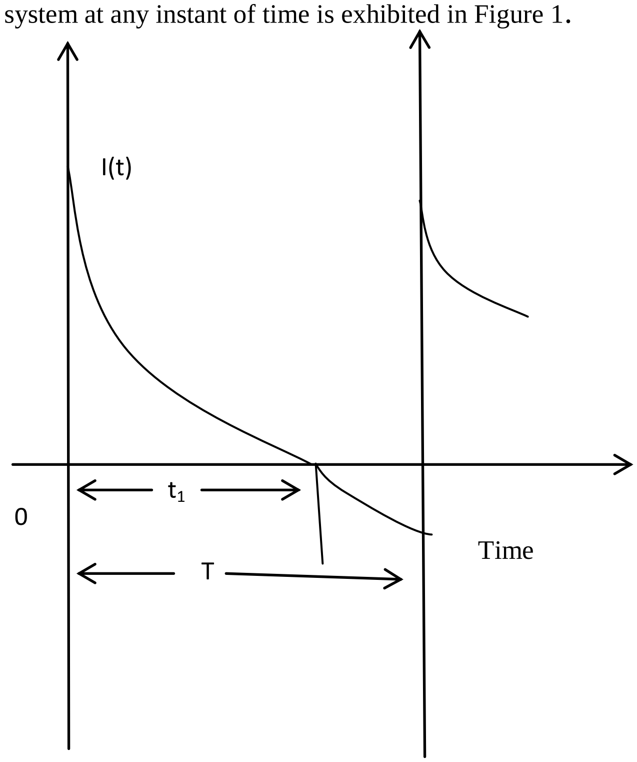

Figure 1: Graphical Representation of Inventory Model (Inventory level I(t) vs. time)

Replenishment is made at time t=0 and the inventory level falls during the period [0, t_1]. The inventory level is zero at t_1. So shortages are allowed during the time interval [t_1,T] and demand during this period is partially backlogged.

The rate of change of the inventory during the positive stock period $(0,t_1)$ and shortage period $(t_{1,} T)$ is represented by the following differential equations:

$$\frac{dI_1(t)}{dt}+\theta(t)I_1(t)=-D(t),\ 0\le t\le t_1 \qquad (1)$$

$$\frac{dI_1(t)}{dt}=-D(t), \qquad t_1\le t\le T \qquad (2)$$

The initial inventory level is I_0 unit at time $t=0$; from $t=0$ to $t=t_1$, at this time, shortage is accumulated which is partially backlogged at the rate β. Thus, boundary conditions are as follows:

$I_1(0)=I_0$, $\quad I_1(t_1)=0$, $\quad I_2(t_1)=0$.

The solutions of equation (1) and (2) with boundary conditions are as follows

$$I_1(t)=ae^{-\theta t}\int_t^{t_1} e^{(\theta-b)t}dt,\ 0\le t\le t_1 \qquad (3)$$

$$I_2(t)=-\beta a\left[\left(T-t_1\right)-\tfrac{b}{2}\left(T^2-t_1{}^2\right)\right] \qquad (4)$$

Using equation (3), we get the following

$$I_1(0)=I_0=a\int_0^{t_1} e^{(\theta-b)t}dt \qquad (5)$$

Inventory is available in the system during the time interval $(0,t_1)$. Hence, the cost for holding inventory in stock is computed for time period $(0,t_1)$ only.

Holding cost is as follows:

$$HC=\int_0^{t_1} h(t)I_1(t)dt$$

$$HC=ah\left[\tfrac{1}{2}t_1{}^2+\tfrac{\theta}{6}t_1{}^3-\tfrac{b}{3}t_1{}^3+\tfrac{b\theta}{8}t_1{}^4\right] \qquad (6)$$

Shortage due to stock out is accumulated in the system during the interval $(t_{1,} T)$.The optimum level of shortage is present at $t=T$; therefore, the total shortage cost during this time period is as follows

$$SC=l\int_{t_1}^{T}-I_2(t)dt$$

$$=\beta al(T-t_1)^2-\tfrac{1}{2}\beta abl(T-t_1)^2(T+t_1) \qquad (7)$$

Due to stock out during $(t_{1,} T)$, shortage is accumulated but not all customers are willing to wait for the next lot size to arrive. Hence, this results in some loss of sale which accounts to loss in profit.Lost sale cost is calculated as follows:

$$LSC=S\int_{t_1}^{T}(1-\beta)D(t)dt$$

$$=S(1-\beta)[a(T-t_1)-\tfrac{ab}{2}(T^2-t_1{}^2) \qquad (8)$$

Purchase cost is as follows:

$$PC=C(I_0+\int_{t_1}^{T}\beta D(t)dt)$$

$$=Ca(t_1+\tfrac{\theta-b}{2}t_1{}^2)+\beta aC(T-t_1)-\tfrac{1}{2}\beta abC(T^2-t_1{}^2)$$
(9)

Thus, total cost is as follows:

$$TC(t_1,T)=OC+PC+HC+SC+LSC$$

$$\Rightarrow TC(t_1,T)=\begin{bmatrix} A+Ca(t_1+\tfrac{\theta-b}{2}t_1{}^2)+\beta aC(T-t_1)-\tfrac{1}{2}\beta abC \\ (T^2-t_1{}^2)+ah\left(\tfrac{1}{2}t_1{}^2+\tfrac{\theta-2b}{6}t_1{}^3+\tfrac{b\theta}{8}t_1{}^4\right)+\beta al \\ (T-t_1)^2-\tfrac{1}{2}\beta abl(T-t_1)^2(T+t_1)+S(1-\beta) \\ [a(T-t_1)-\tfrac{ab}{2}(T^2-t_1{}^2)] \end{bmatrix}$$
(10)

To minimize the total cost $TC(t_1,T)$ per unit time, the optimal value of T and t1 can be obtained by solving the following equations

$$\frac{\partial TC}{\partial t_1}=0 \text{ and } \frac{\partial TC}{\partial T}=0 \qquad (11)$$

Solving equation (11) we get

$$\Rightarrow \begin{bmatrix} Ca[1+(\theta-b)t_1]-\beta aC+\beta abCt_1)+ \\ ah\left(t_1+\frac{\theta-2b}{2}t_1{}^2+\frac{b\theta}{2}t_1{}^3\right)-2\beta al(T-t_1)+ \\ \frac{1}{2}\beta abl(T-t_1)^2+Sa(1-\beta)(-1+bt_1) \end{bmatrix}=0$$

(12)

$$\Rightarrow \beta aC-\beta abCT+2\beta al(T-t_1)-\tfrac{3}{2}\beta abl(T-t_1)^2$$
$$+Sa(1-\beta)(1-bT)=0$$

(13)

4.2 Fuzzy Model

Let $\tilde{a}=(a_1,a_2,a_3,a_4),\tilde{b}=(b_1,b_2,b_3,b_4),$ $\tilde{h}=(h_1,h_2,h_3,h_4),\tilde{C}=(C_1,C_2,C_3,C_4)$ $\tilde{l}=(l_1,l_2,l_3,l_4),\tilde{S}=(S_1,S_2,S_3,S_4)$ are trapezoidal fuzzy number**.**

Total cost of the system in fuzzy sense is given by

$$\Rightarrow \tilde{TC}(t_1,T)=\begin{bmatrix} A+\tilde{C}\tilde{a}(t_1+\frac{\tilde{\theta}-\tilde{b}}{2}t_1{}^2)+\beta\tilde{a}\tilde{C}(T-t_1) \\ -\frac{1}{2}\beta\tilde{a}\tilde{b}\tilde{C}(T^2-t_1{}^2)+\tilde{a}\tilde{h} \\ \left(\frac{1}{2}t_1{}^2+\frac{\tilde{\theta}-2\tilde{b}}{6}t_1{}^3+\frac{\tilde{b}\tilde{\theta}}{8}t_1{}^4\right)+\beta\tilde{a}\tilde{l}(T-t_1)^2 \\ -\frac{1}{2}\beta\tilde{a}\tilde{b}\tilde{l}(T-t_1)^2(T+t_1) \\ +\tilde{S}(1-\beta)[\tilde{a}(T-t_1)-\frac{\tilde{a}\tilde{b}}{2}(T^2-t_1{}^2)] \end{bmatrix}$$

(14)

(i) By Graded Mean Integration, total cost is given by

$$TC_G(t_1,T)=\frac{1}{6}\begin{pmatrix} TC_{G_1}(t_1,T)+2TC_{G_2}(t_1,T)+ \\ 2TC_{G_3}(t_1,T)+TC_{G_4}(t_1,T) \end{pmatrix}$$

(15)

Where

$$\Rightarrow TC_{G_1}(t_1,T)=\begin{bmatrix} A+C_1a_1(t_1+\frac{\theta_1-b_1}{2}t_1{}^2)+\beta a_1C_1(T-t_1)-\frac{1}{2} \\ \beta a_1b_1C_1(T^2-t_1{}^2)+a_1h_1\left(\frac{1}{2}t_1{}^2+\frac{\theta_1-2b_1}{6}t_1{}^3+\frac{b_1\theta_1}{8}t_1{}^4\right) \\ +\beta a_1l_1(T-t_1)^2-\frac{1}{2}\beta a_1b_1l_1(T-t_1)^2(T+t_1) \\ +S_1(1-\beta)[a_1(T-t_1)-\frac{a_1b_1}{2}(T^2-t_1{}^2)] \end{bmatrix}$$

$$\Rightarrow TC_{G_2}(t_1,T)=\begin{bmatrix} A+C_2a_2(t_1+\frac{\theta_2-b_2}{2}t_1{}^2)+\beta a_2C_2(T-t_1)-\frac{1}{2}\beta a_2 \\ b_2C_2(T^2-t_1{}^2)+a_2h_2\left(\frac{1}{2}t_1{}^2+\frac{\theta_2-2b_2}{6}t_1{}^3+\frac{b_2\theta_2}{8}t_1{}^4\right) \\ +\beta a_2l_2(T-t_1)^2-\frac{1}{2}\beta a_2b_2l_2(T-t_1)^2(T+t_1) \\ +S_2(1-\beta)[a_2(T-t_1)-\frac{a_2b_2}{2}(T^2-t_1{}^2)] \end{bmatrix}$$

$$\Rightarrow TC_{G_3}(t_1,T)=\begin{bmatrix} A+C_3a_3(t_1+\frac{\theta_3-b_3}{2}t_1{}^2)+\beta a_3C_3(T-t_1)-\frac{1}{2}\beta a_3 \\ b_3C_3(T^2-t_1{}^2)+a_3h_3\left(\frac{1}{2}t_1{}^2+\frac{\theta_3-2b_3}{6}t_1{}^3+\frac{b_3\theta_3}{8}t_1{}^4\right) \\ +\beta a_3l_3(T-t_1)^2-\frac{1}{2}\beta a_3b_3l_3(T-t_1)^2(T+t_1) \\ +S_3(1-\beta)[a_3(T-t_1)-\frac{a_3b_3}{2}(T^2-t_1{}^2)] \end{bmatrix}$$

$$\Rightarrow TC_{G_4}(t_1,T)=\begin{bmatrix} A+C_4a_4(t_1+\frac{\theta_4-b_4}{2}t_1{}^2)+\beta a_4C_4(T-t_1)-\frac{1}{2}\beta a_4 \\ b_4C_4(T^2-t_1{}^2)+a_4h_4\left(\frac{1}{2}t_1{}^2+\frac{\theta_4-2b_4}{6}t_1{}^3+\frac{b_4\theta_4}{8}t_1{}^4\right) \\ +\beta a_4l_4(T-t_1)^2-\frac{1}{2}\beta a_4b_4l_4(T-t_1)^2(T+t_1) \\ +S_4(1-\beta)[a_4(T-t_1)-\frac{a_4b_4}{2}(T^2-t_1{}^2)] \end{bmatrix}$$

To minimize total cost function per unit time $TC_G(t_1,T)$ the optimal value of t_1 and T can be obtained by solving the following equations

$$\frac{\partial TC_G(t_1,T)}{\partial t_1}=0 \text{ and } \frac{\partial TC_G(t_1,T)}{\partial T}=0$$

(16)

By solving equation (16) we get

$$\Rightarrow \frac{1}{6}\left[\begin{array}{l}
\left(\begin{array}{l} C_1a_1[1+(\theta_1-b_1)t_1]-\beta a_1C_1+\beta a_1b_1C_1t_1+a_1h_1 \\ \left(t_1+\frac{\theta_1-2b_1}{2}t_1^{\,2}+\frac{b_1\theta_1}{2}t_1^{\,3}\right)-2\beta a_1l_1(T-t_1) \\ +\frac{1}{2}\beta a_1b_1l_1(T-t_1)^2+S_1a_1(1-\beta)(-1+b_1t_1) \end{array}\right) \\
+2\left(\begin{array}{l} C_2a_2[1+(\theta_2-b_2)t_1]-\beta a_2C_2+\beta a_2b_2C_2t_1 \\ +a_2h_2\left(t_1+\frac{\theta_2-2b_2}{2}t_1^{\,2}+\frac{b_2\theta_2}{2}t_1^{\,3}\right)-2\beta a_2l_2 \\ (T-t_1)+\frac{1}{2}\beta a_2b_2l_2(T-t_1)^2+S_2a_2 \\ (1-\beta)(-1+b_2t_1) \end{array}\right) \\
+2\left(\begin{array}{l} C_3a_3[1+(\theta_3-b_3)t_1]-\beta a_3C_3+\beta a_3b_3C_3t_1 \\ +a_3h_3\left(t_1+\frac{\theta_3-2b_3}{2}t_1^{\,2}+\frac{b_3\theta_3}{2}t_1^{\,3}\right)-2\beta a_3l_3(T-t_1) \\ +\frac{1}{2}\beta a_3b_3l_3(T-t_1)^2+S_3a_3(1-\beta)(-1+b_3t_1) \end{array}\right) \\
+\left(\begin{array}{l} C_4a_4[1+(\theta_4-b_4)t_1]-\beta a_4C_4+\beta a_4b_4C_4t_1 \\ +a_4h_4\left(t_1+\frac{\theta_4-2b_4}{2}t_1^{\,2}+\frac{b_4\theta_4}{2}t_1^{\,3}\right)-2\beta a_4l_4(T-t_1) \\ +\frac{1}{2}\beta a_4b_4l_4(T-t_1)^2+S_4a_4(1-\beta)(-1+b_4t_1) \end{array}\right)
\end{array}\right]=0$$

(17)

And

$$\Rightarrow \frac{1}{6}\left[\begin{array}{l}
\left(\begin{array}{l} \beta a_1C_1-\beta a_1b_1C_1T+2\beta a_1l_1(T-t_1)-\frac{3}{2}\beta a_1b_1l_1 \\ (T-t_1)^2+S_1a_1(1-\beta)(1-b_1T) \end{array}\right) \\
+2\left(\begin{array}{l} \beta a_2C_2-\beta a_2b_2C_2T+2\beta a_2l_2(T-t_1)-\frac{3}{2} \\ \beta a_2b_2l_2(T-t_1)^2+S_2a_2(1-\beta)(1-b_2T) \end{array}\right) \\
+2\left(\begin{array}{l} \beta a_3C_3-\beta a_3b_3C_3T+2\beta a_3l_3(T-t_1)-\frac{3}{2} \\ \beta a_3b_3l_3(T-t_1)^2+S_3a_3(1-\beta)(1-b_3T) \end{array}\right) \\
+\left(\begin{array}{l} \beta a_4C_4-\beta a_4b_4C_4T+2\beta a_4l_4(T-t_1)-\frac{3}{2} \\ \beta a_4b_4l_4(T-t_1)^2+S_4a_4(1-\beta)(1-b_4T) \end{array}\right)
\end{array}\right]=0$$

(18)

(ii) By Signed Distance Method, total cost is

$$TC_S(t_1,T)=\frac{1}{4}\left(\begin{array}{l} TC_{S_1}(t_1,T)+TC_{S_2}(t_1,T)+ \\ TC_{S_3}(t_1,T)+TC_{S_4}(t_1,T) \end{array}\right)$$

(19)

Where

$$\Rightarrow TC_{S_1}(t_1,T)=\left[\begin{array}{l} A+C_1a_1(t_1+\frac{\theta_1-b_1}{2}t_1^{\,2})+\beta a_1C_1(T-t_1)-\frac{1}{2}\beta a_1 \\ b_1C_1(T^2-t_1^{\,2})+a_1h_1\left(\frac{1}{2}t_1^{\,2}+\frac{\theta_1}{6}t_1^{\,3}-\frac{b_1}{3}t_1^{\,3}+\frac{b_1\theta_1}{8}t_1^{\,4}\right) \\ +\beta a_1l_1(T-t_1)^2-\frac{1}{2}\beta a_1b_1l_1(T-t_1)^2(T+t_1) \\ +S_1(1-\beta)[a_1(T-t_1)-\frac{a_1b_1}{2}(T^2-t_1^{\,2}) \end{array}\right]$$

$$\Rightarrow TC_{S_2}(t_1,T)=\left[\begin{array}{l} A+C_2a_2(t_1+\frac{\theta_2-b_2}{2}t_1^{\,2})+\beta a_2C_2(T-t_1)-\frac{1}{2}\beta a_2 \\ b_2C_2(T^2-t_1^{\,2})+a_2h_2\left(\frac{1}{2}t_1^{\,2}+\frac{\theta_2}{6}t_1^{\,3}-\frac{b_2}{3}t_1^{\,3}+\frac{b_2\theta_2}{8}t_1^{\,4}\right) \\ +\beta a_2l_2(T-t_1)^2-\frac{1}{2}\beta a_2b_2l_2(T-t_1)^2(T+t_1)+S_2 \\ (1-\beta)[a_2(T-t_1)-\frac{a_2b_2}{2}(T^2-t_1^{\,2}) \end{array}\right]$$

$$\Rightarrow TC_{S_3}(t_1,T)=\left[\begin{array}{l} A+C_3a_3(t_1+\frac{\theta_3-b_3}{2}t_1^{\,2})+\beta a_3C_3(T-t_1)-\frac{1}{2}\beta a_3b_3C_3 \\ (T^2-t_1^{\,2})+a_3h_3\left(\frac{1}{2}t_1^{\,2}+\frac{\theta_3}{6}t_1^{\,3}-\frac{b_3}{3}t_1^{\,3}+\frac{b_3\theta_3}{8}t_1^{\,4}\right)+\beta a_3 \\ l_3(T-t_1)^2-\frac{1}{2}\beta a_3b_3l_3(T-t_1)^2(T+t_1)+S_3(1-\beta) \\ [a_3(T-t_1)-\frac{a_3b_3}{2}(T^2-t_1^{\,2}) \end{array}\right]$$

$$\Rightarrow TC_{S_4}(t_1,T)=\left[\begin{array}{l} A+C_4a_4(t_1+\frac{\theta_4-b_4}{2}t_1^{\,2})+\beta a_4C_4(T-t_1)-\frac{1}{2}\beta a_4b_4 \\ C_4(T^2-t_1^{\,2})+a_4h_4\left(\frac{1}{2}t_1^{\,2}+\frac{\theta_4}{6}t_1^{\,3}-\frac{b_4}{3}t_1^{\,3}+\frac{b_4\theta_4}{8}t_1^{\,4}\right) \\ +\beta a_4l_4(T-t_1)^2-\frac{1}{2}\beta a_4b_4l_4(T-t_1)^2(T+t_1)+S_4 \\ (1-\beta)[a_4(T-t_1)-\frac{a_4b_4}{2}(T^2-t_1^{\,2}) \end{array}\right]$$

To minimize total cost function per unit time $TC_S(t_1,T)$ the optimal value of t_1 and T can be obtained by solving the following equations

$$\frac{\partial TC_S(t_1,T)}{\partial t_1}=0 \text{ and } \frac{\partial TC_S(t_1,T)}{\partial T}=0$$

(20)

Equation (20) is equivalent to

$$\Rightarrow \frac{1}{4}\left[\begin{array}{l}\left(\begin{array}{l}C_1a_1[1+(\theta_1-b_1)t_1]-\beta a_1C_1+\beta a_1b_1C_1t_1\\ +a_1h_1\left(t_1+\frac{\theta_1-2b_1}{2}t_1^{\ 2}+\frac{b_1\theta_1}{2}t_1^{\ 3}\right)-2\beta a_1l_1\\ (T-t_1)+\frac{1}{2}\beta a_1b_1l_1(T-t_1)^2+S_1a_1(1-\beta)\\ (-1+b_1t_1)\end{array}\right)\\ +\left(\begin{array}{l}C_2a_2[1+(\theta_2-b_2)t_1]-\beta a_2C_2+\beta a_2b_2\\ C_2t_1+a_2h_2\left(t_1+\frac{\theta_2-2b_2}{2}t_1^{\ 2}+\frac{b_2\theta_2}{2}t_1^{\ 3}\right)-2\\ \beta a_2l_2(T-t_1)+\frac{1}{2}\beta a_2b_2l_2(T-t_1)^2+S_2\\ a_2(1-\beta)(-1+b_2t_1)\end{array}\right)\\ +\left(\begin{array}{l}C_3a_3[1+(\theta_3-b_3)t_1]-\beta a_3C_3+\beta a_3b_3\\ C_3t_1+a_3h_3\left(t_1+\frac{\theta_3-2b_3}{2}t_1^{\ 2}+\frac{b_3\theta_3}{2}t_1^{\ 3}\right)-2\\ \beta a_3l_3(T-t_1)+\frac{1}{2}\beta a_3b_3l_3(T-t_1)^2+S_3\\ a_3(1-\beta)(-1+b_3t_1)\end{array}\right)\\ +\left(\begin{array}{l}C_4a_4[1+(\theta_4-b_4)t_1]-\beta a_4C_4+\beta a_4\\ b_4C_4t_1+a_4h_4\left(t_1+\frac{\theta_4-2b_4}{2}t_1^{\ 2}+\frac{b_4\theta_4}{2}t_1^{\ 3}\right)\\ -2\beta a_4l_4(T-t_1)+\frac{1}{2}\beta a_4b_4l_4(T-t_1)^2\\ +S_4a_4(1-\beta)(-1+b_4t_1)\end{array}\right)\end{array}\right]=0$$

(21)

And

$$\Rightarrow \frac{1}{4}\left[\begin{array}{l}\left(\begin{array}{l}\beta a_1C_1-\beta a_1b_1C_1T+2\beta a_1l_1(T-t_1)-\frac{3}{2}\beta a_1b_1l_1\\ (T-t_1)^2+S_1a_1(1-\beta)(1-b_1T)\end{array}\right)\\ +\left(\begin{array}{l}\beta a_2C_2-\beta a_2b_2C_2T+2\beta a_2l_2(T-t_1)-\frac{3}{2}\beta a_2\\ b_2l_2(T-t_1)^2+S_2a_2(1-\beta)(1-b_2T)\end{array}\right)\\ +\left(\begin{array}{l}\beta a_3C_3-\beta a_3b_3C_3T+2\beta a_3l_3(T-t_1)-\frac{3}{2}\beta a_3\\ b_3l_3(T-t_1)^2+S_3a_3(1-\beta)(1-b_3T)\end{array}\right)\\ +\left(\begin{array}{l}\beta a_4C_4-\beta a_4b_4C_4T+2\beta a_4l_4(T-t_1)-\frac{3}{2}\beta a_4\\ b_4l_4(T-t_1)^2+S_4a_4(1-\beta)(1-b_4T)\end{array}\right)\end{array}\right]=0$$

(22)

5. Numerical Example

Consider an inventory system with following parametric values.

5.1 Crisp Model

The following numerical values of the parameter in proper unit were considered as input for numerical analysis of the model.

$A=2000$, $C=10$, $a=200$, $b=2$, $\theta=0.4$, $h=0.2$, $\beta=0.8$, $l=4$, $S=6$. The output of the model by using Mathematica 5.1 mathematical software is $t_1=0.283038$, $T=0.556074$ and TC=2525.3

5.2 Fuzzy Model

If A=2000, $\tilde{a}=(185, 195, 205, 215)$, $\tilde{b}=(1, 2, 3, 4)$, $\tilde{C}=(7, 9, 11, 13)$, $\beta=0.8$, $\tilde{\theta}=(0.1, 0.2, 0.3, 0.4)$, $\tilde{h}=(0.1, 0.16, 0.22, 0.28)$, $\tilde{l}=(1,3,5,7)$, $\tilde{S}=(5, 6.5, 8, 9.5)$ by using Mathematica 5.1 mathematical software we get by Graded Mean Integration Method is $t_1=0.241301$, $T=0.391626$ and TC_{GD}=2383.43 and by Signed Distance Method is $t_1=0.229905$, $T=0.379559$ and TC_{SD}=2372.56.

6.Sensitive Analysis

Table-1 (Graded Mean Integration Method)

Parameter	% of Change	t_1	T	TC_{GD}
$\tilde{C}$	+50	0.071109	0.383132	2539.28
	+25	0.175812	0.389681	2465.92
	-25	0.288627	0.391838	2293.97
	-50	0.325126	0.390934	2199.56
$\tilde{a}$	+50	0.241301	0.391626	2575.14
	+25	0.241301	0.391626	2479.29
	-25	0.241301	0.391626	2287.57
	-50	0.241301	0.391626	2191.71
$\tilde{b}$	+50	0.144639	0.256055	2250.68
	+25	0.183454	0.30966	2303.28
	-25	0.339015	0.532525	2520.46
	-50	0.543457	0.831165	2807.75
$\tilde{\theta}$	+50	0.198296	0.394181	2383.28
	+25	0.218225	0.393306	2383.44
	-25	0.267822	0.388731	2382.94
	-50	0.297844	0.38406	2381.4
$\tilde{h}$	+50	0.239993	0.391742	2383.55
	+25	0.240649	0.391684	2383.49

	-25	0.241951	0.391568	2383.37
	-50	0.242597	0.39151	2383.3
$\tilde{l}$	+50	0.293314	0.396487	2391.02
	+25	0.27206	0.39449	2387.77
	-25	0.193209	0.387233	2377.48
	-50	0.109431	0.379933	2369.36
$\tilde{S}$	+50	0.267944	0.386886	2389.96
	+25	0.265436	0.390109	2388.27
	-25	0.219841	0.394372	2378.14
	-50	0.185391	0.396905	2369.95

A three dimensional graph is shown

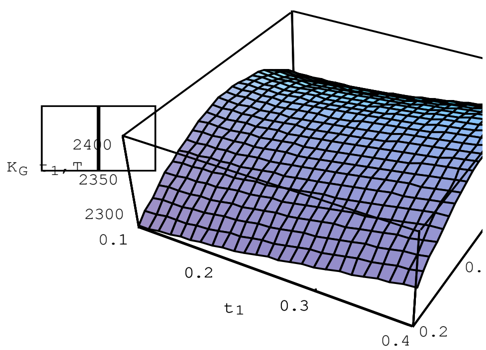

Figure 2: Total average cost (Graded Mean Integration method) vs. t_1 and T

Table-2 (Signed Distance Method)

Parameter	% of Change	t_1	T	TC_{SD}
$\tilde{C}$	+50	0.051264	0.369347	2522.44
	+25	0.162808	0.377229	2452.05
	-25	0.277638	0.379949	2285.89
	-50	0.314132	0.379154	2194.18
$\tilde{a}$	+50	0.229905	0.379559	2558.83
	+25	0.229905	0.379559	2465.7
	-25	0.229905	0.379559	2279.42
	-50	0.229905	0.379559	2186.14
$\tilde{b}$	+50	0.136735	0.248253	2243.64
	+25	0.174229	0.300191	2294.75
	-25	0.323691	0.515875	2505.46
	-50	0.519361	0.804319	2783.56
$\tilde{\theta}$	+50	0.187489	0.381562	2372.08
	+25	0.20707	0.380936	2372.38
	-25	0.256336	0.377013	2372.32
	-50	0.286454	0.372718	2371.11
$\tilde{h}$	+50	0.228625	0.379656	2372.66
	+25	0.229266	0.379608	2372.61
	-25	0.23051	0.379511	2372.51
	-50	0.231173	0.379461	2372.45
$\tilde{l}$	+50	0.281741	0.384479	2380.13
	+25	0.260552	0.382456	2376.88
	-25	0.182052	0.375126	2366.68
	-50	0.099004	0.36781	2358.81
$\tilde{S}$	+50	0.257594	0.375128	2379.33
	+25	0.245744	0.377221	2376.45
	-25	0.207055	0.382023	2366.97
	-50	0.168957	0.383936	2358.1

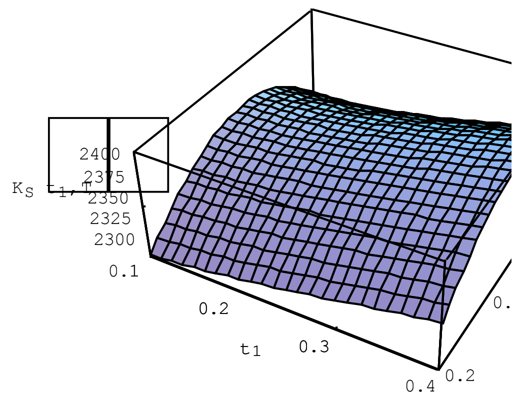

Figure 3: Total average cost (Signed Distance Method) vs. t_1 and T

Based on results presented in Table 1, the following features are observed

- Total cost for Graded Mean Integration increases rapidly with increase in the value of the model parameter $\tilde{C}$, $\tilde{a}$, $\tilde{l}$, $\tilde{S}$.
- Total fuzzy cost TC_{GD} decreases with increase of $\tilde{b}$
- There are negligence changes in fuzzy total cost TC_{GD} when model parameters $\tilde{\theta}$ and $\tilde{h}$ increases

From Table 2, the following observations can be made

- When $\tilde{C}$, $\tilde{a}$, $\tilde{l}$, $\tilde{S}$ increases then TC_{SD} increases rapidly
- Total fuzzy cost for signed distance method (TC_{SD}) decreases with increase in value of $\tilde{b}$ and almost insensitive for changes in $\tilde{\theta}$, $\tilde{h}$.

7. Conclusion and Analysis

This paper represents a fuzzy inventory model for deteriorating items with allowable shortages in which demand is a decreasing exponential function. The demand, deterioration rate, inventory holding cost, shortage cost and lost sale cost are represented by trapezoidal fuzzy numbers. For defuzzificationgraded mean representation and signed distance method are employed to evaluate the optimal time period of positive stock t_1 and total cycle length T which minimizes the total cost. By given numerical example it has been tested that signed distance method gives minimum cost as compared to graded mean representation.

The proposed crisp model can be enriched by taking stochastic fluctuating demand patterns. In further, the fuzzy model can be changed by considering other type of membership functions such as piecewise linear hyperbolic and pentagonal fuzzy number.

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