On Estimation of Population Coefficient of Variation in Presence of Measurement Errors

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Abstract: *The present article addresses the effects of measurement errors on the estimation of population coefficient of variation C_Y of the study variable Y. Bias and mean squared error (MSE) of the proposed estimator are derived upto the first order of approximation under simple random sampling design. A theoretical efficiency comparison is made between the proposed estimator and the usual coefficient of variation estimator in presence of measurement errors. Based on large sample approximations the optimal condition is obtained under which the proposed estimator performs better than the conventional estimator in presence of measurement errors. Theoretical results are verified by the simulation study using R software.*

Key words:*Estimation, Coefficient of variation, Measurement errors, Simulation.*

I. INTRODUCTION

Coefficient of variation (C.V) is powerful statistical tool which is extensively used in sampling theory, quality control, biological studies, biometric and agricultural experiments and economic studies for measuring the fluctuations, stability and inequality. Coefficient of variation is a unitlessrelative measure of dispersion. Therefore it is widely used for the comparison of various data sets having different measurement units. Coefficient of variation has almost stable rate of change from one survey to another. Hence sometimes it is used as prior information in sampling theory for estimating population parameters. The coefficient of variation expressed in percentages indicates quickly the extent of variability present in the data. The C.V is also common in applied probability fields such as renewal theory, queuing theory and reliability theory. The C.V is also used in multiple time scales and the life time (Kordonsky and Gertsbakh(1997))

The research on C.V dates back to the work of McKay (1932), Pearson(1932), and Fieller(1932) where they have studied a numerical approximation to the distribution of the sample C.V (in the case of normality). Later on it was extended by Hendrick and Robey (1936) and Koopmans et al.(1964). Nairy and Rao (2003) and the references cited there discusses the various tests for testing the equality of C.V's of independent normal distributions. The research work on the C.V of the normal distribution is fast growing and one of the recent references is that of Mahmoudvand Hassani (2007) who proposed two new confidence intervals for the C.V in a normal distribution. Compared to the research work on C.V of the normal distribution, research on C.V of a finite population is of recent origin. The estimation of C.V in finite population was initially discussed by Das and Tripathi (1981a, b). Since then various researchers have attempted the estimation of C.V which include the works of Rajyaguru and Gupta (2002, 2006), Tripathi et al.(2002) , Patel and Shah (2009), among others. Following the idea of Srivastava (1971, 80) and Das and Tripathi (1980), Tripathi et al.(2002) constructed a general class of estimators of C.V. This class of estimators is a hybrid class in the sense that a regression type of estimators is used to construct a general class of ratio estimators of C.V. The ratio/product and regression estimators of C.V constructed from the sample C.V are members of this class. Archana and Rao (2011) have proposed six new regression type estimators for estimation of population coefficient of variation.

Instatistical data analysis it is a very common assumption that observations are collected without any error, that is, observed values are the correct measure of their true values. But in practice this assumption is not usually met. Observations are generally contaminated with various types of errors due to manyunavoidable causes like non response of respondents, faulty demarcations of sampling units, faulty preparation of questionnaires, inappropriate interview techniques or interaction of all these factors. Measurement errors or response errors are the discrepancy between observed value and the true value of the characteristic under study. Such errors have serious consequences on the estimation of population parameters in terms of increased bias and variability. Thus the study of measurement errors is essential so that improved estimation techniques can be developed which can provide the efficient and reliable estimates of population parameters in presence of measurement errors in sample surveys. Many authors includingShalabh (1997), Maneesha and Singh (2001), Singh and Karpe (2009), Misra and Yadav

(2015) and Misra et al.(2016) have studied the effects of measurement errors on estimation of population parameters. In the present article we are dealing with the estimation of population coefficient of variation in presence of measurement errors.

II. Notations and Methodology

Consider a finite population U = {$U_1, U_2 \ldots U_N$} of N distinct and identifiable units. Let (Y, X) be the study and auxiliary variable respectively. Suppose that we are given a set of n paired observations obtained through simple random sampling procedure on two characteristics Y and X. It is assumed that x_i and y_i for the i^{th} sampling unit are recorded instead of true values X_i and Y_i . The observational or measurement errors are defined as

$$u_i = y_i - Y_i$$

$$v_i = x_i - X_i$$

which are assumed to be stochastic with mean zero but possibly different variances σ_u^2 and σ_v^2.

Let the population means of (X,Y) characteristics be (μ_X, μ_Y) and population variances be (σ_X^2, σ_Y^2) respectively. Further, let ρ be the population correlation coefficient between X and Y. Let $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ be the unbiased estimators of population means μ_X and μ_Y respectively. We note that $s_x^2 = \frac{1}{n-1}\sum_{i=n}^{n}(x_i - \bar{x})^2$ and $s_y^2 = \frac{1}{n-1}\sum_{i=n}^{n}(y_i - \bar{y})^2$ are not unbiased estimators of the population variances σ_X^2 and σ_Y^2. In presence of measurement errors the expected value of s_y^2 is given by

$$E(s_y^2) = \sigma_Y^2 + \sigma_u^2$$

Let error variances σ_u^2 and σ_v^2 are known a prior then unbiased estimators of population variance in presence of measurement errors are

$$\hat{\sigma}_y^2 = s_y^2 - \sigma_u^2 > 0$$

$$\hat{\sigma}_x^2 = s_x^2 - \sigma_v^2 > 0$$

Consider the following approximation,

$$\hat{\sigma}_Y^2 = \sigma_Y^2\ (1 + e_0), \qquad \bar{y} = \mu_Y(1 + e_1)$$
$$\bar{x} = \mu_x(1 + e_2), \qquad \hat{\sigma}_X^2 = \sigma_X^2\ (1 + e_3),$$
$$\hat{\sigma}_{XY} = \sigma_{XY}(1 + e_4)$$

Such that $E(e_0) = E(e_1)) = E(e_2) = E(e_3) = E(e_4) = 0$

From Singh and Karpe (2009), we have

$$E(e_0^2) = \frac{A_Y}{n},\ E(e_1^2) = \frac{C_Y^2}{n}\left(1 + \frac{\sigma_u^2}{\sigma_Y^2}\right) = \frac{C_Y^2}{n\theta_Y},$$
$$E(e_2^2) = \frac{C_X^2}{n}\left(1 + \frac{\sigma_v^2}{\sigma_X^2}\right) = \frac{C_X^2}{n\theta_X}$$

$$E(e_0 e_1) = \frac{\gamma_{1Y}^* C_Y}{n},\ \text{where}, \gamma_{1Y}^* = \gamma_{1Y} + \gamma_{1u}\left(\frac{\sigma_u^3}{\sigma_Y^3}\right),$$
$$E(e_1 e_2) = \frac{\rho C_X C_Y}{n}$$

$$\theta_Y = \frac{\sigma_Y^2}{\sigma_Y^2 + \sigma_u^2}, \theta_X = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_v^2}, A_y = \gamma_{2y} + \gamma_{2u}\frac{\sigma_u^4}{\sigma_y^4} + 2(1 + \frac{\sigma_u^2}{\sigma_y^2})^2$$

$$E(e_0 e_2) = \frac{\lambda C_X}{n}\quad,\quad \lambda = \frac{\mu_{12(X,Y)}}{\sigma_X \sigma_Y^2}$$

III.Usual Estimator for Estimating Population Coefficient of Variation in Presence of Measurement Errors

For estimating population coefficient of variation in presence of measurement errors the usual estimator is given as

$$\hat{C}_Y = \frac{\hat{\sigma}_Y}{\bar{y}} \qquad (1)$$

Substituting values of $\hat{\sigma}_Y^2$ and $\bar{Y}$ in equation (1)

$$\hat{C}_Y = \frac{(\hat{\sigma}_Y^2)^{1/2}}{\bar{y}}$$

$$\hat{C}_Y - C_Y = C_Y\left[\frac{1}{2}e_0 - \frac{1}{8}e_0^2 - e_1 - \frac{1}{2}e_0 e_1 + e_1^2\right] (2)$$

Taking expectation on both sides, the Bias of $\hat{C}_Y$ up to the terms of order O($\frac{1}{n}$), is obtained as

$$\text{Bias}(\hat{C}_Y) = \frac{C_Y}{n}\left[\frac{C_Y^2}{\theta_Y} - \frac{1}{2}\left\{\gamma_{IY} + \gamma_{1u}\left(\frac{1-\theta_Y}{\theta_Y}\right)^{3\backslash 2}\right\}C_Y - \frac{A_Y}{8}\right](3)$$

Theorem 2.MSE of $\hat{C}_Y$ up to the terms of order O($\frac{1}{n}$) is given as

$$\text{MSE}\ (\hat{C}_Y) = \frac{C_Y^2}{n}\left[\frac{C_Y^2}{\theta_Y} - \left\{\gamma_{IY} + \gamma_{1u}\left(\frac{1-\theta_Y}{\theta_Y}\right)^{3\backslash 2}\right\}C_Y + \frac{A_Y}{4}\right]$$

Proof. Squaring (2) and taking expectations both sides, we have

$$E(\hat{C}_Y - C_Y)^2 = E\left[C_Y^2\left\{\frac{1}{4}E(e_0^2) + E(e_1^2) - E(e_0 e_1)\right\}\right]$$

$$MSE(\hat{C}_Y) = \frac{C_Y^2}{n}\left[\frac{C_Y^2}{\theta_Y} - \left\{\gamma_{1Y} + \gamma_{1u}\left(\frac{1-\theta_Y}{\theta_Y}\right)^{3\backslash 2}\right\}C_Y + \frac{A_Y}{4}\right] \qquad (4)$$

IV. Proposed Estimator under Measurement Errors

Archana and Rao (2011), has given the following estimator for estimating population coefficient of variation as,

$$\hat{C}_{Y_1} = \frac{s_y}{\bar{y}+b_1(\bar{X}-\bar{x})} \qquad (5)$$

where, b_1 is the estimator of B_1, the regression coefficient of $\bar{y}$ on $\bar{x}$.

Motivated by Archana and Rao (2011)and adopting their estimator under measurement errors, we have proposed the following estimator for population coefficient of variation under simple random sampling without replacement scheme,

$$\hat{C}_{Y_1} = \frac{\hat{\sigma}_Y}{\bar{y} + b_1(\mu_X - \bar{x})} \qquad (6)$$

where, $b_1 = \frac{\hat{\sigma}_{xy}}{\hat{\sigma}_x^2}$ = Regression coefficient

Estimator (6) can also be expressed as,

$$\hat{C}_{Y_1} = \frac{(\hat{\sigma}_Y^2)^{1/2}}{\bar{y} + b_1(\mu_X - \bar{x})} \qquad (7)$$

Now, the denominator $\bar{y} + b_1(\mu_X - \bar{x})$, can be expressed as,

$$= \mu_Y(1+e_1) + \frac{\hat{\sigma}_{XY}}{\hat{\sigma}_X^2}(-e_2\mu_X)$$

Taking first order approximation, we have

$$= \mu_Y\left[1 + e_1 + \frac{\mu_X}{\mu_Y}\frac{\sigma_{XY}}{\sigma_X^2}(e_2 + e_2e_4 - e_2e_3)\right] \qquad (8)$$

Substituting this value in (6),

$$\hat{C}_{Y_1} = \frac{(\hat{\sigma}_Y^2)^{1/2}}{\mu_Y\left[1 + e_1 + \frac{\mu_X}{\mu_Y}\frac{\sigma_{XY}}{\sigma_X^2}(e_2 + e_2e_4 - e_2e_3)\right]}$$

$$= \frac{[\sigma_Y^2(1+e_o)]^{1/2}}{\mu_Y\left[1 + e_1 + \frac{\mu_X}{\mu_Y}\frac{\sigma_{XY}}{\sigma_X^2}(e_2 + e_2e_4 - e_2e_3)\right]}$$

taking first order approximation, we have

$$\hat{C}_{Y_1} - C_Y = C_Y\Bigg[e_1^2 + \left(\frac{\mu_X}{\mu_Y}\right)^2\left(\frac{\sigma_{XY}}{\sigma_X^2}\right)^2 e_2^2 + 2\left(\frac{\mu_X}{\mu_Y}\right)\left(\frac{\sigma_{XY}}{\sigma_X^2}\right)^2 e_1e_2 + \frac{1}{2}e_0 - \frac{1}{8}e_0^2 - e_1 - \frac{\mu_X}{\mu_Y}\frac{\sigma_{XY}}{\sigma_X^2}(e_2 + e_2e_4 - e_3e_2) - \frac{1}{2}e_0e_1 - \frac{1}{2}\frac{\mu_X}{\mu_Y}\frac{\sigma_{XY}}{\sigma_X^2}e_0e_2 + \cdots\ldots\Bigg] \qquad (9)$$

Theorem 3: Bias of the proposed estimator $\hat{C}_{Y_1}$ is given as

$$Bias(\hat{C}_{Y_1}) = \frac{C_Y}{n}\left[\frac{C_Y^2}{\theta_Y} - \frac{1}{2}\left\{\gamma_{1Y} + \gamma_{1u}\left(\frac{1-\theta_Y}{\theta_Y}\right)^{\frac{3}{2}}C_Y - \frac{A_Y}{8}\right\}\right] + 2\rho^3C_Y^2\frac{\sigma_Y}{\mu_X} + \rho^2\frac{C_Y^2}{\theta_X} - \rho\frac{C_Y}{\sigma_X}\left\{\frac{\mu_{2100}}{\sigma_{XY}} - \frac{\mu_{3000}}{\sigma_X^2}\right\} - \frac{1}{2}\rho\left(\frac{\sigma_Y}{\mu_X}\right)\mu_{2100}$$

Proof: Taking expectation in equation (9) on both sides, we get the bias of $\hat{C}_{Y_1}$ up to the terms of order $O\left(\frac{1}{n}\right)$,

$$E(\hat{C}_{Y_1} - C_Y) = C_Y\Big[E(e_1^2) + \left(\frac{\mu_X}{\mu_Y}\right)^2\left(\frac{\sigma_{XY}}{\sigma_X^2}\right)^2 E(e_2^2) + 2\left(\frac{\mu_X}{\mu_Y}\right)\left(\frac{\sigma_{XY}}{\sigma_X^2}\right)^2 E(e_1e_2) + \frac{1}{2}E(e_0) - \frac{1}{8}E(e_0^2) - E(e_1) - \frac{\mu_X}{\mu_Y}\frac{\sigma_{XY}}{\sigma_X^2}\{E(e_2) + E(e_2e_4) - E(e_3e_2)\} - \frac{1}{2}E(e_0e_1) - \frac{1}{2}\frac{\mu_X}{\mu_Y}.\frac{\sigma_{XY}}{\sigma_X^2}E(e_0e_2)$$

$$Bias(\hat{C}_{Y_1}) = \frac{C_Y}{n}\left[\frac{C_Y^2}{\theta_Y} - \frac{1}{2}\left\{\gamma_{1Y} + \gamma_{1u}\left(\frac{1-\theta_Y}{\theta_Y}\right)^{\frac{3}{2}}C_Y - \frac{A_Y}{8}\right\}\right] + 2\rho^3C_Y^2\frac{\sigma_Y}{\mu_X} + \rho^2\frac{C_Y^2}{\theta_X} - \rho\frac{C_Y}{\sigma_X}\left\{\frac{\mu_{2100}}{\sigma_{XY}} - \frac{\mu_{3000}}{\sigma_X^2}\right\} - \frac{1}{2}\rho\left(\frac{\sigma_Y}{\mu_X}\right)\mu_{2100} \qquad (10)$$

Theorem 4: Mean squared error (MSE) of the proposed estimator is given as

$$MSE(\widehat{C}_{Y_1}) = \frac{C_Y^2}{n}\left[\frac{C_Y^2}{\theta_Y} - \left\{\gamma_{1Y} + \gamma_{1u}\left(\frac{1-\theta_Y}{\theta_Y}\right)^{3\backslash 2}\right\}C_Y + \frac{A_Y}{4}\right] - \frac{C_Y^2}{n}\left[\left(\frac{\mu_X}{\mu_Y}\right)B_1 C_X\left\{\frac{\mu_{12}(X,Y)}{\sigma_X\sigma_Y^2} - \left(\frac{\mu_X}{\mu_Y}\right)B_1\frac{C_X}{\theta_X} - 2\rho C_Y\right\}\right]$$

$$\text{where} B_1 = \frac{\sigma_{XY}}{\sigma_X^2}$$

Proof: Squaring equation (9)and taking expectation both sides we get the $MSE(\widehat{C}_{Y_1})$upto the first order approximation,

$$E(\widehat{C}_{Y_1} - C_Y)^2 = \mathbf{E}\left[C_Y^2\left\{\frac{1}{4}e_0^2 + e_1^2 + \left(\frac{\mu_X}{\mu_Y}\right)^2 B_1^2 e_2^2 - e_0 e_1 - \left(\frac{\mu_X}{\mu_Y}\right)B_1(e_0 e_2) + 2B_1\left(\frac{\mu_X}{\mu_Y}\right)e_1 e_2\right\}\right]$$

$$MSE(\widehat{C}_{Y_1}) = \frac{C_Y^2}{n}\left[\frac{C_Y^2}{\theta_Y} - \left\{\gamma_{1Y} + \gamma_{1u}\left(\frac{1-\theta_Y}{\theta_Y}\right)^{3\backslash 2}\right\}C_Y + \frac{A_Y}{4}\right] - \frac{C_Y^2}{n}\left[\left(\frac{\mu_X}{\mu_Y}\right)B_1 C_X\left\{\frac{\mu_{12}(X,Y)}{\sigma_X\sigma_Y^2} - \left(\frac{\mu_X}{\mu_Y}\right)B_1\frac{C_X}{\theta_X} - 2\rho C_Y\right\}\right] \qquad (11)$$

$MSE(\widehat{C}_{Y_1})$ can also be expressed as,

$$MSE(\widehat{C}_{Y_1}) = MSE(\widehat{C}_Y) - \frac{C_Y^2}{n}\left[\left(\frac{\mu_X}{\mu_Y}\right)B_1 C_X\left\{\frac{\mu_{12}(X,Y)}{\sigma_X\sigma_Y^2} - \left(\frac{\mu_X}{\mu_Y}\right)B_1\frac{C_X}{\theta_X} - 2\rho C_Y\right\}\right] \qquad (12)$$

V.Theoretical Efficiency Comparison

The proposed estimator $\widehat{C}_{Y_1}$ will be more efficient than the usual estimator$\widehat{C}_Y$ if

$$MSE(\widehat{C}_{Y_1}) - MSE(\widehat{C}_Y) < 0$$

which gives optimality condition

$$\rho < \frac{\lambda}{2C_Y} - \left(\frac{\mu_X}{\mu_Y}\right) \qquad (13)$$

If any data set satisfies the condition (13), then the proposed estimator $\widehat{C}_{Y_1}$ will be more efficient than the usual estimator $\widehat{C}_Y$ for that data set.

VI .Simulation Study

We demonstrate the performances of both estimators through simulation study by generating a sample from Normal distribution using R software. The auxiliary information on variable X has been generated by N (5, 10) population. This type of population is very relevant in most socio-economic situations with one interest and one auxiliary variable.

The description of this data is as follows

X = N(5,10), Y = X + N(0,1), y = Y+N(1,3), x =X+N(1,3),

n=5000,$\mu_X = 4.95$, $\mu_Y = 4.93$, $\sigma_X^2 = 99.38$, $\sigma_Y^2 = 100.12$, $\sigma_u^2 = 25.57$, $\sigma_v^2 = 24.28$, $\rho_{XY} = 0.99$

$C_X = 2.012$, $C_Y = 2.029$, $\lambda = -0.038$, $A_X = 3.05$, $A_Y = 3.11$

Using the above values the MSEs of both the estimators are given as,

$MSE(\widehat{C}_Y) = 41.55$

$MSE(\widehat{C}_{Y_1}) = 35.75$

From the expressions of MSE'sthe percent relative efficiency (PRE) of the proposed estimator $\widehat{C}_{Y_1}$ over the usual coefficient variation estimator $\widehat{C}_Y$ in presence of measurement errors is 116%.

VII. Concluding Remarks

(i) From the results of MSEs of both estimators it is observed that the proposed estimator $\widehat{C}_{Y_1}$ is more efficient than the usual coefficient of variation estimator$\widehat{C}_Y$ in presence of measurement errors.

(ii) Percent relative efficiencies of both estimators show that the proposed estimator $\widehat{C}_{Y_1}$ is 116% more efficient than the usual coefficient of variation estimator $\widehat{C}_Y$ in presence of measurement errors.

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