Hydromagnetic Heat and Mass Transfer on Oscillatory Flow of VISCO-Elastic Fluid in a Vertical Channel

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Abstract

In this paper, a theoretical analysis is carried out to study the visco-elastic effects on hydromagnetic heat and mass transfer in a vertical channel. The two vertical plates are in porous medium and non-uniform wall temperatures. A magnetic field of uniform strength is applied in the direction perpendicular to the plates. The visco-elastic fluid flow is characterized by second order fluid. The analytical solution to the coupled non-liear equations governing the motion are obtained by regular perturbation technique. The effects of different flow parameters on skin friction are analyzed and illustrated graphically.

Key words: *oscillatory flow, visco-elastic, heat and mass transfer, skin-friction.*

1. Introduction

The phenomenon of oscillatory flow along with heat and mass transfer of a conducting fluid has attracted the attention of many researchers due to its importance in many areas such as biological and industrial processes. Also some fluids can emit and absorb thermal radiations, it is of interest to study the effect of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing thermal radiations. This is of interest because heat transfer by thermal radiations is becoming of greater importance when we are concerned with space application and higher operating temperature. A list of key references in the vast literature concerning this field are given in[3-14].

The various industries for example, chemical and hydrometallurgical industries require the study of heat and mass transfer along with chemical reaction. The effect of chemical reaction on heat and mass transfer has been studied under different conditions by several authors [see 15-23].

In this study, an attempt has been made to extend the problem studied by Kumari et al..[23] to the case of visco-elastic fluid characterized by second-order fluid.

The constitutive equation for the incompressible Second-order fluid is of the form

$$\sigma = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \qquad (1)$$

Where σ is the stress tensor, $A_n (n = 1,2,3)$ are the kinematic Rivlin-Ericksen tensors; μ_1, μ_2, μ_3 are the material coefficients describing the viscosity, elasticity and cross-viscosity respectively. The material coefficients μ_1, μ_2, μ_3 are taken constants with μ_1 and μ_3 as positive and μ_2 as negative [Coleman and Markovitz (1964)] [1].The equation (1) was derived by Coleman and Noll[2] from that of simple fluids by assuming that the stress is more sensitive to the recent deformation than to the deformation that occurred in the distant past.

2. FORMULATION OF THE PROBLEM

We consider the unsteady oscillatory MHD flow of an electrically conducting incompressible visco-elastic fluid between two infinite vertical parallel plates. The fluid is assumed to be a radiating, optically thin and heat absorbing. The x^*- axis is taken along the vertical plates in upward direction, the y^*- axis is perpendicular to the wall of the channel. A strong transverse magnetic field of uniform strength is applied in a direction parallel to the y^*- axis. The plate at $y^*=0$ is oscillating in its own plane, while the other plate at $y^*=h$ is moving with a constant velocity in the direction. It is assumed that the temperature and the concentration at the wall $y^*=0$, while and T_1^* and C_1^* are constant temperature and concentration at $y^*=h$. Under these assumptions, the unsteady flow is governed by the following system of equations

Equation of motion:

$$\frac{\partial u}{\partial t} = \frac{\mu_1}{\rho}\frac{\partial^2 u}{\partial y^2} + \frac{\mu_2}{\rho}\frac{\partial^3 u}{\partial t \partial y^2} - \frac{v_1 u}{k} - \frac{\sigma B_0^2 u}{\rho} + g\beta(T - T_0) + g\beta^*(T - T_0) \qquad (2)$$

Equation of Energy:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q}{\partial y^*} - \frac{Q_0}{\rho C_p}\left(T^* - T^*_0\right) \qquad (3)$$

Concentration equation:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} - D_1(C - C_0) \qquad (4)$$

The relevant boundary conditions in non-dimensional form are given b

$$u' = U_0(1+\varepsilon e^{i\omega' t}),\ T' = T_1' + \varepsilon(T_1' - T_0')e^{i\omega' t},$$
$$C' = C_1' + \varepsilon(C_1' - C_0')e^{i\omega' t} \quad at \quad y' = 0$$
$$u' = u_p',\ T' = T_1',\ C' = C_1' \quad at\ y^* = h \qquad (5)$$

Where ε is the scalar constants and ε<<1. As the fluid is optically thin with a relatively low density and according to [10] radiative heat flux is given by

$$\frac{\partial q}{\partial y^*} = 4\alpha^2 (T^* - T_0^*) \qquad (6)$$

Introducing the following non-dimensional quantities:

$$y = \frac{y^*}{h}, t = \frac{t^* v_1}{h^2}, \omega = \frac{h^2\omega^*}{v_1}, u = \frac{u^*}{U_0}, \theta = \frac{T^* - T_0^*}{T_1^* - T_0^*}, C = \frac{C^* - C_0^*}{C_1^* - C_0^*}, u_p = \frac{u_p^*}{U_0},$$
$$Gr = \frac{g\beta h^2 (T_1^* - T_0^*)}{v_1 U_0} Gm = \frac{g\beta^* h^2 (C_1^* - C_0^*)}{v_1 U_0}, M^2 = \frac{\sigma B_0^2 h^2}{\rho v_1}, R^2 = \frac{4\alpha^2 h^2}{\kappa},$$
$$K^2 = \frac{h^2}{K^*}, S = \frac{Q_0 h^2}{\kappa}, \Pr = \frac{\rho v_1 C_p}{\kappa}, Sc = \frac{v_1}{D}, Kr = \frac{D_1 h^2}{v_1} \qquad (7)$$

Using (6), and the above non-dimensional quantities in (7), the equations (2)-(5),becomes,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + D_0 \frac{\partial^3 u}{\partial t \partial y^2} - (k^2 + M^2)u + Gr\theta + GmC \qquad (8)$$

$$\Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - R^2\theta - S\theta \qquad (9)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - KrC \qquad (10)$$

The corresponding boundary conditions are

$$u = 1 + \varepsilon e^{i\omega t}, \theta = 1 + \varepsilon e^{i\omega t}, C = 1 + \varepsilon e^{i\omega t} \qquad at \quad y = 0$$
$$u = u_p, \theta = 1,\ C = 1 \qquad at \quad y = 1 \qquad (11)$$

3. Solution of the Problem

In order to solve the equations (8) -(10), we assume that the unsteady flow is superimposed on the mean steady flow so we have,

$$u(y,t) = u_0(y) + \varepsilon^{i\omega t} u_1(y) + o(\varepsilon^2)$$
$$\theta(y,t) = \theta_0(y) + \varepsilon^{i\omega t} \theta_1(y) + o(\varepsilon^2)$$
$$C(y,t) = C_0(y) + \varepsilon^{i\omega t} C_1(y) + o(\varepsilon^2) \qquad (12)$$

Substituting equations(12) in to equation(8)-(10), neglecting the higher order terms of $o(\varepsilon^2)$, the following equations are obtained;

$$u_0'' - (k^2 + M^2)u_0 + Gr\theta_0 + GmC_0 = 0 \qquad (13)$$
$$u_1'' + D_0 i\omega u_1'' - (k^2 + M^2 + i\omega)u_1 + Gr\theta_1 + GmC_1 = 0 \qquad (14)$$
$$\theta_0'' - (R^2 + S)\theta_0 = 0 \qquad (15)$$
$$\theta_1'' - (R^2 + S + i\Pr\omega)\theta_1 = 0 \qquad (16)$$
$$C_0'' - KrScC_0 = 0 \qquad (17)$$
$$C_1'' - (KrSc + i\omega Sc)C_1 = 0 \qquad (18)$$

The corresponding boundary conditions are

$$u_0 = 1, u_1 = 1, \theta_0 = 1, \theta_1 = 1, C_0 = 1, C_1 = 1 \qquad at \quad y = 0$$
$$u_0 = u_p, u_1 = 0, \theta_0 = 1, \theta_1 = 0, C_0 = 1, C_1 = 0 \qquad at \quad y = 1 \qquad (19)$$

4.Results and Discussions

The expression for the skin friction at the plate y=0 is

$$C_f = \left(\frac{d^2u_0}{dy^2} + \varepsilon e^{i\omega t}\frac{d^2u_1}{dy^2} + D_0(i\omega\varepsilon e^{i\omega t}\frac{d^2u_1}{dy^2})\right)\frac{d^2u_0}{dy^2}$$

The purpose of this study is to bring out the effects of visco-elastic parameter on hydro-magnetic, heat and mass transfer characteristics as the effects of other parameter have been discussed by Kumari *etal*.[23]. The non-Newtonian effect is exhibited through the parameter α. The corresponding results for Newtonian fluid is obtained by setting D_0=0 and it is worth mentioning that these results show conformity with earlier results.

In order to understand the physics of the problem, analytical results are discussed with the help of graphical illustrations. The parameters U_p=.5,ε=.001,ω=0.5are kept fixed throughout the discussions.

Figures 1-7 portray the nature of viscous drag formed during the motion of Newtonian and non-Newtonian fluids against time. Figure 1-4, depict that the profile of shearing stress with the variation of magnetic parameter(M),Schmidt number(Sc),heat source parameter(S) and radiation parameter(R). From these figures it is observed that the profile of shearing stress follow an diminishing trend with the increasing values of those parameters.But a complete reverse trend is observed in Figures 5-7. Figures 5,6 and 7 represent the viscous drag against various values of Grashof number for heat and mass transfer(Gr and Gm), radiation parameter(R), and permeability parameter(K).In all the cases it is pragmatic that shearing stress profile shows an increasing trend with the increasing values of those parameters.
It is also observed from the expression of temaperatue(θ) and concentration© that the temperature field and concentration field are not significantly affected by the visco-elastic parameters.

5. Conclusion

The visco-elastic effects on hydromagnetic heat and mass transfer on oscillatory flow of visco-elastic fluid in a vertical channel are studied in this paper. Some of the important conclusions this paper are as follows
i) The shearing stress is prominently affected by the visco-elastic parameter

ii)The effect of flow parameters on shearing stress are prominent throughout the flow .

iii) The temperature field and concentration field are not significantly affected by the visco-elastic parameters.

Nomenclature

B_0 Strength of applied magnetic field, kg/s^2
C Dimensionless concentration
C_p Specific heat at constant pressure, J/kgK
D Chemical molecular diffusivity, m^2/s
D_1 Rate of chemical reaction
Ec Eckert number
Sc Schmidt number
T Fluid temperature,K
t Time,s
Gc Solutal Grashof number
Gr Thermal Grashof number
g Acceleration due to gravity, m/s^2
h Distance between the plates, m
K Permeability of porous medium
Kr Chemical reaction parameter
κ Thermal conductivity, W/mK
M magnetic parameter
Pr Prandtl number
Q_0 Heat absorption parameter
q Heat flux,W/m^2
R radiation parameter
S Heat source parameter
U_0 Mean flow velocity,m/s
u Dimensionless velocity.m/s
u_p Wall dimensionless velocity
α Mean radiation absorption coefficient
β Thermal expansion coefficient, K^{-1}
β^* Solutal expansion coefficient, K^{-1}
θ Dimensionless temperature
σ Stefan-Boltzmann constant
ρ Density,kg/m^3
ν Kinematic viscosity,m^2/s
ω Frequency of the oscillation
D_0 visco-elastic parameter

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REFERENCES

[1] Coleman, B.D.and Noll, W. (1960). An approximation theorem for functional with applications in continuum mechanics, Archs ration, MechAnalysis. 6, 355.
[2] Coleman, B. D. and Markovitz, H. (1964). Incompressible second order fluid. Adv. Appl. Mech. 8, 69.
[3] Misra, J. C., and Adhikary, S. D., 2016, "MHD oscillatory channel flow, heat and mass transfer in a physiological fluid in presence of chemical reaction," Alexandria Eng. J., 55, pp. 287–297.
[4] Muthucumaraswamy, R., and Manivannan, K., 2007, "Mass transfer effects on vertical oscillating plate with heat flux," Theoret. Appl. Mech., 34(4), pp. 309–322.
[5] Prasad, V. R., and Reddy, N. B., 2008, "Radiation and mass transfer effects on an unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with viscous dissipation," Indian J. of Pure Appl. Phys., 46, pp. 8192.
[6] Salawu, S.O., and Dada, M.S., 2016, "Radiative heat transfer of variable viscosity and thermal conductivity effects on inclined magnetic field with dissipation in a non-Darcy medium," J. Nigerian Mathe. Society, 35, pp. 93–106.
[7] Ahmed, S., Batin, A., and Chamkha, A. J., 2015, "Numerical/Laplace transform analysis for MHD radiating heat/mass transport in a Darcian porous regime bounded by an oscillating vertical surface," Alexandria Eng. J., 54, pp. 45–54.
[8] Chand, K., Singh, K. D., and Kumar, S., 2013, "Oscillatory free convective flow of viscous fluid through porous medium in a rotating vertical channel," Proc. Ind. Natn. Sci. Acad. DOI. 10.1007/s40010-013-0095-3.
[9] Sharma, P. R., Sharma, K., and Mehta, T., 2014, "Radiative and free convective effects on MHD flow through a porous medium with periodic wall temperature and heat generation or absorption, Int. J. of Mathe. archive, 5(9), pp. 119–128.
[10] Cogley, A. C. L., Vincenti, W. G., and Giles E. S., 2007, "Differential approximation for radiative heat transfer in a non-linear equation grey gas near equilibrium," AIAA Journal, 6, pp. 551–553.
[11] El-Hakien, M. A., 2000, "MHD oscillatory flow on free convection radiation through a porous medium with constant suction velocity," J. of Magnetism & Magnetic Materials, 220(2), pp. 271–276.
[12] Hamza, M. M., Isah, B. Y., and Usman, H., 2011, "Unsteady heat transfer to MHD oscillatory flow through a porous medium under slip condition," Int. J. Computer Appl., 33(4), pp. 12–17.
[13] Harisingh, Naik S., Rao, K. R., and Murthy, M. V. R., 2014, "Thermal radiation effect on an unsteady hydro-magnetic free convective oscillatory Couette flow of a viscous fluid embedded in a porous medium," Int. J. of Emerging Trends & Tech. in Comp. Sci., 3(4), pp. 6–12.
[14] Makinde, O. D., and Mhone, P. Y., 2005, "Heat transfer to MHD oscillatory flow in a channel filled with porous medium," Rom. J. Phys., 50, pp. 931–938.
[15] Balamurugan, K. S., Ramaprasad, J. L., and Kumar Varma, S. V., 2015, "Unsteady MHD free convective flow past a moving vertical plate with time dependent suction and chemical reaction in a slip flow regime," Int. Conf. comp. heat mass transfer, proced. eng., 127, pp. 516–523.
[16] Ibrahim, F.S., Elaiw, A.M., and Bakr, A.A., 2008, "Effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi–infinite vertical permeable moving plate with heat source and suction," Comm. Nonlinear Science and Numerical Simul., 13, pp. 1056–1066.
[17] Ibrahim, S. M., Gangadhar, K., and Reddy, N. B., 2015, "Radiation and mass transfer effects on MHD oscillatory flow in a channel filled with porous medium in the presence of chemical reaction," J. Appl. Fluid Mechanics, 8(3), pp. 529–537.
[18] Kataria, H. R., and Patel, H. R., 2016, "Radiation and chemical reaction effects on MHD Casson fluid flow past an oscillating vertical plate embedded in porous medium," Alexandria Eng. J., 55, pp. 583–595.
[19] Malapati, V., and Polarapu, P., 2015 "Unsteady MHD free convective heat and mass transfer in a boundary layer flow past a vertical permeable plate with thermal radiation and chemical reaction," Int. conference on comp. heat and mass transfer, 127, pp. 791–799.
[20] Mansour, M. A., ElAnssary, N. F., and Aly, A. M., 2008 "Effects of chemical reaction and viscous dissipation on MHD natural convection flows saturated in porous media," Int. J. Appl. Math. Mech., 4, 6076.
[21] Mohammed Ibrahim, S., Gangabhar, K., and Bhaskar Reddy., N., 2015 "Radiation and mass transfer effects on MHD oscillatory flow in a channel filled with porous medium in the presence of chemical reaction," J. Appl. Fluid Mechanics, 8(3), pp. 529–537.
[22] Sekhar, D. V., and Reddy, G. V., 2012, "Effects of chemical reaction on MHD free convective oscillatory flow past a porous plate with viscous dissipation and heat sink," Advances in Appl. Science Research, 3 (5), pp. 3206–3215.
[23] Kumari,K. and Goyal,M. "Viscous Dissipation and Mass transfer effects on MHD oscillatory flow in a vertical channel with porous medium" Advances in Dynamical Systems and Applications,12(2);pp205-216.

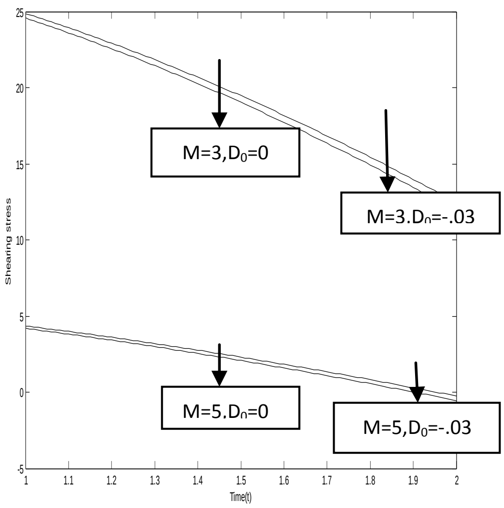

Figure 1:Variation of shearing stress (C_f) against time(t) for Gr=5,Gm=2,Pr=.71,R=1,Sc=.3,Kr=2,R=.5,S=2,K=1.5, U_p=.5,ε=.001,ω=0.5.

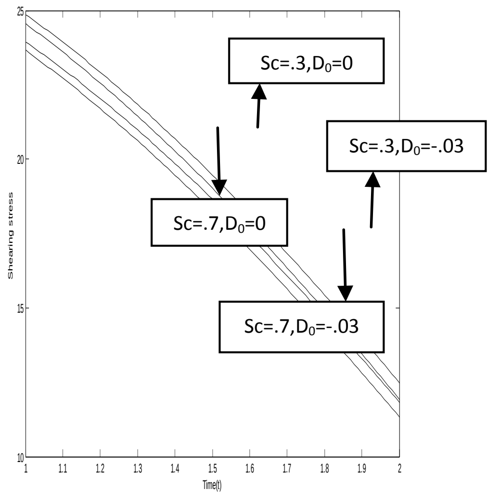

Figure 2:Variation of shearing stress (C_f) against time(t) for M=4,Gr=5,Gm=2,Pr=.71,R=1,Kr=2,R=.5, S=2,K=1.5,U_p=.5,ε=.001,ω=0.5.

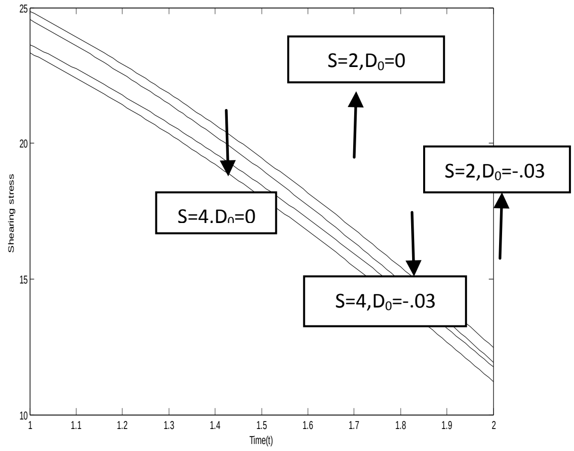

Figure 3:Variation of shearing stress (C_f) against time(t) for M=4,Gr=5,Gm=2,Pr=.71,R=1,Sc=.3,Kr=2,R=.5,K=1.5,U_p=.5,ε=.001,ω=0.5.

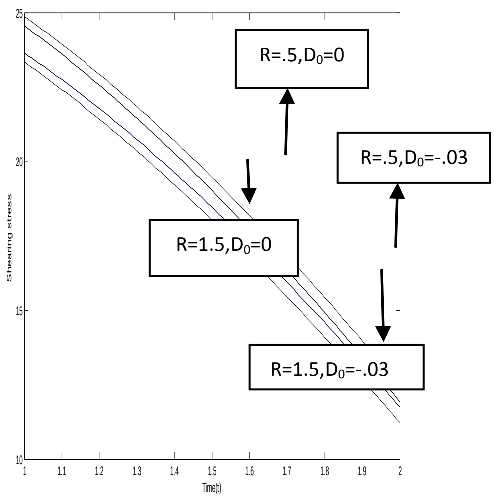

Figure 4:Variation of shearing stress (C_f) against time(t) for M=4, Gr=5,Gm=2,Pr=.71,R=1,Sc=.3,Kr=2,S=2,K=1.5,U_p=.5,ε=.001,ω=0.5.

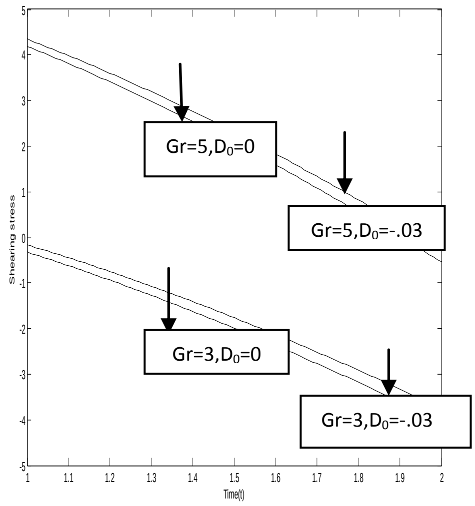

Figure5: Variation of shearing stress (C_f) against time(t) for M=4,Gm=2,Pr=.71,R=1,Sc=.3,Kr=2,R=.5,S=2,K=1.5, U_p=.5,ε=.001,ω=0.5.

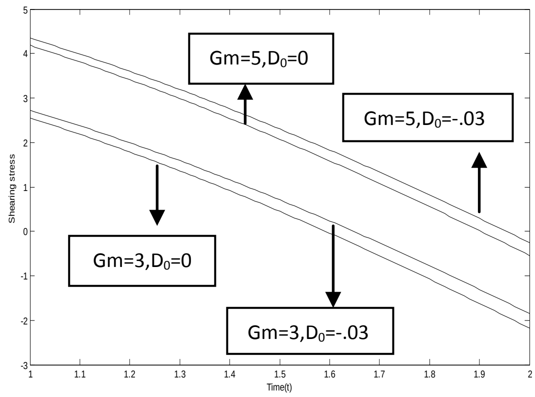

Figure 6: Variation of shearing stress (C_f) against time(t) for M=4,Gr=5,Pr=.71,R=1,Sc=.3,Kr=2,R=.5,

S=2,K=1.5,U_p=.5,ε=.001,ω=0.5.

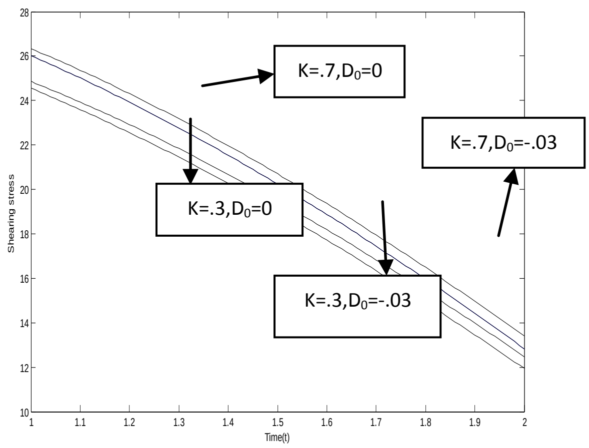

Figure 7: Variation of shearing stress (C_f) against time(t) for Gr=5,Gm=2,Pr=.71,R=1,Sc=.3,Kr=2,R=.5,S=2,U_p=.5, ε=.001,ω=0.5.