# On fuzzy soft convergence

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***Abstract:*** *In this research paper, we continue the study on fuzzy soft Filters and Nets investigate the properties and theorems of fuzzy soft Frechet Filter, fuzzy soft Union and Intersection filters, Fuzzy soft Supremum, Fuzzy soft Infinimum of the set of filters. Then discuss the fuzzy soft directed system and define and discuss the proportions of fuzzy soft Cluster point of a net. Which are important for further research on soft topology. This research not only can form the theoretical basis for further application of topology on soft sets but also learn about the practical applications of modeling and simulation in diverse area of Engineering.*

**Keywords:** *Fuzzy soft sets, Fuzzy soft Filter, Fuzzy soft Nets, Fuzzy soft Cluster point, Fuzzy soft union.*

## I.INTRODUCTION

Fuzzy sets and Fuzzy Convergences, found in the 1965 by Lotfi A. Zadeh [10] professor of Electrical engineering and Computer Science at Berkeley, U.S.A can be viewed as a broad conceptual framework enclosing the classical sets and fuzzy logic. In the last five decades significant progress has been made in the develop of fuzzy sets and fuzzy logic theory and their applications in all branches of science, engineering and socio economic sciences. This theory brought a paradigmatic change in mathematics. But there exists difficulty, how to set the membership function in each particular case. The theory of fuzzy soft sets is more generalized concept than the theory of fuzzy soft sets but this theory has same difficulties. In 1999, Molodtsov [3] initiated the concept of soft sets. Molodtsov [3], is free of the difficulties present in this theories. Pie and Mio [4] discussed the relationship between soft sets is a parameterized classifications of the objects of the universe. The topological structures of soft set theories dealing with uncertainties were first studied by Cheang [2] .He introduce the notion of fuzzy topology and also studied some basic properties. K. Borgohain [7, 8, 9] 2014 studied fuzzy soft separation axioms, fuzzy soft compact spaces. K. Borgohain[10] gives the new concept of fuzzy soft topological space. Some mathematicians [6],[11],[12 ],[13][15] studied on the compact fuzzy soft topological space. My main aim in this research paper is to develop the basic properties of fuzzy soft separated spaces and establish several equivalent forms of fuzzy soft topological spaces.

## II. PRELIMINARIES

Definition 2.1[ 13]A pair (F,A) is called a fuzzy soft set over U where F:A $\rightarrow \tilde{P}$(U) is a mapping from a into $\tilde{P}$(U)

Definition2.2 [11] A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subset of the set U. In other words, the soft set is a parameterized family of subsets of the set U. Every set F($\varepsilon$), $\varepsilon \in$ F from this family may be considered as the set of $\varepsilon$-elements of the soft set(F,E) or as the set of $\varepsilon$-approximate elements of the soft set.

Definition2.3 [10] Let U be an initial universe set and E be the set of parameters. Let $A \subseteq E$, a pair (F,A) is called a fuzzy soft set over U . Where F is a mapping given by $F : A \rightarrow I^U$, Where $I^U$ denotes the collection of all fuzzy subset of U.

Definition 2.4[10] If T is a fuzzy soft topology on (U, E) , then (U,E,T) is said to be a fuzzy soft topological space . Also each member of T is called a fuzzy soft open set in (U, E, T)

Definition 2.5[13] The complement of a fuzzy soft set (F,A) is denoted by $(F,A)^C$ and is defined by

$(F, A)^C = (F^C, A)$. Where $F^C$: A $\rightarrow \tilde{P}$(U) is mapping given by

$$F^C(\alpha) = U—F(\alpha)=[F(\alpha)]^C \ \forall \alpha \in A$$

Definition 2.6[6]

Union of two fuzzy soft sets (F, A) and (G, B) over a common universe U is a fuzzy soft set (H,C) where C= A $\tilde{\cup}$ B $\forall$ ε Є C, H (ε) = F(ε ) if ε Є A-B, G (B) if ε Є B-A, F (ε) $\tilde{\cup}$ G (ε) if ε Є A $\tilde{\cap}$ B

And is written as( F,A) $\tilde{\cup}$ (G,B) = (H,C)

Definition 2.7[9]Intersection of two fuzzy soft sets (F,A) and (G,B) over a common universe U is a fuzzy soft set

(H,C), Where C= A $\tilde{\bigcap}$ B , ε Є C, H(ε )= F(ε) $\tilde{\bigcap}$ G(ε) and is written as (F,A) $\tilde{\bigcap}$ (G,B) = (H,C)

Definition 2.8[10] Let (X,E,T) be fuzzy soft topological space. A fuzzy soft set is called fuzzy soft closed if its complement is a member of $\tilde{T}$.

Definition2.9 [12] if T is a fuzzy soft topology on (X,E), the triple (X,E,T) is said to be fuzzy soft topological space. Each member of T is called fuzzy soft open set in (X,E,T).

Definition2.10 [9] Let (X,E,T) be a fuzzy soft topological space. Let (F,A) be fuzzy soft set over (X,E).Then the fuzzy soft closure of (F,A) denoted by $\overline{(F,A)}$ is defined as the intersection of all fuzzy soft closed sets which contains (F,A).

Definition2.11 [8] Let A $\tilde{\subseteq}$ E. Then the mapping $F_A$: E $\rightarrow \tilde{P}$(U), defined by $F_A$(e)=$\mu^e$ $F_A$ (a fuzzy subset of U) is called fuzzy soft over (U,E), Where $\mu^e$ $F_A$=$\bar{O}$ if e$\in$E-A and $\mu^e$ $F_A \neq \bar{O}$ if e$\in A$. The set of all fuzzy soft set over (U,E) is denoted by FS(U,E)

Definition2.12[8] Let (X,E,T) be a fuzzy soft topological space. Let (F,A) be a fuzzy soft set over (X,E). Then the fuzzy soft interior of (F,A) denoted by $(F,A)^o$, is defined as union of all fuzzy soft open sets contained in (F,A). That is $(F,A)^o = \tilde{\bigcup}\{(G,B):(G,B)$ is fuzzy soft open and $(G,B)\tilde{\subseteq}(F,A)\}$

Definition 2.13[13]

A fuzzy soft topology T on (U,E) is a family of fuzzy soft sets over (U,E) satisfying the following properties

(i)$\bar{\emptyset},\bar{E} \in \tilde{T}$

(ii) If $F_A$, $G_B \in$ T then $F_A \tilde{\cap}$ $G_B \in \tilde{T}$

(iii) If $F^{\alpha}A_{\alpha} \in$ T for all $\alpha \in \Delta$ ,then $\tilde{\cup} F^{\alpha}A_{\alpha} \in \tilde{T}$

## III. MAIN RESULTS

Definition3.1 Let (X,E,T) be a non empty fuzzy soft sets and $(\boxed{F},E)$ be fuzzy soft non-empty subsets of (X,E,T). Then collection of $(\boxed{F},E)$ is said to be a fuzzy soft filter on (X,E,F) if satisfies the followings axioms

(i $\tilde{\phi} \notin (\boxed{F},E)$ i. e the empty fuzzy soft sets does not belongs to $(\boxed{F},E)$

(ii) If $\overline{(F,E)} \in (\boxed{F},E)$ and $(\boxed{H},E) \tilde{\supseteq} \overline{(F,E)}$ then $(\boxed{H},E) \in (\boxed{F},E)$ i.e every fuzzy soft super set of a member of $(\boxed{F},E)$ is a member of $(\boxed{F},E)$.

(iii) if $\overline{(F,E)} \in (\boxed{F},E)$ and $(\boxed{H},E) \in \overline{(F,E)}$ then $\overline{(F,E)} \tilde{\cap} (\boxed{H},E) \in (\boxed{F},E)$ i.e the intersection of any two fuzzy soft members of $(\boxed{F},E)$ is $(\boxed{F},E)$.

Example3.1 Let X= {a,b,c} then write whether the followings collection are fuzzy soft filter or not. If not fuzzy soft filter then why

$F_1$={X}, $F_2$={X,{a,b}}, $F_3$={X,{a},{a,b},{a,c}}

All the above of collections are fuzzy soft filters on X as they clearly satisfy the axioms of fuzzy soft filter.

$F_4$={X,{a,b},{b,c}} it is not a fuzzy soft filter on X as {a,b}$\tilde{\cap}${b,c} = {b} which does not belongs to $F_4$ and hence axiom $F_3$ is not satisfied.

$F_5$={X, {a},{b}{a,b},{b,c}}

It is not a Fuzzy soft filter on X as {a} $\in F_5$ and {a,c } $\tilde{\supseteq}$ {a} but {a,c} does not belongs to $F_5$ and hence axiom belongs to hold good.

Definition3.2 If (X,E,T) be an Fuzzy soft infinite set and *F* be a collection of all those non empty fuzzy soft subsets of X whose complements are finite then *F* is a filter on X called the fuzzy soft co-finite filter.

Let *F*={A: X− A finite fuzzy soft sets}

*F* is certainly non -empty as X − X =Φ i.e fuzzy soft finite and hence X ∈ *F*

$F_1$ The fuzzy soft complement i .e X− Φ= X i.e fuzzy soft infinite and hence Φ ∉ *F*

$F_2$ Let A ∈ *F* so that X− A finite fuzzy soft sets .If B $\tilde{\supseteq}$ A then X− B $\tilde{\subseteq}$ X− A and since X− A is fuzzy soft finite and X− B is also fuzzy soft finite and hence B ∈ *F*

$F_3$ Let A, B ∈ *F*, So that X− B and X− A are both fuzzy soft finite and hence X− B $\tilde{\cup}$ X− A is also fuzzy soft finite set.

Definition3.3 Let (Y,T,E) be fuzzy soft topological space and $N_x$ be the collection of all fuzzy soft nbds of a point x∈ X .Then $N_x$ is a fuzzy soft filter on X is called the fuzzy soft neighborhood filter of x.

Definition3.4 Fuzzy soft Comparison of filters: If $F_1$ and $F_2$ be two fuzzy soft filters on X then $F_1$ is said to be fuzzy soft coarser then $F_2$ or $F_2$ is said to be fuzzy soft finer than $F_1$ if and only if $F_1 \tilde{\subseteq}$ $F_2$ ,if $F_1 \tilde{\subseteq}$ $F_2$ and at the same time $F_1 \neq$ $F_2$ then $F_1$ is strictly coarser than $F_2$ or $F_2$ is fuzzy soft strictly finer than $F_1$

Definition3.5 Fuzzy soft Comparable filters: I case either $F_1 \tilde{\subseteq}$ $F_2$ or $F_2 \tilde{\subseteq}$ $F_1$ then the fuzzy soft filters $F_1$ and $F_2$ are said to be fuzzy soft comparable.

Proposition3.1 Let (X,E,T) be a fuzzy soft infinite set and *F* be a fuzzy soft filter on X such that $\tilde{\bigcap}$ {A:A ∈ *F*

}=Φ then $F$ is fuzzy soft finer than the fuzzy soft co finite filter c.

Proof: Let C={B: X-B is fuzzy soft finite} is the fuzzy soft co finite filter. We are to prove that $F$ is fuzzy soft finer than $C$ i. e $C \in F$. Suppose that it is not true that $C \not\subset F$ this means that there exists a member B $\in C$ such that B $\notin F$. Since B $\in C$ therefore X-B is fuzzy soft finite and let it be $\{x_1, x_2, \ldots\ldots\ldots, x_n\}$

Again $\tilde{\bigcap}$ {A:A $\in F$ }=Φ therefore there exist $A_i \in F$ such that $x_i \notin A_i$

As $F$ is fuzzy soft filter then $\tilde{\bigcap} A_i \in F$. Thus G$\in F$ and G will not contain any element of X-B $=\{x_1, x_2, \ldots\ldots\ldots, x_n\}$

Since G$\in F$ and B is fuzzy soft super set of G so that B$\in F$, But this is a contradiction to our supposition that B$\notin F$. Hence $F$ is fuzzy soft finer than fuzzy soft co finite filter $C$.

Proposition3.2 Let X be a fuzzy soft non empty set then the intersection of an arbitrary non empty collection of fuzzy soft filters on X is itself a fuzzy soft filter on X.

Proof: Let $\{F_\alpha : \alpha \in \Lambda\}$ be any fuzzy soft non empty collection of filters on X. Again let $\tilde{\bigcap}\{F_\alpha : \alpha \in \Lambda\}$

We are to prove that F is a fuzzy soft filter on X.

Since $F_\alpha$ is a fuzzy soft filter, X $\in F_\alpha$ and hence X $\in F$ their intersection. So that $F$ is fuzzy soft non empty.

If Φ $\notin F_\alpha$, So does not belongs to $F$ their intersection.

Let A$\in F$ and B$\supset$A so that A$\in F_\alpha$ where $F_\alpha$ is a fuzzy soft filter. Hence B$\supset$A for each $F_\alpha$. We conclude that B$\in F_\alpha$. This means B$\in$ $F=\tilde{\bigcap}\{F_\alpha : \alpha \in \Lambda\}$

Also Let A$\in F$ and B$\in F$ so that A$\in F_\alpha$ and B$\in F_\alpha$, Again as each $F_\alpha$ is a fuzzy soft filter therefore A$\in F_\alpha$, B$\in F_\alpha$ this imply A$\tilde{\bigcap}$ B$\in F_\alpha$. Hence A$\tilde{\bigcap}$ B$\tilde{\subset} F$.

Hence $F=\tilde{\bigcap}\{F_\alpha : \alpha \in \Lambda\}$ is a fuzzy soft filter on X.

Definition3.6 Let X be any fuzzy soft non empty set. A fuzzy soft filter base on X is a non empty family B of subset of X satisfying the following properties

i) $\phi \notin B$ ii)if $F \in B$ and $H \in B$ then there exists a $G \in B$ such that $G \tilde{\subset} F \tilde{\cap} H$

Example 3.1 The fuzzy soft family B={A} where A is any non void subset of a non void set X is a fuzzy soft filter base on X

Solution: Here B consists a single member A. Since A is non –void, $\phi \notin B$ and $A \tilde{\subset} A \tilde{\cap} A$. So B is a fuzzy soft filter base on X.

Proposition3.2 Every fuzzy soft filter is a filter base.

Proof: Let F be fuzzy soft filter and $\phi \notin F$ which is condition of fuzzy soft base filter.

Again let $F_1 \in$F and $F_2 \in$F then $F_1 \tilde{\cap} F_2 \in F$ and since $F_1 \tilde{\cap} F_2 \tilde{\supset} F_1 \tilde{\cap} F_2$ the condition of fuzzy soft base filter also satisfied. Hence every fuzzy soft base filter is a fuzzy soft base filter.

Definition3.7 Let F be a fuzzy soft filter on a set X and let $A \tilde{\subset} X$. Then F is said to be fuzzy soft eventually in A iff $A \in F$

Definition3.8 A fuzzy soft filter on a set X is said to be fuzzy soft frequently in a fuzzy soft subset A of X iff A intersects every member of F, that is $A \tilde{\cap} F \neq \phi, \forall F \in$F

Remarks 3.1 i) F is fuzzy soft frequently in a if F is fuzzy soft eventually in A but non- conversely. For $A \in F \Rightarrow A \tilde{\cup} F = \varnothing \forall F \in$F

For the converse, consider the fuzzy soft filter F={{b,c},X} on X ={a,b,c} and fuzzy soft set A={b}. The fuzzy soft filter F is fuzzy soft frequently in A Since $A \notin F$

Definition3.9 Let (X,T,E) be a fuzzy soft topological space and $F$ be fuzzy soft a filter on X. Then $F$ is said to be fuzzy soft convergent to a point x$\in$X iff $F$ is fuzzy soft eventually in each neighborhood of x, that is ,iff every nbd of x is a member of $F$ and we say that x is fuzzy soft limit point and written as $F \to x$

Definition3.10 A fuzzy soft filter base B on X is said to be converge to a point $x \in X$ iff the filter whose base is B converges to x and we say that x is a limit point of B

Example 3.2 A discrete space X is only fuzzy soft convergent filter are nbd filter.

Proposition3.3 I an indiscrete space X , Every fuzzy soft filter on X converges to every point of X.

Proof: Let $F$ be any fuzzy soft filter on X and let x be any point of X. The neighborhood filter of x in this case is {X}. Since X$\in F$, it follows that $F$ converges to x. Since x was an arbitrary point of X, We see that $F$ converges to every point of X.

Definition3.11 Fuzzy soft supremum of the set of filters : Let M be a non empty collection of fuzzy soft filters on a non empty fuzzy soft set X. and M={F: F is a fuzzy soft filter on X} then fuzzy soft filter F is said to be fuzzy soft supremum of M iff

i) F is fuzzy soft finer than every other fuzzy soft filter in M
ii) If $F^{/}$ is any other fuzzy soft filter on X which is fuzzy soft finer than every other fuzzy soft filter in M then F is coarser than $F^{/}$

Definition3.12 The fuzzy soft filter F is said to be fuzzy soft infinimum of M if and only if i) F is fuzzy soft coarser than every other fuzzy soft filter in M ii) If $F^{/}$ is any other fuzzy soft filter on X which is fuzzy soft coarser than every other fuzzy soft filter in M then F is fuzzy soft finer than $F^{/}$

Example 3.3 Let X= {a,b,c} , we have the following fuzzy soft filters.

$F_1$={X}

$F_2$={{a,b}X}

$F_3=\{\{b,c\},X\}$
$F_4=\{\{c,a\}X\}$
$F_5=\{\{a\},\{a,b\},\{a,c\},X\}$
$F_6=\{\{b\},\{a,b\},\{b,c\},X\}$
Let $M_1=\{ F_1, F_2, F_3, F_4\}$

Clearly $F_1$ is the fuzzy soft infimum of $M_1$, as it is the only fuzzy soft filter on X which is fuzzy soft coarser than every member of $M_1$. But $M_1$ has no fuzzy soft supremum as there is no fuzzy soft filter in $M_1$ which is fuzzy soft finer than each member of $M_1$

## IV. CONCLUSION

Now a day, the study of fuzzy mathematics is a branch of scientific knowledge about quantity, structure, space and change, spread into more than a hundred branches to nourish other applied and natural sciences. The ensuring fuzzy mathematics in conjunction with technology is being able to solve several problems related to various fields, including social and behavioral sciences, arrangements, design patterns, sequences and schedules etc. In this regards I have continued to study the fuzzy soft filters. Introduced Fuzzy soft Comparison of filters, Comparable filters and have established several interesting propositions, theorems, examples. I hope that the findings in this research work will help researchers enhance and promote the father study on fuzzy soft topology to carry out a general framework for their applications in practical life.

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