Anti Fuzzy Meet Subsemilattice

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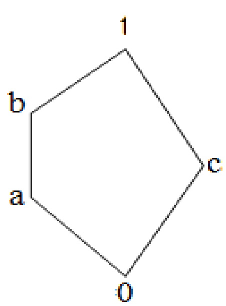
Abstract: *In this paper, we made an attempt to define and study some properties of Anti Fuzzy meet subsemilattice and we introduce some definitions and theorems on the union and intersection of Anti Fuzzy meet subsemilattice.*

Key words: *Fuzzy meet semilattice, Fuzzy meet subsemilattice,Anti fuzzy meet subsemilattice,Anti ,Fuzzy level meet subsemilattice.*

Introduction: The notion of Fuzzy sets was introduced by Zadeh,L.A.[38] in 1965.He has initiated fuzzy set theory as a modification of ordinary set theory. In this paper we define Anti fuzzy subsemilattice,Anti Fuzzy level meet subsemilattice and some related theorems.

Definition 1: Let A be a Fuzzy meet semilattice. A fuzzy subset $T_\mu : A \to [0,1]$ of a fuzzy meet semilattice A is called a anti Fuzzy meet subsemilattice of A if $\forall x, y \in A,\ T_\mu(x \wedge y) \le \max\{T_\mu(x), T_\mu(y)\}$

Example 1: Let A={0,a,b,c,1}. Let $T_\mu : A \to [0,1]$ be a Fuzzy subset in A defined by $T_\mu(0) = 0.4, T_\mu(a) = 0.7, T_\mu(b) = 0.6,$ $T_\mu(c) = 0.5, T_\mu(1) = 0.9$



Thus, T_μ is an anti fuzzy meet subsemilattice.

Definition 2: Let T_μ be any anti fuzzy meet subsemilattice of a fuzzy meet semilattice A and let $t \in [0,1]$. Then $T_{\mu t} = \{x \in A / T_\mu(x) \ge t\}$ is called anti fuzzy level meet subsemilattice of T_μ.

Example 2: From Example(1), Let t=0.6.Then $T_{\mu t}$ ={a,1}. Then $T_{\mu t}$ is an anti fuzzy level meet subsemilattice of T_μ.

Remark: $T_{\mu t} \subseteq T_{\mu S}$ whenever t>s

Definition 3: Let $T_{\mu 1}$ and $T_{\mu 2}$ be any two anti fuzzy meet subsemilattices of a fuzzy meet semi lattice A. Then $T_{\mu 1}$ is said to be contained in $T_{\mu 2}$ if $T_{\mu 1}(x) \le T_{\mu 2}(x), \forall x \in A$ and is denoted by $T_{\mu 1} \subseteq T_{\mu 2.}$

Definition 4: Let $T_{\mu 1}$ and $T_{\mu 2}$ be any two anti fuzzy meet subsemilattices of a fuzzy meet semi lattice A. If $T_{\mu 1}(x) = T_{\mu 2}(x), \forall x \in A$, then $T_{\mu 1}$ and $T_{\mu 2}$ are said to be equal and it is written as $T_{\mu 1} = T_{\mu 2}$.

Definition 5: The complement of a anti fuzzy meet subsemilattice T_μ of a fuzzy meet semilattice A symbolized by $\sim T_\mu(x) = 1 - T_\mu(x), \forall x \in A$.

Definition 6: The intersection of two anti fuzzy meet subsemilattices $T_{\mu 1}$ and $T_{\mu 2}$ of a fuzzy meet semilattice A is defined as $[T_{\mu 1} \cap T_{\mu 2}](x) = \min\{T_{\mu 1}(x), T_{\mu 2}(x)\}, \forall x \in A$.

Definition 7: The union of two anti fuzzy meet subsemilattices $T_{\mu 1}$ and $T_{\mu 2}$ of a fuzzy meet semilattice A is defined as $[T_{\mu 1} \cup T_{\mu 2}](x) = \max\{T_{\mu 1}(x), T_{\mu 2}(x)\}, \forall x \in A$.

Lemma 1: Let T_{μ} be an anti fuzzy meet subsemilattice of a fuzzy meet semilattice A and let t,s $\in \mathrm{Im} T_{\mu}$ .Then $T_{\mu t} = T_{\mu s}$ if t=s.

Proof:

If t=s ,then $T_{\mu t} = T_{\mu s}$.

Conversely,

Let $T_{\mu t} = T_{\mu s}$.

Since t $\in \mathrm{Im} T_{\mu}$, $\exists$ x $\in$ A such that $T_{\mu}(x) = t,$

$\Rightarrow$ t $\in T_{\mu s}$

Hence t= $T_{\mu}(x) \geq s$ -------------- (1)

Similarly,

it can be proved that s $\geq$ t----------- (2)

Then from (1) and (2), t=s.

Theorem 1: Two anti fuzzy meet subsemilattices T_{μ} and T_{θ} of a fuzzy meet semilattice A such that the card Im $T_{\mu} < \infty$ are equal iff Im T_{μ} =Im T_{θ} and $F_{S\mu} = F_{S\theta}$ where $F_{S\mu}$ ={ $T_{\mu t} / T_{\mu t}$ is an anti fuzzy meet subsemilattices of A for all t $\in \mathrm{Im} T_{\mu}$ } and $F_{S\theta} = \{T_{\theta t} / T_{\theta t}$ is an anti fuzzy level meet subsemilattice of A for all t $\in$ Im $T_{\theta}\}$.

Proof: Let T_{μ} and T_{θ} be two anti fuzzy meet subsemilattices of a fuzzy meet semilattice A such that the card Im $T_{\mu} < \infty$

Assume that T_{μ} and T_{θ} are equal

(ie) $T_{\mu}(x) = T_{\theta}(x)$ ------------------------(1)

$$\Rightarrow T_{\mu}(x) \in \mathrm{Im} T_{\mu}$$
$$\Rightarrow T_{\theta}(x) \in \mathrm{Im} T_{\mu}$$
$$But T_{\theta}(x) \in \mathrm{Im} T_{\theta}$$
$$\Rightarrow \mathrm{Im} T_{\mu} \subseteq \mathrm{Im} T_{\theta} ----------(2)$$

Similarly ,

it can be proved that $\mathrm{Im} T_{\theta} \subseteq \mathrm{Im} T_{\mu}$ ------- (3)

(2)and(3) $\Rightarrow \mathrm{Im} T_{\mu} = \mathrm{Im} T_{\theta} --------(4)$

$$Let T_{\mu t} \in F_{S\mu} and, x \in T_{\mu t}, t \leq T_{\mu}(0)$$
$$\Rightarrow T_{\mu}(x) \geq t, t \in \mathrm{Im} T_{\mu}$$
$$\Rightarrow T_{\theta}(x) \geq t, t \in \mathrm{Im} T_{\theta}, by (1) and (4).$$
$$\Rightarrow x \in T_{\theta t}, t \leq T_{\mu}(0) = T_{\theta}(0), by (1)$$

$$T_{\mu t} \subseteq T_{\theta t}$$

Similarly, it can be proved that $T_{\theta t} \subseteq T_{\mu t}$

$$Hence, T_{\mu t} = T_{\theta t}$$
$$\Rightarrow T_{\theta t} \in F_{S\mu} but, T_{\theta t} \in F_{S\theta}$$
$$\Rightarrow F_{S\theta} \subseteq F_{S\mu} --------------------(5)$$
$$Similarly, F_{S\mu} \subseteq F_{S\theta} ----------------(6)$$
$$(5) \& (6) \Rightarrow F_{S\mu} = F_{S\theta} --------------(7)$$

Equation (4) and (7) completes the proof.

Conversely, assume that Im T_{μ} =Im T_{θ} and $F_{S\mu} = F_{S\theta}$

To prove: T_{μ} and T_{θ} are equal.

Suppose $T_{\mu}(x) \neq T_{\theta}(x)$ for $x \in A$

Then either $\operatorname{Im} T_\mu \neq \operatorname{Im} T_\theta$ or $F_{S\mu} \neq F_{S\theta}$.

This is a contradiction

Hence $T_\mu(x) = T_\theta(x), \forall x \in A.$

Therefore $T_\mu and, T_\theta$ are equal.

Theorem 2

If B is a fuzzy meet semilattice of A, B≠A ,Then the anti fuzzy meet subsemilattice T_μ of A and is defined by

$$T_\mu(x) = \begin{cases} s, if & x \in B \\ t, if & x \in A \sim B \end{cases}$$

where s,t $\in$[0,1], s>t is an anti fuzzy meet subsemilattice of A.

Proof:

Let x,y $\in$ A

To prove: T_μ is an anti fuzzy meet subsemi lattice of A.

(ie)To Prove: .$T_\mu(x \wedge y) \leq \max\{T_\mu(x), T_\mu(y)\}$

It is proved by considering exhaustive three cases.

Case(i)

Let x,y $\in$ B. $T_\mu(x) = s, T_\mu(y) = s.$

As x,y $\in B$, $x \wedge y \in B$,since B is a fuzzy meet semilattice.

$$T_\mu(x \wedge y) \leq \max\{T_\mu(x), T_\mu(y)\}$$

= max{s.s}

=s

Case(ii):

Let x $\in$ B,y $\in$ A~B. $T_\mu(x) = s, T_\mu(y) = t$

As x $\in$ B,y $\in$ A~B, $x \wedge y \in A$

Now $T_\mu(x) = s > t = T_\mu(y)$

(ie) $T_\mu(x) > T_\mu(y)$

$$x \wedge y \in B \Rightarrow x \wedge y = \omega, for\, \omega \in B$$
$$T_\mu(\varpi) = T_\mu(x \wedge y) \leq \max\{T_\mu(x), T_\mu(y)\}$$

=max{s,s}

=s

$$x \wedge y \in A \sim B \Rightarrow x \wedge y = \omega, for\, \varpi \in A \sim B$$
$$T_\mu(\omega) = T_\mu(x \wedge y) \leq \max\{T_\mu(x), T_\mu(y)\}$$

=max{t,t}

=t

Hence T_μ is an anti fuzzy meet subsemi lattice.

Case(iii)

Let x, y $\in$ A~B, $T_\mu(x) = t, T_\mu(y) = t$

As x,y $\in$ A~B, $x \wedge y \in A \sim B$ or B

If x $\wedge$ $y \in A$~B, then

$$T_\mu(\omega) = T_\mu(x \wedge y) \leq \max\{T_\mu(x), T_\mu(y)\}$$

=max{t,t}

=t

If $x \wedge y \in B$,then

$$T_\mu(x \wedge y) \leq \max\{T_\mu(x), T_\mu(y)\}$$

=max{s,s}

=s

Therefore T_μ is an anti fuzzy meet subsemilattice.

Hence T_μ is an anti fuzzy meet subsemilattice of A in all the three cases.

Proposition 1:

A non empty fuzzy meet subset C of A is a fuzzy meet semilattice of A iff χ_C is an anti fuzzy meet subsemilattice of A.

Proof:

χ_C is nothing but characteristic function of the anti fuzzy meet subsemi lattice of C.

(ie) $\chi_C(x) = \begin{Bmatrix} 1 & x \in C \\ 0 & x \in A \sim C \end{Bmatrix}$

Where s,t $\in [0,1]$

Then by previous theorem,the proof is complete.

Theorem 3:

The intersection of two anti fuzzy meet subsemilattices of a fuzzy meet semilattice A is also an anti fuzzy meet subsemilattice of A.

Proof:

Let A be a fuzzy meet semilattice.

Let $T_{\mu 1}$ and $T_{\mu 2}$ be any two anti fuzzy meet subsemilattices of A.

T.P: $T_{\mu 1} \cap T_{\mu 2}$ is an anti fuzzy meet subsemilattice of A.

Let a,b $\in T_{\mu 1} \cap T_{\mu 2}$

Then a,b $\in T_{\mu 1}$ and a,b $\in T_{\mu 2}$.

$\Rightarrow a \wedge b \in T_{\mu 1} and, a \wedge b \in T_{\mu 2}$

Hence $a \wedge b \in T_{\mu 1} \cap T_{\mu 2}$.

Therefore $T_{\mu 1} \cap T_{\mu 2}$ is an anti fuzzy meet subsemilattice of A.

Theorem 4: The union of anti fuzzy meet subsemilattices of a fuzzy meet semilattice A is also an anti fuzzy meet subsemilattice of A iff one contained in the other.

Proof:

Let A be a fuzzy meet semilattice.

Let $T_{\mu 1}$ and $T_{\mu 2}$ be any two anti fuzzy meet subsemilattices of A such that one contained in the other.

$T_{\mu 1} \subseteq T_{\mu 2}$ or $T_{\mu 2} \subseteq T_{\mu 1}$

$\Rightarrow T_{\mu 1} \cup T_{\mu 2} = T_{\mu 1}$ or $T_{\mu 1} \cup T_{\mu 2} = T_{\mu 2}$

Then $T_{\mu 1} \cup T_{\mu 2}$ is an anti fuzzy meet subsemilattice of A.

Conversely, suppose $T_{\mu 1} \cup T_{\mu 2}$ is an anti fuzzy meet subsemilattice of A.

T.P: $T_{\mu 1} \subseteq T_{\mu 2}$ or $T_{\mu 2} \subseteq T_{\mu 1}$

Suppose $T_{\mu 1}$ is not contained in $T_{\mu 2}$

Then there exist an element a,b such that

$$a \in T_{\mu 1} and, a \notin T_{\mu 2} ------------(1)$$
$$b \in T_{\mu 2} and, b \notin T_{\mu 1} ------------(2)$$

Clearly, a,b $\in T_{\mu 1} \cup T_{\mu 2}$

Since $T_{\mu 1} \cup T_{\mu 2}$ is an antifuzzy meet subsemilattice of A, $a \wedge b \in T_{\mu 1}$ or $T_{\mu 2}$

Case(i)

Let $a \wedge b \in T_{\mu 1}$

Since a $\in T_{\mu 1}$ and $a' \in T_{\mu 1}$

$a' \vee (a \wedge b) = (a' \vee a) \wedge b = 1 \wedge b = b \in T_{\mu 1}$

Which is a contradiction to $b \notin T_{\mu 1,} by(2)$

Case(ii)

Let $a \wedge b \in T_{\mu 2}$

Since $b \in T_{\mu 2} and, b' \in T_{\mu 2}$

Hence
$(a \wedge b) \vee b' = a \wedge (b \vee b') = a \wedge 1 = a \in T_{\mu 2}$

Which is a contradiction to the assumption $a \notin T_{\mu 2} by(1)$

Hence the assumption that $T_{\mu 1}$ is not contained in $T_{\mu 1}$ and $T_{\mu 2}$ is not contained in $T_{\mu 1}$ is false.

Therefore either $T_{\mu 1} \subseteq T_{\mu 2}$ or $T_{\mu 2} \subseteq T_{\mu 1}$.

Theorem 5 :

Let T_θ be any fuzzy meet subsemilattice of a fuzzy meet semilattice A Such that Im T_θ = {0, t}, where $t \in [0,1]$. If $T_\theta = T_\mu \cup T_\sigma$, where T_μ, and T_σ are fuzzy meet subsemilattices of A, then either $T_\mu \subseteq T_\sigma$, or $T_\sigma \subseteq T_\mu$.

Proof

Suppose $T\mu \not\subset T_\sigma$ or $T_\sigma \not\subset T\mu$ then there exist some x, y $\in$A such that

$T_\mu(x) > T_\sigma(x)$ and $T_\sigma(y) > T_\mu(y)$

Then $t = T_\theta(x) = [T_\mu \cup T_\sigma](x)$.

$= \max \{T_\mu, (x), T_\sigma(x)\}$

$= T_\mu(x) \geq 0$, Since $T_\mu(x) > T_\sigma(x)$

and $t = T_\theta(y) = [T_\mu \cup T_\sigma](y)$

$= \max [T_\mu(y), T_\sigma(y)\}$

$= T_\sigma(y) \geq 0$, Since $T_\sigma(y) > T_\mu(y)$

Therefore $T_\theta(x) = t = T_\sigma(y)$

$\Rightarrow T_\mu(x) = t = T_\sigma(y)$

$\Rightarrow T_\sigma(y) < T_\sigma(x)$ and $T_\mu(x) < T_\mu(y)$

Then $T_\mu(x \wedge y) \leq \max \{T_\mu(x), T_\mu(y)\}$

$= T_\mu(y) < t$ …………………..(1)

and $T_\sigma(x \wedge y) \leq \max \{T_\sigma(x), T_\sigma(y)\}$

$= T_\sigma(x) < t$……………………..…(2)

hence $t = T_\theta(x \wedge y) = [T_\mu \cup T_\sigma](x \wedge y)$

$= \max \{T_\mu(x \wedge y), T_\sigma(x \wedge y)\}$

$= \max\{T_\mu(y), T_\sigma(x)\}$

$< t$ by (1) and (2) which is a contradiction.

Therefore, if $T_\theta = T\mu \cup T_\sigma$, then either $T_\mu \subseteq T_\sigma$ or $T_\sigma \subseteq T_\mu$.

Conclusion: Thus in this paper, we have defined Anti fuzzy meet subsemilattice and some related theorems.

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