Optimal Ordering Policy for Deteriorating Items with Price and Credit Period Sensitive Demand under Default Risk

[1]Dhir Singh & [2]Amit Kumar
[1]*Dept. of Mathematics, Shaheed Heera Singh Govt. Degree College, Dhanapur (Chandauli), India*
[2]*Dept. of Mathematics, Govt. Degree College, Nadhabhood(Sahaswan),Badaun, India*

Abstract
In this paper, an inventory model for deteriorating items with selling price and credit period sensitive demand is developed. The default risk associated with sales revenue of the retailer is also taken into consideration. Here, shortages are allowed and partially backlogged. To make the study close to reality, the holding cost is considered as a time dependent function. This study provides a procedure to develop the total retailer's profit function per unit time of the system and optimal ordering quantity per cycle for the retailer. Finally, the model is illustrated with a numerical example and to study the effect of changes of different system parameters on the total retailer's profit per unit time of the system, sensitivity analysis is performed by changing one parameter at a time and preserving the other parameters at their original values.

Keywords: *Inventory, deterioration, price and credit period sensitive demand, and default risk.*

1. Introduction

Most of the existing inventory models under trade credit financing are assuming that buyer pays instantly purchasing cost of the items as soon as the items are received. However, such assumption is not necessarily what happens in the real world. In practice, the retailer offers a delay period known as the trade credit period to her customers to pay for more purchasing. And this practice may be proved to be an excellent strategy in today's competitive scenario. But at the same time, there may be a chance of default risk for the retailer if buyer can't pay his dues. Obviously, the longer delay period may increase the default risk.

During the past few years, many articles dealing with a range of inventory models under trade credit financing have appeared in various national and international journals. At the earliest, Goyal [8] established an inventory model for a single item under permissible delay in payments when selling price equal to the purchase cost. Aggarwal and Jaggi [3] extended Goyal [8] model for the deteriorating items. Jamal et al. [9] further generalized the Goyal [8] model to allow the shortages. Teng [14] extended Goyal [8] model by considering the difference between selling price and purchasing cost. Abad [1] developed an optimal pricing and lot sizing inventory model for a reseller considering selling price dependent demand. Abad [2] formulated optimal lot sizing policies for perishable goods in a finite production inventory model with partial backlogging and lost sales. Dye *et al.* [6] determined optimal selling price and lot size with a variable rate of deterioration and exponential partial backlogging. Kumari *et al.* [10] presented two warehouse inventory model for deteriorating items with partial backlogging under the conditions of permissible delay in payments. Chang *et al.*[4] investigated a partial backlogging inventory model for non instantaneous deteriorating items. They assumed that the demand of the items are stock dependent, and proposed a mathematical model to find the minimum total relevant cost. Liao *et al.* [11] investigated a distribution free newsvendor model with balking and lost sales penalty. Teng and Lou [15] proposed the demand rate is an increasing function of the trade credit period. Lou and Wang [12] studied optimal trade credit and order quantity by considering trade credit with a positive correlation of market sales, but are negatively correlated with credit risk.

Wu et al. [16] explored optimal credit period and lot size by considering delayed payment time dependent demand under default risk for deteriorating items with expiration dates. Dye and Yang [7] discussed the sustainable trade credit and replenishment policy with credit-linked demand and credit risk considering the carbon emission constraints. Chen and Teng [5] extended Teng and Lou's model [15] to consider time varying deteriorating items and default risk rates under two levels of trade credit. Wu and Zhao [17] discussed two retailer–supplier supply chain models with default risk under the trade credit policy. Singh and Singh [13] recently developed an optimal inventory policy for deteriorating items with stock level and selling price dependent demand under the permissible delay in payments.

However, none of the paper discusses the optimal ordering policy by considering the demand of the products to a credit period sensitive and selling price dependent involving default risk. Therefore, we develop an economic order quantity model for the retailer under the following scenario: (1) the retailer provides a trade credit period to his customer, (2) the retailer's trade credit period to the buyer not only increases sales and revenue but also increases the default risk, (3) the demand varies simultaneously with price and the length of credit period that is offered to the customers, (4) shortages are allowed and partially backlogged, and (5) To make the study close to reality, the holding cost is considered as a linear function of time. Finally, the model is illustrated with a numerical example and to study the effect of changes of different system parameters on the order quantity and the total retailer's profit per unit time of the system, sensitivity analysis is performed.

2. Assumptions

The following assumptions are used in developing the mathematical inventory model.

1. The product considered in this model is deteriorating in nature and there is no repair or replacement of the deteriorated units during the complete cycle.
2. The demand $D(s,n)$ of the products is a linear function of the selling price and the credit period. For simplicity, the demand rate $D(s,n)$ may be given by $D(s,n) = a - bs + cn$, where $a, b,$ and c are non-negative parameters. Also, s is the selling price of the product and n is the credit period offered by the retailer.
3. Shortages are allowed and backlogged partially. The backlogging rate depends on the length of the waiting time for the next replenishment. For simplicity, the backlogging rate of negative inventory is given by $B(T-t) = e^{-\delta(T-t)}$, Where δ is known as backlogging parameter with $0 < \delta < 1$ and $(T-t)$ is the waiting time up to the next replenishment.
4. The deterioration rate $\theta(t)$ of the product is defined as a two-parameter Weibull function $\theta(t) = \alpha\beta t^{\beta-1}$, where $0 \le \alpha < 1, \beta \ge 1$, α is known as the scale parameter and β is the shape parameter.
5. Replenishment is instantaneous and lead time is zero.
6. The default risk to the retailer depends on the credit period offered by his/her to their customers. For simplicity, the rate of the default risk with respect to the credit period offered by the retailer is taken as $f(n) = 1 - n^{-\lambda}$, where λ is non-negative default parameter.
7. The planning horizon is infinite and the inventory system involves only one product.
8. Holding cost $h(t)$ per unit per unit time is assumed to be an increasing function of time and expressed as $h(t) = h_1 + h_2 t$ where $h_1, h_2 \ge 0$

3. Notations

The following notations are used in developing the mathematical inventory model.

i. a,b,c : Demand parameters
ii. h_1, h_2 : Holding cost parameters
iii. α, β : Deterioration parameters
iv. δ : Backlogging parameter
v. λ : Default risk parameter
vi. p : Purchasing price per unit
vii. s : Selling price per unit
viii. k : Shortage cost per unit
ix. l : Lost sales cost per unit
x. d : Deterioration cost per unit
xi. O : Ordering cost per order
xii. $I(t)$: Inventory level at any time t
xiii. Q_1 : Initial inventory level at $t = 0$
xiv. Q_2 : Maximum backordered quantity during stock out period
xv. Q : Order quantity per cycle
xvi. v : The time at which inventory level becomes zero
xvii. T : Cycle time
xviii. n : Allowable trade credit period offered by the retailer to his customers
xix. Z : Total profit of the retailer per unit time

4. Mathematical Modelling

Here, the retailer receives the stock at the initial time $t = 0$. During the time interval $[0,v]$, the inventory level is depleted as a result of cumulative effects of demand and deterioration until it reduces to zero. At $t = v$, the inventory level becomes zero, and thereafter shortages occur. During the time interval $[v,T]$, shortages are accumulated due to demand, until it reaches to a maximum shortage level Q_2 which is partially backlogged. At $t = T$, the retailer receives the new stock to clear the previous backlog and to continue the process. The behavior of the inventory level over time during a given cycle is shown in Fig.1. Therefore, the inventory level at any instant of time t is described by the following differential equations:

$$I'(t) + \theta(t)I(t) = -D(s,n), \qquad 0 \le t \le v \quad (1)$$

$$I'(t) = -B(T-t)D(s,n), \qquad v \le t \le T \quad (2)$$

With the boundary condition $I(v) = 0$

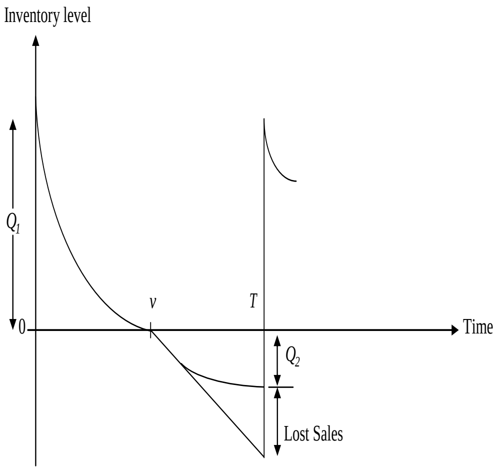


Fig.1: The graphical representation of behavior of the inventory level over time

The solutions of the above differential equations are given by

$$I(t)=\begin{pmatrix}a-bs\\+cn\end{pmatrix}\left\{\begin{matrix}(v-t)+\dfrac{\alpha}{(\beta+1)}\\ \begin{pmatrix}v^{(\beta+1)}-(\beta+1)vt^{\beta}\\+\beta t^{(\beta+1)}\end{pmatrix}\end{matrix}\right\}$$

where $0\le t\le v$ (3)

$$I(t)=(a-bs+cn)\left\{(v-t)+\delta\left\{\begin{matrix}T(v-t)\\-\dfrac{1}{2}\left(v^2-t^2\right)\end{matrix}\right\}\right\}$$

where $v\le t\le T$ (4)

With the help of equations (3) and (4), one can get the initial inventory level

$$Q_1=I(0)=(a-bs+cn)\left\{v+\frac{\alpha}{(\beta+1)}v^{(\beta+1)}\right\} \quad (5)$$

and the maximum backordered quantity is

$$Q_2=-I(T)=(a-bs+cn)\left\{\begin{matrix}(T-v)\\-\dfrac{\delta}{2}(T-v)^2\end{matrix}\right\} \quad (6)$$

Hence, the order quantity per cycle is given by

$$Q=Q_1+Q_2$$

$$Q=(a-bs+cn)\left\{\begin{matrix}T+\dfrac{\alpha}{(\beta+1)}v^{(\beta+1)}\\-\dfrac{\delta}{2}(T-v)^2\end{matrix}\right\} \quad (7)$$

The total profit of the retailer per unit time of the system compromises the following components:

- **Sales Revenue:** Since Q_2 is the total backordered quantity of the system, the retailer's sales revenue considering default risk per cycle is given by

$$SR=s\left(1-n^{-\lambda}\right)\left\{\int_0^v D(s,n)dt+Q_2\right\}$$

$$SR=s(a-bs+cn)\left(1-n^{-\lambda}\right)\left\{\begin{matrix}T-\dfrac{\delta}{2}\\(T-v)^2\end{matrix}\right\} \quad (8)$$

- **Ordering Cost:** The ordering cost per cycle

$$OC=O \quad (9)$$

- **Purchasing Cost:** The purchasing cost per cycle is given by

$$PC=pQ$$

$$PC=p\begin{pmatrix}a-bs\\+cn\end{pmatrix}\left\{\begin{matrix}T+\dfrac{\alpha}{(\beta+1)}v^{(\beta+1)}\\-\dfrac{\delta}{2}(T-v)^2\end{matrix}\right\} \quad (10)$$

- **Deterioration Cost:** Since Q is the total order quantity per cycle, the deterioration cost per cycle is

$$DC=d\left\{Q_1-\int_0^v D(s,n)dt\right\}$$

$$DC=\frac{d\alpha(a-bs+cn)}{(\beta+1)}v^{(\beta+1)} \quad (11)$$

- **Holding Cost:** The cost associated with the holding of the stock is calculated as

$$HC=\int_0^v\left(h_1+h_2t\right)I(t)\,dt$$

$$=\begin{pmatrix}a-bs\\+cn\end{pmatrix}\left\{\begin{matrix}h_1\left\{\begin{matrix}\dfrac{1}{2}v^2+\dfrac{\alpha\beta}{(\beta+1)(\beta+2)}\\v^{(\beta+2)}\end{matrix}\right\}+\\h_2\left\{\begin{matrix}\dfrac{1}{6}v^2+\\\dfrac{\alpha\beta v^{(\beta+1)}}{2(\beta+1)(\beta+2)(\beta+3)}\end{matrix}\right\}\end{matrix}\right\} \quad (12)$$

- **Shortage Cost:** The shortage cost of the retailer is

$$SC=k\int_v^T\{-I(t)\}dt$$

$$SC=k(a-bs+cn)\left\{\begin{matrix}\dfrac{1}{2}(T-v)^2-\dfrac{\delta}{3}\\(T-v)^3\end{matrix}\right\} \quad (13)$$

- **Lost Sales Cost:** During the stock-out period, shortages are accumulated due to demand, but all customers do not wait up to the next arrival lot and some of them make their purchasing from other retailers. So, the cost associated with the lost sales per cycle can be calculated as

$$LSC = l\int_{v}^{T}(1-B(T-t))D(s,n)dt$$

$$LSC = \frac{l(a-bs+cn)\delta}{2}\left(T^2-v^2\right) \quad (14)$$

Hence the total profit of the retailer per unit time of the system is given by

$$Z = \frac{1}{T}\begin{Bmatrix} SR-OC-PC-DC \\ -HC-SC-LSC \end{Bmatrix} \quad (15)$$

The goal is to maximize the total profit Z of the retailer per unit time with respect to the critical time v and the cycle time T. The nonlinearity of the objective functions in (15) does not allow us to obtain the closed form solution. We analyze the model with numerical values for the inventory parameters in the next section.

5. Numerical Example

To illustrate the model developed here, consider the following parameter values in appropriate units:

$$a=100, b=0.2, c=3, h_1=2, h_2=0.6, \alpha=0.01,$$
$$\beta=2, n=3, \delta=0.02, \lambda=7, p=100, s=160,$$
$$k=6, l=7, d=4, \text{and } O=400.$$

With the help of Mathematica software and above data, we have the following optimal solutions:

$v=1.3249, \quad T=1.9689, \quad Q=77.14,$

$and\ Z=4239.99.$

The Concavity of the Retailer profit function Z with respect to v and T is also shown in figure 2.

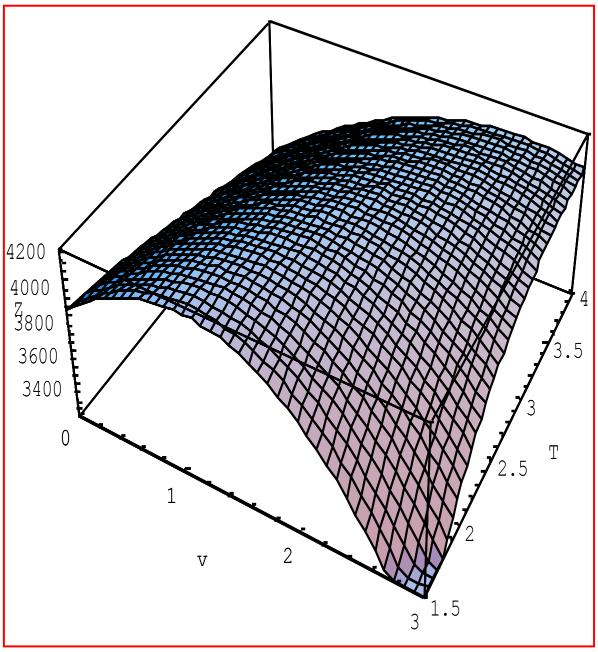


Fig.2

6. Sensitivity Analysis and its Graphical Representation

Table 1: Sensitivity Analysis with respect to the trade credit period

Credit period	% change in credit period	v	T	Q	Z
n	-20	1.3370	1.9899	75.34	4115.46
	-10	1.3309	1.9793	76.24	4182.00
	0	1.3249	1.9689	77.14	4239.99
	10	1.3189	1.9588	78.04	4294.69
	20	1.3130	1.9488	78.94	4348.02

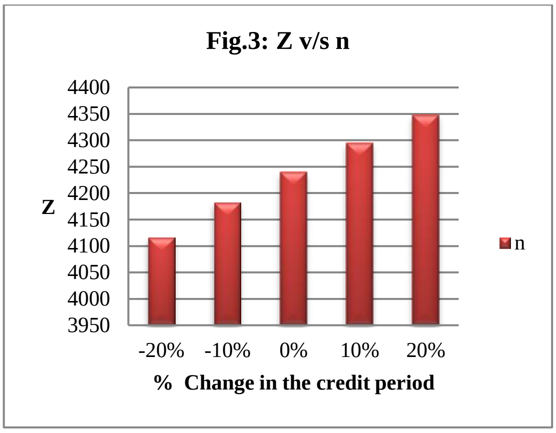


Table 2: Sensitivity Analysis with respect to the holding cost parameters

Parameter	% change in the parameter	v	T	Q	Z
h_1	-20	1.400	2.0165	77.20	4254.41
	-10	1.3616	1.9921	77.17	4247.04
	0	1.3249	1.9689	77.14	4239.99
	10	1.2898	1.9468	77.11	4233.23
	20	1.2563	1.9256	77.08	4226.75
h_1	-20	1.3320	1.9734	77.14	4241.37
	-10	1.3284	1.9711	77.14	4240.67
	0	1.3249	1.9689	77.14	4239.99
	10	1.3213	1.9667	77.13	4239.30
	20	1.3178	1.9645	77.13	4238.62

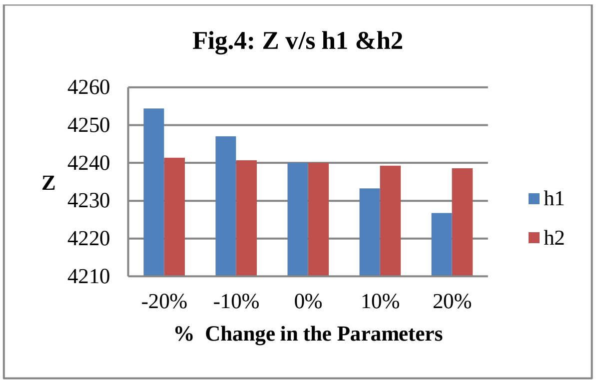


Table 3: Sensitivity Analysis with respect to the deterioration parameters

Parameter	% change in the parameter	V	T	Q	Z
α	-20	1.3881	2.0187	77.12	4246.75
	-10	1.3550	1.9925	77.13	4243.27
	0	1.3249	1.9689	77.14	4239.99
	10	1.2972	1.9474	77.14	4236.87
	20	1.2716	1.9276	77.15	4233.92
β	-20	1.3742	2.0186	77.17	4239.24
	-10	1.3480	1.9922	77.15	4239.65
	0	1.3249	1.9689	77.14	4239.99
	10	1.3043	1.9482	77.12	4240.27
	20	1.2860	1.9298	77.11	4240.52

Fig.5: Z v/s α & β

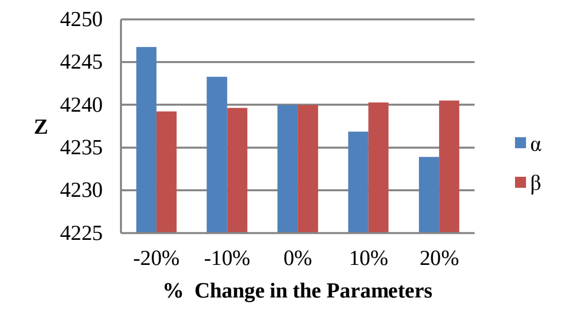


Table 4: Sensitivity Analysis with respect to the backlogging parameter

Parameter	% change in the parameter	V	T	Q	Z
δ	-20	1.3199	1.9870	77.15	4243.08
	-10	1.3224	1.9778	77.14	4241.51
	0	1.3249	1.9689	77.14	4239.99
	10	1.3273	1.9603	77.13	4238.49
	20	1.3296	1.9519	77.12	4237.03

Fig.6: Z v/s δ

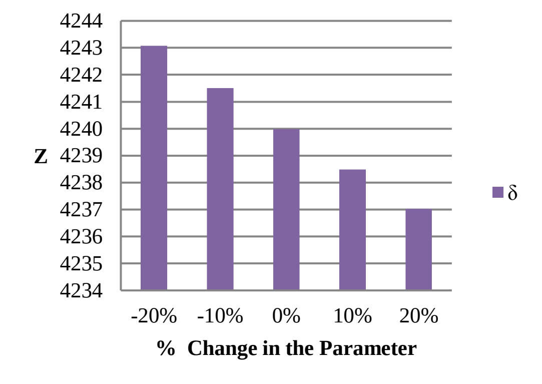


Table 5: Sensitivity Analysis with respect to the default risk parameter

Parameter	% change in the parameter	V	T	Q	Z
λ	-20	1.3247	1.9692	77.14	4219.44
	-10	1.3248	1.9690	77.14	4233.48
	0	1.3249	1.9689	77.14	4239.99
	10	1.3249	1.9689	77.14	4243.00
	20	1.3249	1.9689	77.14	4244.40

Fig.7: Z v/s λ

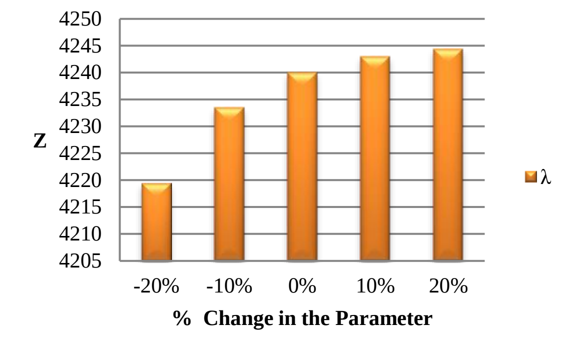
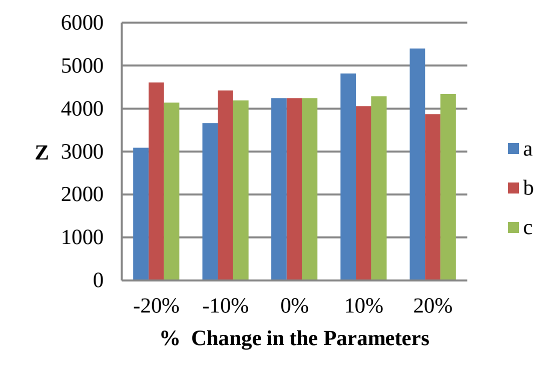


Table 6: Sensitivity Analysis with respect to the demand parameters

Parameter	% change in the parameter	V	T	Q	Z
a	-20	1.4877	2.2490	57.13	3089.45
	-10	1.3982	2.0940	67.13	3663.77
	0	1.3249	1.9689	77.14	4239.99
	10	1.2633	1.8652	87.14	4817.73
	20	1.2104	1.7772	97.14	5396.73
b	-20	1.2843	1.9005	83.54	4609.58
	-10	1.3040	1.9337	80.34	4424.71
	0	1.3249	1.9689	77.14	4239.99
	10	1.3469	2.0063	73.93	4055.41
	20	1.3702	2.0460	70.73	3871.01
c	-20	1.3371	1.9896	75.34	4136.14
	-10	1.3309	1.9792	76.24	4188.06
	0	1.3249	1.9689	77.14	4239.99
	10	1.3189	1.9588	78.04	4291.92
	20	1.3130	1.9489	78.94	4343.87

Fig.8: Z v/s a, b, & c



7. Conclusion

In this paper, an inventory model for deteriorating items with selling price and credit period sensitive demand has been developed. The default risk associated with sales revenue of the retailer is also taken into consideration. Here, shortages are allowed and partially backlogged. To make the study close to reality, the holding cost is considered as a time dependent function. Finally, the model has been illustrated with a numerical example and to study the effect of changes of different system parameters on the total retailer's profit per unit time of the system, sensitivity analysis has been performed by changing one parameter at a time and preserving the other parameters at their original values.

As a future scope of research this model can be studied under more realistic environments such as inflationary environment and fuzzy environment. To make it more relevant to today's scenario certain constraints on storage space and budget can be implemented.

References

[1] Abad, P. L. (2001), "*Optimal price and order size for a reseller under partial backordering*," Computers and Operations Research, 28 (1), 53-65.

[2] Abad, P. L. (2003), "*Optimal pricing and lot sizing under conditions of perishability, finite production and partial backordering and lost sales*," European Journal of Operational Research, 144 (3), 677-685.

[3] Aggarwal, S. P., & Jaggi, C. K. (1995), "*Ordering policy for deteriorating items under permissible delay in payments*," Journal of the Operational Research Society, 46 (5), 658-662.

[4] Chang, C. T., Teng, J. T., & Goyal, S. K. (2010)., "*Optimal replenishment policies for non instantaneous deteriorating items with stock-dependent demand*," International Journal of Production Economics, 123, 62-68.

[5] Chen, S. C., & Teng, J. T. (2015), "*Inventory and credit decisions for time-varying deteriorating items with up-stream and down-stream trade credit financing by discounted cash flow analysis*," European Journal of Operational Research, 243(2), 566–575.

[6] Dye, C. Y., Hsieh, T. P., & Ouyang, L. Y. (2007), "*Determining optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging*," European Journal of Operational Research, 181, 668-678.

[7] Dye, C. Y., & Yang, C. T. (2015), "*Sustainable trade credit and replenishment decisions with credit-linked demand under carbon emission constraints,*" European Journal of Operational Research, 244(1), 187–200.

[8] Goyal, S. K. (1985), "*Economic order quantity under conditions of permissible delay in payments*.," Journal of Operational Research Society, 36 (4), 335-338.

[9] Jamal, A. M., Sarker, B. R., & Wang, S. (1997), "*An ordering policy for deteriorating items with allowable shortage and permissible delay in payment*," Journal of Operational Research Society, 48(8), 826–833.

[10] Kumari, R., Singh, S.R., & Kumar, N. (2008), "*Two warehouse inventory model for deteriorating items with partial backlogging under the conditions of permissible delay in payment*," International Transactions in Mathematical Sciences and computer, 1 (1), 123-134.

[11] Liao, Y., Banerjee, A., and Yan, C. (2011), "*A distribution free newsvendor model with balking and lost sales penalty*," International Journal of Production Economics, 133 (1), 224-227.

[12] Lou, K. R., Wang, W. C. (2012), "*Optimal trade credit and order quantity when trade credit impacts on both demand rate and default risk*," Journal of the Operational Research Society, 11, 1-6.

[13] Singh, S. R., & Singh, D. (2017), "*Development of an optimal inventory policy for deteriorating items with stock level and selling price dependent demand under the permissible delay in payments and partial backlogging*," Global Journal of Pure and Applied Mathematics, 13, (9), 4813-4836.

[14] Teng, J. T. (2002), "*On the economic order quantity under conditions of permissible delay in payments.*" Journal of the Operational Research Society, 53 (8), 915-918.

[15] Teng, J. T., & Lou, K. R. (2012), "*Seller's optimal credit period and replenishment time in a supply chain with up-stream and down-stream trade credit,*" Journal of Global Optimization, 53(3), 417-430.

[16] Wu, J., Ouyang, L. Y., Barron, L., & Goyal, S. (2014), "*Optimal credit period and lot size for deteriorating items with expiration dates under two level trade credit financing,*" European Journal of Operational Research, 237(1), 898-908.

[17] Wu, C., & Zhao, Q. (2016), "*Two retailer–supplier supply chain models with default risk under trade credit policy*," Springerplus 5(1), 1728.