Availability Measures of Two Unit Parallel Redundant System with Replacement

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ABSTRACT: *For the system which has two units in parallel redundancy, the reliability of repairable systems can be calculated as a result of the numerical solution of a simultaneous set of linear differential equations. The closed form solutions of the transient probabilities are used to obtain the reliability for non-repairable systems. The probabilities so obtained can be utilized to estimate the reliability and MTTF of the system.*

KEYWORDS: *Reliability, Multi-failure, Redundant system, Failure rate, Supplementary variable process.*

INTRODUCTION

The system has two units in parallel redundancy. Since the system can fail partially or completely, a unit is said to be degraded when it fails partially, i.e., a degraded unit works, but not with its normal efficiency. A unit is said to be failed when it fails completely. Initially the system is in good state. There may be 3 states, good, degraded or failed, i.e., system is good when either or both units are in good state. The system is degraded when both units are neither good nor failed. The system fails when both units are failed. The failure and repair time for the system follows exponential distribution.

NOTATIONS

(i) G_0, G_1, G_2 : Good state of the system, D_1,D_2, Degraded states of the system, F : Failed

(ii) $\lambda_1 \neq \lambda'_1$: Failure rates of a good unit to degraded state.

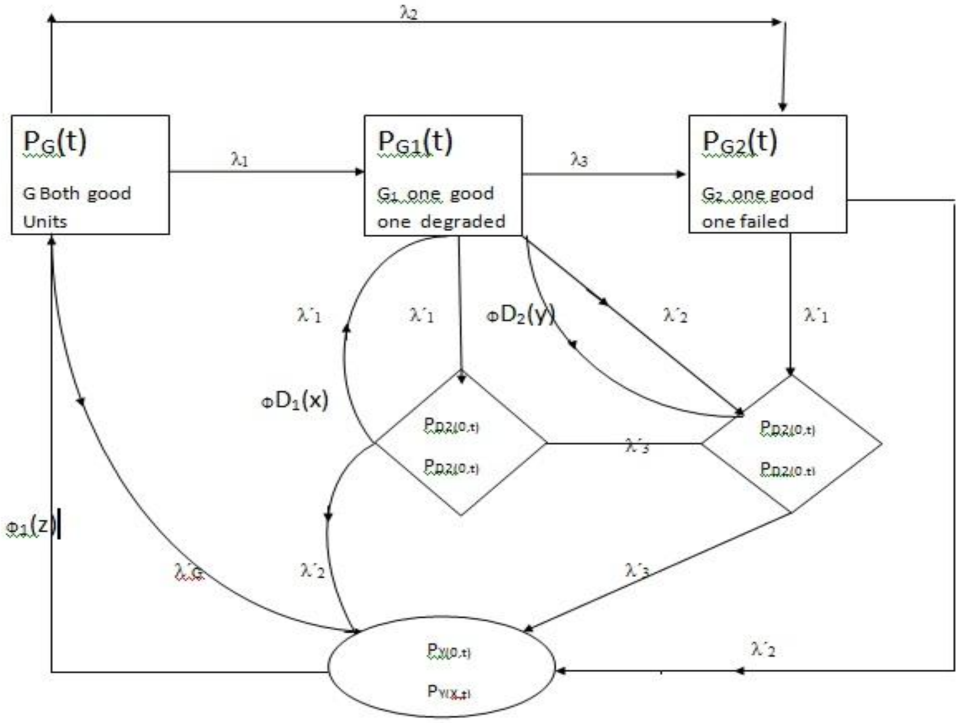
(iii) λ_2, λ'_2 : Failure rates of a good unit to failure state, (iv) λ_3, λ'_3 : Failure rates of a degraded units to failed state.

(v) $S_i(r), \phi_i(r)$: Probability density function and hazard rates for repair time of the system.

(vi) P_i (t) : Probability that the system in state i at time t i = G_0, G_1, G_2, D_1, D_2, F

MATHEMATICAL ANALYSIS

The equations are formed as follows:



$$\left(\frac{\partial}{\partial}+2\lambda_1+2\lambda_2+\lambda_{hG}\right)P_G(t)=\int_0^\infty \phi_F(z)P_F(z,t)dt, \quad (1)$$

$$\left(\frac{\partial}{\partial t}+\lambda_3+\lambda'_1+\lambda'_2\right)P_{G1}(t)=2\lambda_1P_G(t)+\int_0^\infty P_{D1}(x,t)\phi_{D1}(x)dx,$$

$$+\int_0^\infty \phi(D_2Y)P_{D2}(y,t)dy \quad (2)$$

$$\left(\frac{\partial}{\partial t}+\lambda'_1+\lambda'_2\right)P_{G2}(t)=\lambda_3P_{G1}(t)+2\lambda_2P_G(t); \quad (3)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda'_3+\phi_{D1}(x)\right)P_{D1}(x,t)=0; \quad (4)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda'_3+\phi_{D2}(y)\right)P_{D2}(y,t)=0 \quad (5)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}++\phi_F(z)\right)P_F(z,t)=0. \quad (6)$$

Boundary conditions

$$P_{D1}(o,t)=\lambda'_1\,P_{G1}(t); \quad (7)$$

$$P_{D2}(o,t)=\lambda'_1\,P_{G2}(t)+\lambda'2P_{G1}(t)+2\lambda'_3\,P_{D1}(t) \quad (8)$$

$$P_F(o,t)=\lambda'_2\,P_{G2}(t)+\lambda'_3\,P_{D2}(t)+\lambda_{hG}P_G(t). \quad (9)$$

Initial conditions are

$$\left.\begin{aligned} Pi(0)&=1\,when\,i=0\\ &=wheni\neq 0\end{aligned}\right\} \quad (10)$$

SOLUTIONS OF THE PROBLEM

Taking Laplace transform of above equations from (1) to (10) we obtain

$$(s+2\lambda_1+2\lambda_2+\lambda_{hG})P^*{}_G(s)=1+\int_0^\infty P^*{}_F(z,s)\phi_F(z)dz \quad (11)$$

$$(s+\lambda_3+\lambda_1+\lambda_2)P^*{}_{G_1}(s)=2\lambda_1P^*{}_G+\int_0^\infty P^*{}_{D_1}(x,s)\phi_{D_1}(x)dx \quad (12)$$

$$(s+\lambda'_1+\lambda'_2)P^*{}_{G_2}(S)=\lambda_3P^*{}_{G_1}(S)+2\lambda_2P^*{}_G(S); \quad (13)$$

$$\left(s+\frac{\partial}{\partial x}+2\lambda_3+\phi_{D_1}(x)\right)P^*_{D_1}(x,s)=0 \quad (14)$$

$$\left(s+\frac{\partial}{\partial x}+\lambda_3+\phi_{D_2}(y)\right)P^*{}_{D_2}(y,s)=0 \quad (15)$$

$$\left(s+\frac{\partial}{\partial z}+\phi_F(z)\right)P^*{}_F(z,s)=0 \quad (16)$$

Boundary conditions (7) –(9) yield

$$P^*{}_{D_1}(0,s)=\lambda_1 P^*{}_{G_1}(s); \quad (17)$$

$$P^*{}_{D_2}(0,s)=\lambda P^*{}_{G_2}(s)+\lambda_2 P^*{}_{G_1}(s)+2\lambda_3 P^*{}_{D_1}(s); \quad (18)$$

$$P^*{}_F(0,s)=\lambda_2 P^*{}_{G_2}(s)+\lambda_3 P^*{}_{D_2}(s)+\lambda_{hG} P^*{}_G(s) \quad (19)$$

Solving (11) to (16) and using (17) to (19) we obtain

$$P*G2(s)=\frac{\lambda_3 P^*{}_{G_1}(s)+2\lambda_2 P^*{}_G(s)}{s+\lambda'_i+\lambda'_2}, P^*{}_{D_1}(s)=\lambda_1 P^*{}_G(s)Q_{D_1}(s+2\lambda'_3)$$

$$P*D2(s)=\left\{\left(\frac{\lambda_3\lambda_1}{s+\lambda'_1+\lambda'_2}\right)+\lambda_2\right\}P^*{}_{G_1}(s)+\left\{\frac{2\lambda_2\lambda_1}{s+\lambda'_1+\lambda'_2}\right.$$

$$\left.+2\lambda'_3\lambda'_1 Q_{D_1}(S+\lambda'3)\right\}\times Q_{D_2}(s+\lambda'_3) \quad (20)$$

$$P^*{}_F(s)=P^*{}_{G_1}(s)Q_F(s)\left[\frac{\lambda_2\lambda_3}{s+\lambda_1+\lambda_2}+\left(\lambda_3\lambda_2+\frac{\lambda_3\lambda_3\lambda_1}{s+\lambda'_1+\lambda'_2}Q_{D_2}(s+\lambda'_3)\right)\right]$$

$$+P^*{}_G(s)Q_F(s)\left[\lambda_{hG}+\frac{2\lambda_2\lambda_2}{s+\lambda'_1+\lambda'_2}+\lambda_3 Q_{D_2}(s+\lambda'_3)\right) \quad (21)$$

$$P^*{}_G(s)=\frac{1+\int_0^\infty P^*{}_F(z,s)\phi_F(z)dz}{s+2\lambda_1+2\lambda_2+\lambda_{hG}}. \quad (22)$$

Now we get

$$P^*{}_G(s)=P^*{}_{G_1}(s)Q_F(s)\left[\frac{\lambda'_2\lambda'_3}{s+\lambda'_1+\lambda'_2}+\lambda'_3\left(\lambda_2+\frac{\lambda'_3\lambda'_1}{s+\lambda'_1+\lambda'_2}Q_{D_2}(s+\lambda'_3)\right)\right]$$

$$+P_*(s)Q_F(s)\left[\lambda_{hG}+\frac{2\lambda_2\lambda'_2}{s+\lambda'_1+\lambda'_2}+\lambda_3 Q_{D_2}(s+\lambda'_3)\right) \quad (23)$$

$$(s+2\lambda_1+2\lambda_2+\lambda_{hG})P^*{}_G(s)=1+\int_0^\infty P^*{}_F(x,s)\phi_F(z)dz$$

$$1+\frac{\lambda_2\lambda_3 P^*{}_{G_1}(s)+2\lambda_2 P^*{}_G s\lambda'_2}{}Q_F(s)$$

$$+\lambda_3\left[\left(\frac{\lambda_3\lambda'_1}{s+\lambda'_1+\lambda'_2}+\lambda'_2\right)P^*_{G_1}(s)+Q_{D_1}(s+\lambda'_3)P^*_G(s)\right.$$

$$\left.+\left[\frac{2\lambda_2\lambda_1}{s+\lambda'_1+\lambda'_2}+2\lambda'_3\lambda'_1Q_{D_1}(s+2\lambda'_3)Q_{D_2}(s+\lambda'_3)\right]\right.$$

$$+\lambda_{hG}P^*_G(s) \tag{24}$$

which reduces to

$$P^*_G(s)\left[-s+2\lambda_1+2\lambda_2+\lambda_{hG}-\frac{2\lambda_2\lambda'_2}{s+\lambda'_1+\lambda'_2}Q_F(s)-\lambda'_3Q_{D_2}(s+\lambda'_3)\right.$$

$$\left.\left[\frac{2\lambda_2\lambda_1}{s+\lambda'_1+\lambda'_2}+2\lambda'_3\lambda'_1Q_{D1}(s+2\lambda'_3)\right]-\lambda_{hG}\right]$$

$$=1+P^*_G(s)\left[\frac{\lambda'_2\lambda_3}{s+\lambda'_1+\lambda'_2}Q_Fs+\lambda'_3\left(\frac{\lambda_3\lambda'_1}{s+\lambda'_1+\lambda'_2}+\lambda'_2\right)Q_{D1}(s+\lambda'_3)\right] \tag{25}$$

If we have

$$P^*_{up}(s)=P^*_{G_1}(s)+P^*_{G_2}(s)+P^*_{D_1}(s)+P^*_{D_2}(s) \tag{26}$$

$$=P^*_G(s)\left[1+\frac{2\lambda_2}{s+\lambda'_1+\lambda'_2}+\lambda'_1Q_{D_1}(s+2\lambda'_3)\right.$$

$$\left.+\left(\frac{2\lambda_2\lambda'_1}{s+\lambda'_1+\lambda'_2}+2\lambda'_3\lambda'_1Q_{D_1}(s+2\lambda'_3)Q_{D_2}(s+\lambda'_3)\right)\right]$$

$$+P^*_{G_1}(s)\left[1+\frac{\lambda_3}{\lambda'_1+\lambda'_2}+\left(\frac{\lambda_3\lambda'_1}{s+\lambda'_1+\lambda'_2}+\lambda'_2\right)Q_{D_1}(s+\lambda'_3)\right]$$

$$P*_{down}(s)=P^*_F(s) \tag{27}$$

We see that

$$P^*_{up}(s)+P^*_{down}(s)=1 \tag{28}$$

ERGODIC BEHAVIOUR OF THE SYSTEM

Using Abel's theorem

$\lim_{s\to 0} sF^*.(s)=\lim_{t\to\infty}F(t)=F\ say$ we get

$$P_{up}=\lim_{s\to 0} sP^*_{up}(s) \tag{29}$$

Here we use $sP^{*}(s) = P_G$

Thus we have

$$P^{*}_{up} = P_G\left[1 + \frac{2\lambda_2}{\lambda_1 + \lambda_2} + \lambda'_1 Q_{D_1}(2\lambda'_3)\right.$$

$$+ Q_{D_2}(\lambda'_3)\left(\frac{2\lambda_2\lambda'_1}{\lambda'_1 + \lambda'_2} + 2\lambda'_3\lambda'_1 Q_{D_1}(2\lambda'_3)\right)$$

$$+ P_{G_1}\left(1 + \frac{\lambda 3}{\lambda'_1 + \lambda'_2} + \left(\frac{\lambda'_3\lambda'_1}{\lambda'_1 + \lambda'_2}\right) Q_{D_1}(\lambda'_3)\right) \tag{30}$$

Similarly,

$$P*_{down} =_{s \to 0} \left(sP^{*}_{down}(s)\right)$$

$$P_{G_1} Q_F(0)\left(\frac{\lambda'_2\lambda_3}{\lambda'_1 + \lambda'_2} + \left(\lambda' 3\lambda' 2 + \frac{\lambda_3\lambda'_3\lambda'_1}{\lambda'_1 + \lambda'_2}\right) Q_{D_2}(\lambda'_3)\right)$$

$$+ P_{G_1} QF(0)\left(\lambda_{hG} + \frac{2\lambda_2\lambda_1}{\lambda_1 + \lambda'_2} + \lambda'_3 Q_{D_2}(\lambda'_3)\right)$$

$$+ \left(\frac{2\lambda_2\lambda'_1}{\lambda'_1 + \lambda'_2} + 2\lambda'_3\lambda'_1 Q_{D_1}(2\lambda'_3)\right) \tag{31}$$

Here $Q_F(0)$ is mean time to repair to the failed state of the system.

INTERPRETATIONS OF RESULTS

If we take particular values of $\lambda_1, \lambda_2, \lambda_3, \lambda'_1, \lambda'_2, \lambda'_3$, we see that results are similar i.e., availability of system decreases with time but as time progresses total decrease is low and expected profit v/s time will decrease rapidly.

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