

# Computational Procedure for Calculating Sherwood Number Values in Horizontal Porous Channel Geometry by using the Numerical Method Finite Element Method

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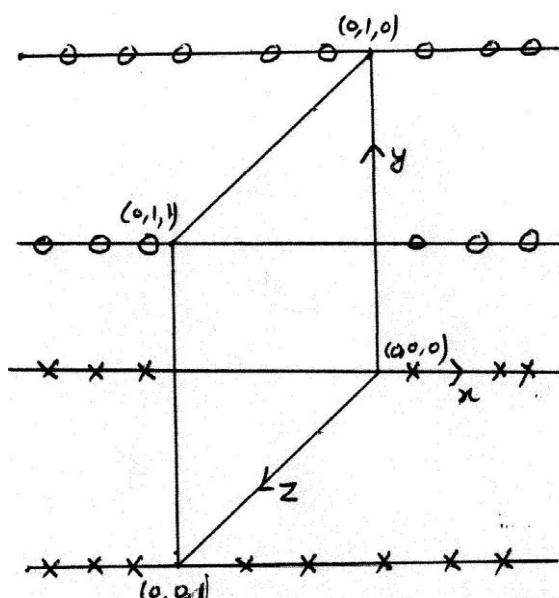
**Abstract** — In this paper we analyzed the computational procedure for calculating Sherwood number values by Mathematica 4.1 Software Commands in Horizontal Porous Channel Geometry by using the Numerical Method Finite Element Method (FEM).

**Keywords** — Sherwood number, Horizontal Porous Channel, Finite Element Method.

## I. INTRODUCTION

Free convection heat transfer between horizontal parallel plate channels is of considerable interest to engineers because of its application to various engineering problems. Needs for buoyancy driven ventilation appear in a variety of engineering applications, ranging from cooling of electronic components and solar energy applications to cooling of nuclear reactor fuel elements. For an efficient application of natural convection to cooling processes it is necessary to fully understand the mechanisms of heat dissipation inside parallel-plate channels. Premchandran et al [1] investigated Conjugate mixed convection with surface radiation from a horizontal channel with protruding heat sources. Dogan et al studied the Investigation of mixed convection heat transfer in a horizontal channel with discrete heat sources at the top and at the bottom.

$\rho$	Density
$\mu$	Coefficient of Kinematic viscosity
K	Permeability Coefficient
$k_1$	Coefficient of thermal conductivity
$\rho_o$	Mean density
$T_o$	Mean temperature
$C_o$	Mean Concentration
$C_p$	Specific heat at constant pressure
$\beta$	Coefficeint of thermal expansion
$\beta^*$	Volumetric Coefficent of expansion with mass fraction concentration
Q	Strength of the heat source
$D_1$	Molecular Diffusivity
$K_{11}$	Cross diffusivity



## Nomenclature

u	Velocity
T	Temperature
C	Concentration
p	Pressure

Fig.a Schematic diagram of the Horizontal Channel

## II. FORMULATION OF THE PROBLEM

The equations governing the flow, heat, Shear Stress and mass transfer with Soret effect are

$$\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial y^2} - \frac{u}{k} + \nu_o \frac{\partial u}{\partial y} = \frac{1}{\mu} \frac{\partial p}{\partial x} \quad (1)$$

$$0 = \frac{\partial p}{\partial y} - \rho g \quad (2)$$

$$\begin{aligned} \rho_o C_p (u \frac{\partial T}{\partial z} - \nu_o \frac{\partial T}{\partial y}) &= \\ k_1 \left( \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \\ \frac{\rho_o \nu}{k} [(\frac{\partial u}{\partial z})^2 + (\frac{\partial u}{\partial y})^2] + \\ \frac{\rho_o \nu}{k} u^2 & \\ u \frac{\partial C}{\partial x} &= D_1 \left( \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) + \\ k_{21} \left( \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) & \end{aligned} \quad (3)$$

$$\rho = \rho_0 [1 - \beta(T - T_0) - \beta^*(C - C_0)] \quad (5)$$

The flow being unidirectional, in view of the equation of continuity  $u = u(y, z)$ .

The heat mass flux being constant along the channel

$$\frac{dT}{dx} = A, \frac{dC}{dx} = B \text{ on the wall } y = b$$

where A and B are the uniform temperature and concentration gradients respectively. Hence the temperature and the concentration in the flow field may chosen to be

$$\begin{aligned} T &= Ax + T_1(y, z) \\ C &= Bx + C_1(y, z). \end{aligned}$$

The boundary conditions are

$$u = 0 \quad \text{on} \quad y = b$$

$T = T_1$ ,  $C = C_1$  on  $y = b$  at the entry  $x = 0$  and

$$\begin{aligned} T &= Ax + T_1, C = Bx + C_1 \text{ on} \\ y &= b, x \neq 0 \end{aligned} \quad (6)$$

In view of the symmetry w.r.t. the central line  $y = 0$

$$\frac{\partial u}{\partial y} = 0, \frac{\partial T}{\partial y} = 0 \text{ and } \frac{\partial C}{\partial y} = 0 \text{ on } y = 0 \quad (7)$$

We introduce the following non-dimensional variables as follows.

$$z = z^* b, y = y^* b, T = T_0 + \theta^* (T_1 - T_0)$$

$$u^* = \frac{\nu u}{\beta g b^2 (T_1 - T_0)}, C = C_0 + C^* (C_1 - C_0)$$

Substituting these non-dimensional variables in equations (1) – (4) and making use of the Boussinesque approximation, the governing dimensionless equations on elimination of p reduce to (dropping the asterisk).

$$\frac{\partial}{\partial y} \left( \frac{\partial^2 u}{\partial z^2} \right) + \frac{\partial^3 u}{\partial y^3} - D^{-1} \frac{\partial u}{\partial y} + S \frac{\partial^2 u}{\partial y^2} = N_1 + N_2 \quad (8)$$

$$\begin{aligned} \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} + PS \frac{\partial \theta}{\partial y} - \alpha \theta + \\ PEcG(u_y^2 + u_z^2) + PEcGD^{-1} u^2 &= N_1 PGU \end{aligned} \quad (9)$$

$$\begin{aligned} Sc(-S \frac{\partial C}{\partial y}) + N_2 u &= \left( \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) + \\ \frac{S_0 Sc}{N} \left( \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) & \end{aligned} \quad (10)$$

The corresponding boundary conditions in the non-dimensional form are

$$\begin{aligned} u &= 0 \quad \text{on} \quad y = 1, \theta = 1, C = 1 \text{ at } x = 0 \\ \text{on} \quad y &= 1 \end{aligned} \quad (11)$$

$$\theta = 1 + N_1 x, C = 1 + N_2 x, x \neq 0 \text{ on } y = 1 \quad (12)$$

$$\frac{\partial u}{\partial y} = 0, \frac{\partial \theta}{\partial y} = 0 \text{ and } \frac{\partial C}{\partial y} = 0 \text{ on } y = 0 \quad (13)$$

In view of the two dimensionality and symmetry of the flow w.r.t. the midplane of the channel we analyse the flow features in a domain in the upper half of the channel bounded by the impermeable wall lying between two parallel planes normal to the wall at unit distance apart. The finite element analysis with quadratic approximation functions is carried out using eight noded serendipity elements.

### III.FINITE ELEMENT ANALYSIS OF THE PROBLEM

we now make a finite element analysis of the given problem governed by (8), (9) & (10) subject to the conditions (11), (12) and (13). We get the following equations

$$\begin{aligned} & \int_{\Omega_i} \left[ \sum_{k=1}^8 u_k^i \left\{ \frac{\partial^2 N_k^i}{\partial y^2} \frac{\partial N_j^i}{\partial z} + \frac{\partial^2 N_k^i}{\partial z^2} \frac{\partial N_j^i}{\partial y} + \right. \right. \\ & D^{-1} N_k^i \frac{\partial N_j^i}{\partial y} - S N_j^i \frac{\partial^2 N_k^i}{\partial y^2} \left. \right\} + \\ & (N_1 + N_2) N_j^i \right] d\Omega_i = Q_j^i \quad (14) \end{aligned}$$

$$\begin{aligned} & \int_{\Omega_i} \left[ \sum_{k=1}^8 \theta_k^i \left\{ \frac{\partial N_k^i}{\partial y} \frac{\partial N_j^i}{\partial y} + \frac{\partial N_k^i}{\partial z} \frac{\partial N_j^i}{\partial z} + \right. \right. \\ & P S N_j^i \frac{\partial N_k^i}{\partial y} - \alpha N_j^i N_k^i \left. \right\} + \\ & \left. + \int_{\Omega_i} \sum_{k=1}^8 u_k^i \left\{ P E c G \left( \left( \frac{\partial N_k^i}{\partial y} \right)^2 + \left( \frac{\partial N_k^i}{\partial z} \right)^2 \right) + N_j^i \right\} \right. \\ & D^{-1} (N_k^i)^2 - N_1 P G N_k^i \} d\Omega_i = (Q^T)_j^i \quad (15) \end{aligned}$$

$$\begin{aligned} & \int_{\Omega_i} \left[ \sum_{k=1}^8 C_k^i \left\{ \frac{\partial N_k^i}{\partial y} \frac{\partial N_j^i}{\partial y} + \frac{\partial N_k^i}{\partial z} \frac{\partial N_j^i}{\partial z} + \right. \right. \\ & S S_c N_j^i \frac{\partial N_k^i}{\partial y} \} d\Omega_i \\ & + \frac{S c S_o}{N} \int_{\Omega_i} \sum_{k=1}^8 \theta_k^i \left\{ \frac{\partial N_k^i}{\partial y} \frac{\partial N_j^i}{\partial y} + \frac{\partial N_k^i}{\partial z} \frac{\partial N_j^i}{\partial z} \right\} d\Omega_i \\ & - \int_{\Omega_i} S_c N_2 \sum_{k=1}^8 u_k^i (N_j^i N_k^i) d\Omega_i = (Q^C)_j^i \quad (16) \end{aligned}$$

where

$$Q_j^i = \oint_{\Gamma_i} [N_j^i \frac{\partial^2 u^i}{\partial y^2} n_y + N_j^i \frac{\partial u^i}{\partial y} n_y + N_j^i \frac{\partial u^i}{\partial z} n_z] d\Gamma_i$$

$$(Q^T)_j^i = \oint_{\Gamma_i} [N_j^i \frac{\partial \theta^i}{\partial y} n_y + N_j^i \frac{\partial \theta^i}{\partial z} n_z] d\Gamma_i ,$$

$$j = 1, 2, \dots, 8.$$

$$\begin{aligned} (Q^C)_j^i = & \oint_{\Gamma_i} [N_j^i \{ N \frac{\partial C^i}{\partial y} + S_0 S c \frac{\partial \theta^i}{\partial y} \} n_y + \\ & N_j^i \{ N \frac{\partial C^i}{\partial z} + \\ & + S_0 S c \frac{\partial \theta^i}{\partial z} \} n_z] d\Gamma_i \quad j=1,2,\dots,8. \end{aligned}$$

Choosing different  $N_k^i$ 's corresponding to each element  $e_i$  the equation (14),(15) and (16) results in twenty four equations for three sets of unknown  $(u_k^i)$ ,  $(\theta_k^i)$  and  $(C_k^i)$ , viz

$$(a_{kj}) (u_k^i) = (Q_j^i) \quad (17)$$

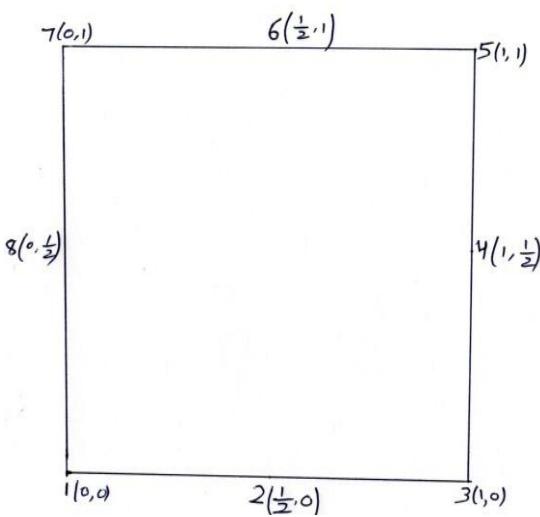
$$(b_{kj}) (\theta_k^i) + (d_{kj}) (u_k^i) = (Q_j^T)_j^i \quad (18)$$

$$\begin{aligned} (m_{kj}) (C_k^i) + (l_{kj}) (u_k^i) = & (n_{kj}) (\theta_k^i) + (Q_j^C)_j^i \\ (j, k = 1, 2, \dots, 8) \end{aligned} \quad (19)$$

where

$(a_{kj})$ ,  $(b_{kj})$ ,  $(d_{kj})$ ,  $(m_{kj})$ ,  $(n_{kj})$  and  $(l_{kj})$  are  $8 \times 8$  stiffness matrices and  $(Q_j^i)$ ,  $(Q_j^T)_j^i$  and  $(Q_j^C)_j^i$  are  $8 \times 1$  column matrices. Repeating the process with each of mn elements and making use of global coordinates and inter element continuity conditions as well as the boundary conditions to assemble the element matrices, we obtain global matrices for the unknown  $u$ ,  $\theta$  and  $C$  at the respective global nodes which are ultimately determined on solving the matrix equation.

For computational purpose ,we choose a serendipity element with  $(0,0)$ ,  $(0,1)$ ,  $(1,0)$  and  $(1, 1)$  as its vertices. The eight nodes of the element are as shown in Fig .(b) and the quadratic interpolation function at these nodes are



**Fig.b Eight noded Rectangular Serendipity element**

$$\begin{aligned}
 N_1 &= -2(y-1)(z-1)(z+y-\frac{1}{2}) ; \\
 N_2 &= 4(z-1)(y-1)(z) ; \\
 N_3 &= -2(y-1)(z)(z-y-\frac{1}{2}) ; \\
 N_4 &= -4(y-1)(z)(y) ; \\
 N_5 &= 2zy(z+y-\frac{3}{2}) ; \\
 N_6 &= -4zy(z-1) ; \\
 N_7 &= 2y(z-1)(z-y+\frac{1}{2}) ; \\
 N_8 &= 4y(z-1)(y-1)
 \end{aligned}$$

Substituting these shape functions in (17) and integrating over the element domain the matrix for the global nodes of  $u$  viz.,  $U_i$  ( $i = 1, 2, \dots, 8$ ) reduces to a  $8 \times 8$  matrix equations and we write in the partitioned form.

The  $8 \times 8$  matrix equation for  $\theta_j$  ( $j = 1, 2, \dots, 8$ ) and we write in the partitioned form.

Similarly the  $8 \times 8$  matrices equations for  $C_j$  ( $j = 1, 2, \dots, 8$ ) and we write in the partitioned form.

The boundary conditions (essential boundary conditions on the primary variables) are

$$U_5 = U_6 = U_7 = 0, \theta_5 = \theta_6 = \theta_7 = 1 \text{ and}$$

$$C_5 = C_6 = C_7 = 1 \text{ on } y = 1$$

In view of the symmetry conditions we obtain.

$$\left. \begin{array}{l} Q_1 = Q_2 = Q_3 = Q_4 = Q_8 = 0 \\ Q_1^T = Q_2^T = Q_3^T = Q_4^T = Q_8^T = 0 \\ Q_1^C = Q_2^C = Q_3^C = Q_4^C = Q_8^C = 0 \end{array} \right| \quad \text{on } y = 0$$

Solving the ultimate  $8 \times 8$  matrix we determine the unknown global nodal values of  $U_i$ ,  $\theta_i$  and  $C_i$  ( $i = 1, 2, \dots, 8$ ).

The solution for  $u$ ,  $\theta$  and  $C$  may now be represented as

$$u = \sum_{i=1}^8 U_i N_i, \theta = \sum_{i=1}^8 \theta_i N_i, C = \sum_{i=1}^8 C_i N_i$$

The rate of Sherwood number in the non-dimensional form on the boundary is

$$Sh = \left( \frac{\partial C}{\partial y} \right)_{y=1}$$

The Sherwood number evaluated computationally for different variations in the governing parameters.

#### IV. NUMERICAL COMPUTATION

To find out Sherwood number values using mathematica 4.1 software commands are given below

Shape functions

$$s_1 := -4X(-1+y)X(\frac{y}{2} + \frac{1}{2}(-\frac{1}{2}+z))X(-1+z)$$

$$s_2 := 4X(-1+y)X(-1+z)Xz$$

$$s_3 := -4X(-1+y)X(-\frac{y}{2} + \frac{1}{2}(-\frac{1}{2}+z))Xz$$

$$s_4 := -4X(-1+y)XyXz$$

$$s_5 := 4XyX(\frac{1}{2}(-\frac{1}{2}+y) + \frac{1}{2}(-1+z))Xz$$

$$s_6 := -4XyX(-1+z)Xz$$

$$s_7 := -4XyX(\frac{1}{2}(-1+y) + \frac{1}{2}(\frac{1}{2}-z))X(-1+z)$$

$$s_8 := 4X(-1+y)XyX(-1+z)$$

#### FEM equation for Momentum

$$Do[m = \sum_{i=1}^8 u_i X \int_0^1 \int_0^1 (\partial_y s_i X \partial_{y,y} s_j +$$

$$\partial_z s_i X \partial_{z,z} s_j + (ad + M^2) X s_i X s_j) dy dz / .$$

$$u_5 \rightarrow 0 / . u_6 \rightarrow 0 / . u_7 \rightarrow 0 +$$

$$(n+n) X \int_0^1 \int_0^1 (s_i) dy dz, \{i, 1, 8\}]$$

Next we should find Coefficient matrix for momentum equation by executing the below command

**Table[Coefficient[ $m_i, u_j$ ], {i,1,8}, {j,1,8}]**

Equate this coefficient to M and execute this command

ReplacePart[M,-1,{ {5,5},{6,6},{7,7} }] for getting non singular matrix and this matrix we can take as coefficient matrix

Next we should find constant matrix for momentum equation by executing below two commands

**Do[ $u_i = 0$ , {i,1,8}];**

**Table[ $m_i$ , {i,1,8}]**

#### FEM equation for temperature

**Do[ $t_i = \sum_{j=1}^8 \theta_j X \int_0^1 \int_0^1 (\partial_y s_i X \partial_y s_j + \partial_z s_i X \partial_z s_j - p X s_i s_j + (ad + M^2) X s_i X \partial_y s_j + \alpha X s_i X \partial_z s_j) dy dz -$**

**$p X s_i s_j + (ad + M^2) X s_i X \partial_y s_j + \alpha X s_i X \partial_z s_j) dy dz -$**

**$\sum_{j=1}^8 u_j X \int_0^1 \int_0^1 (p X k X G X s_i X ((\partial_y s_j)^2 + (\partial_z s_j)^2) - p X k X G X (ad + M^2)$**

**$X s_i X ((s_j)^2) + n X p X G X s_i X s_j) dy dz / . u_5 -> 0 / . u_6 -> 0 / . u_7 -> 0 / . \theta_5 -> 1$**

**$/ . \theta_6 -> 1 / . \theta_7 -> 1, \{i,1,8\};$**

Next we should find Coefficient matrix for temperature equation by executing the below command

**Table[Coefficient[ $t_i, \theta_j$ ], {i,1,8}, {j,1,8}]**

Equate this coefficient to M and execute this command

ReplacePart[M,-1,{ {5,5},{6,6},{7,7} }] for getting non singular matrix and this matrix we can take as coefficient matrix

Next we should find constant matrix for temperature equation by executing below two commands

**Do[ $\theta_i = 0$ , {i,1,8}];**

**Table[ $t_i$ , {i,1,8}]**

#### FEM equation for Concentration

**$Do[con = \sum_{j=1}^8 \int_0^1 \int_0^1 (\partial_y s_i X \partial_y s_j + \partial_z s_i X \partial_z s_j) X c_j +$**

**$(\frac{s_o X s_c}{an}) X (-p X s_i s_j + (ad + M^2) X$**

**$s_i X \partial_y s_j + \alpha X s_i X \partial_z s_j) dy dz -$**

**$(\partial_y s_i X \partial_y s_j + \partial_z s_i X \partial_z s_j) X \theta_j -$**

**$((Sc X s_i X \partial_y s_j) X c_j) +$**

**$(Sc X n X s_i X \partial_z s_j) X u) dy dz /.$**

**$u_5 -> 0 / . u_6 -> 0 / . u_7 -> 0$**

**$/ . \theta_5 -> 1 / . \theta_6 -> 1 / . \theta_7 -> 1$**

**$/ . c_5 -> 1 / . c_6 -> 1 / . c_7 -> 1, \{i,1,8\}$**

Next we should find Coefficient matrix for diffusion equation by executing the below command

**Table[Coefficient[ $con, c_j$ ], {i,1,8}, {j,1,8}]**

Equate this coefficient to M and execute this command

ReplacePart[M,-1,{ {5,5},{6,6},{7,7} }] for getting non singular matrix and this matrix we can take as coefficient matrix

Next we should find constant matrix for concentration equation by executing below two commands

**Do[ $c_i = 0$ , {i,1,8}];**

**Table[ $con_i$ , {i,1,8}]**

Coefficient Matrices mom, T, on and Constant matrices A,B,F

**$Mom := \{ -\frac{4}{3} - \frac{ad}{15} - \frac{M^2}{15} +$**

**$\frac{s}{9}, \frac{4}{3} + \frac{7ad}{90} + \frac{7M^2}{90}, -\frac{2}{3} +$**

**$\frac{ad}{60} + \frac{M^2}{60} + \frac{2S}{9}, -\frac{4S}{9}, 0, 0, 0, 0, \frac{4}{3} +$**

**$\frac{ad}{9} + \frac{M^2}{9} - \frac{2S}{9} \}, \dots$**

**$\{ -\frac{4}{3} - \frac{ad}{9} - \frac{M^2}{9} - \frac{8s}{9}, \frac{8}{3} - \frac{2}{9}(ad + M^2),$**

**$-\frac{4}{3} - \frac{4s}{9}, \frac{8s}{9}, 0, 0, 0, \frac{16s}{9} \} \}$**

$$\begin{aligned}
 A &:= -\left\{ \frac{1}{12} \left( -\frac{2}{3} - \frac{2ad}{45} - \frac{2M^2}{45} + \frac{s}{9} \right) \right. \\
 &\quad \left( -n - n \right), \frac{1}{3} \left( -\frac{4}{3} - \frac{7ad}{90} - \frac{7M^2}{90} - \frac{2s}{3} \right) \\
 &\quad (n + n), \dots, \frac{1}{3} \left( -\frac{4}{3} + \frac{ad}{9} + \frac{M^2}{9} - \frac{8s}{9} \right) \\
 &\quad \left. (n + n) \right\} \\
 T &:= \left\{ \left\{ \frac{52}{45} + \frac{ps}{15} + \frac{\alpha}{30}, -\frac{37}{45} - \frac{7ps}{90} - \frac{\alpha}{30}, \frac{1}{2} - \frac{ps}{60} + \frac{\alpha}{90}, -\frac{23}{45} - \frac{2\alpha}{45}, 0, 0, 0, \right. \right. \\
 &\quad \left. \left. 0 - \frac{37}{45} - \frac{ps}{9} - \frac{\alpha}{30} \right\}, \dots, \right. \\
 &\quad \left. \dots, \left\{ -\frac{37}{45} + \frac{ps}{9} - \frac{\alpha}{30}, \frac{2ps}{9} + \frac{\alpha}{9}, -\frac{23}{45} - \frac{2\alpha}{45}, \frac{16}{45} + \frac{4\alpha}{45}, 0, 0, 0, \frac{104}{45} + \frac{8\alpha}{45} \right\} \right\} \\
 B &:= -\left\{ \frac{1}{2} + \frac{5ps}{36} - \frac{\alpha}{60} - \left( \frac{299GKP}{1260} - \frac{11adGKP}{2800} - \frac{11GKM^2P}{2800} + \frac{1}{30} GP_n \right)_1 u - \left( \frac{4GKP}{1260} + \frac{3}{175} GKM^2P - \frac{1}{30} GP_n \right)_2 u - \right. \\
 &\quad \left. \left( \frac{16GKP}{21} - \frac{4}{35} adGKP - \frac{4}{35} GKM^2P + \frac{8}{45} GP_n \right)_8 u \right\} \\
 on &:= \left\{ \left\{ \frac{52}{45} + \frac{S Sc}{15}, -\frac{37}{45} - \frac{7S Sc}{90}, \frac{1}{2} - \frac{S Sc}{60}, -\frac{23}{45}, 0, 0, -1, -\frac{37}{45} + \frac{S Sc}{9}, \right. \right. \\
 &\quad \left. \left. \dots, \left\{ -\frac{37}{45} + \frac{S Sc}{9}, \frac{2 S Sc}{9}, -\frac{23}{45}, \frac{16}{45}, 0, 0, 0, \frac{104}{45} \right\} \right\} \right\}
 \end{aligned}$$

$$\begin{aligned}
 F &:= -\left\{ \frac{1}{2} + \frac{5 S Sc}{36} + \frac{S c S_o}{2 an} + \frac{1}{30} S c n u - \frac{1}{30} S c n u - \frac{2}{45} S c n u + \dots, \right. \\
 &\quad \left. \frac{16 S c S_o}{45 an} \theta^4 + \frac{104 S c S_o}{45 an} \theta^8 \right\} \\
 \text{By Serendipity element the velocity, temperature and concentration equations are given below} \\
 vel &= u_1 X_1 s + u_2 X_2 s + u_3 X_3 s + u_4 X_4 s + u_8 X_8 s; \\
 temp &= \theta_1 X_1 s + \theta_2 X_2 s + \theta_3 X_3 s + \theta_4 X_4 s + \theta_8 X_8 s + \theta_5 X_5 s + \theta_6 X_6 s + \theta_7 X_7 s; \\
 concen &= c_1 X_1 s + c_2 X_2 s + c_3 X_3 s + c_4 X_4 s + c_8 X_8 s + c_5 X_5 s + c_6 X_6 s + c_7 X_7 s; \\
 Sh &:= -4 \left( \frac{1}{2} + \frac{1}{2} \left( -\frac{1}{2} + z \right) \right) (-1 + z) C_1 + 4(-1 + z) z C_2 - 4 \left( -\frac{1}{2} + \frac{1}{2} \left( -\frac{1}{2} + z \right) \right) z C_3 - 4 z C_4 + 4(-1 + z) C_8 + \\
 &\quad + (-2(-1+z)-2(\frac{1}{2}-z)(-1+z)) + 2z + 4 \left( \frac{1}{4} + \frac{1}{2}(-1+z)z \right) - 4(-1+z)z(1+x_n^2)
 \end{aligned}$$

#### Iterations for finding Sherwood number values

$$\begin{aligned}
 ad &= 5; G = 5; S = 0.1; M = 5; \\
 n &= 0.5, n_2 = 0.7; an = 1; p = 0.7; \\
 k &= 0.3; Sc = 1.3; s_o = 0.5; \\
 \alpha &= 2; z = 0.5; x = 1; \\
 \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\} &= Inverse[mom].A; \\
 \{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8\} &= Inverse[T].B; \\
 \{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\} &= Inverse[on].F; \\
 TableForm[Table[Sh]] & \\
 ad &= 6; G = 5; S = 0.1; M = 5; n_1 = 0.5; \\
 n &= 0.7; an = 1; p = 0.7; k = 0.3; \\
 Sc &= 1.3; s_o = 0.5; \alpha = 2; z = 0.5; x = 1;
 \end{aligned}$$

$$\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\} = \text{Inverse}[\text{mom}].A;$$

$$\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8\} = Inverse[T].B;$$

$\{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\} = Inverse[on].F;$

*TableForm[Table[Sh]]*

$$ad = 7; G = 5; S = 0.1; M = 5; n = 0.5;$$

$$n = 0.7; an = 1; p = 0.7; k = 0.3;$$

$$Sc = 1.3; s_o = 0.5; \alpha = 2; z = 0.5; x = 1; \\ \{u, u, u, u, u, u, u, u\} = Inverse[mom].A$$

$$\{ \theta, \theta, \theta, \theta, \theta, \theta, \theta, \theta \} = Inverse[T].B;$$

$\{c, c, c, c, c, c, c, c\} = Inverse[on].F;$

```
TableForm[Table[Sh]]
```

$$ad = 8; G = 5; S = 0.1; M = 5; n = 0.5,$$

$$n = 0.7; an = 1; p = 0.7; k = 0.3;$$

$$Sc = 1.3; s_o = 0.5; \alpha = 2; z = 0.5; x = 1; \\ \{u, u, u, u, u, u, u\} = Inverse[mom] \wedge$$

$\{\theta_1 \theta_2 \theta_3 \theta_4 \theta_5 \theta_6 \theta_7 \theta_8\} = Inverse[T] B$ :

$$\{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\} = \text{Inverse}[op] F$$

Table 11. Summary

Table

## Output

0.583073

0.530001

0.47714

We should calculate Sherwood number values at  $z=0$ ,  $0.5$  and  $1$  levels. At every iteration changing one parameter and remaining other parameter values fixed we get all parameter values at  $z=0$ ,  $0.5$ , and  $1$  levels.

## V. DISCUSSION OF THE NUMERICAL RESULTS

**RESULTS**  
The finite element technique is used to investigate the mixed convective mass transfer flow of a viscous, incompressible fluid through a porous medium in a porous horizontal channel. The flow within the porous medium is taken into account based on the Brinkman extended Darcy model. The flow is unidirectional and Sherwood number is discussed for different variations of the governing parameters viz., the Grashoff number  $G$ , the Darcy parameter  $D^{-1}$ , the buoyancy ratio  $N$ , the Suction parameter  $S$ , the Soret

parameter  $S_o$ , the Buoyancy ration  $N$ , the Schmidt number  $S_c$  and the Eckert number  $Ec$ . The pressure gradient is chosen positive so that negative  $u$  indicates the actual flow while positive  $u$  corresponds to reversal flow.

The concentration distribution ( $C$ ) at different horizontal and vertical levels is shown in figs (1-5) for  $So$ . The variation of  $C$  with Soret parameter  $S_o$  is shown in figs (1-5). At  $y = 0$  level the concentration enhances with  $So$  and it reduces with  $|S_o| (< 0)$  at  $y = 0$  level (fig. 1) and enhances at  $y = 1/2$  level (fig. 2). At the vertical level  $z = 0$  the concentration  $C$  enhances with increase in  $S_o$  and reduces with  $|S_o| (< 0)$  (fig. 3). At the other vertical levels  $z = 1/2$  & 1, we find an increment in  $C$  with  $|S_o| (> 0)$  (fig. 4 & 5).

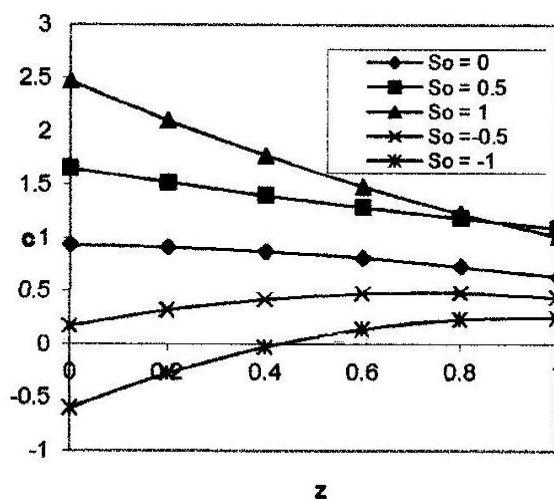


Fig. 1 Variation of C with So at y = 0 level.  
 $D^{-1} = 10^3$ ,  $G = 10^3$ ,  $N = 1$ ,  $Sc = 1.3$ ,  $S = 0.3$ ,

**Ec=0.3,** □

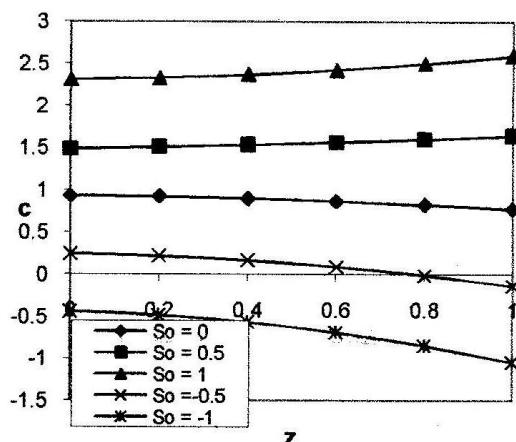


Fig. 2 Variation of C with So at  $y = 0.5$  level.  
 $D^{-1} = 10^3$ ,  $G = 10^3$ ,  $N = 1$ ,  $Sc = 1.3$ ,  $S = 0.3$ ,  
 $Ec = 0.3$ ,  $\square \square \square \square \square \square \square$

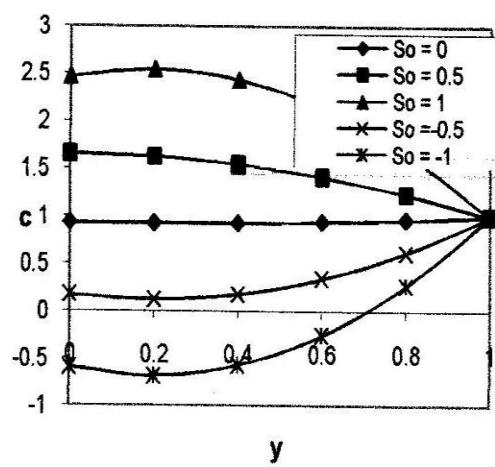


Fig. 3 Variation of C with So at  $Z = 0$  level.  
 $D^{-1} = 10^3$ ,  $G = 10^3$ ,  $N = 1$ ,  $Sc = 1.3$ ,  $S = 0.3$ ,  $Ec = 0.3$ ,  $\square \square \square \square \square \square \square$

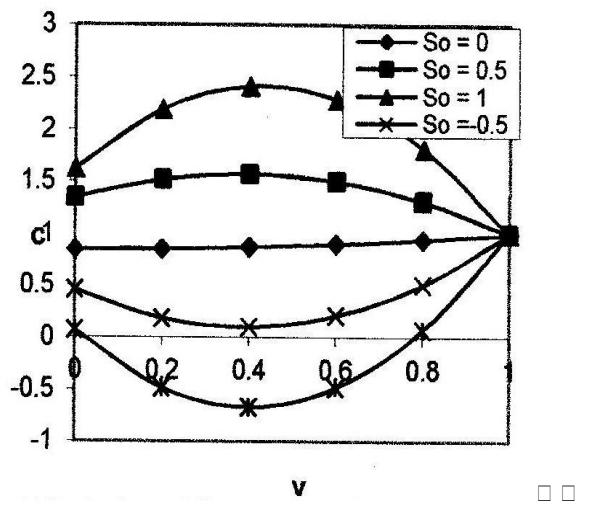


Fig. 4 Variation of C with So at  $Z = 0.5$  level.  
 $D^{-1} = 10^3$ ,  $G = 10^3$ ,  $N = 1$ ,  $Sc = 1.3$ ,  $S = 0.3$ ,  $Ec = 0.3$ ,  $\square \square \square \square \square \square \square$

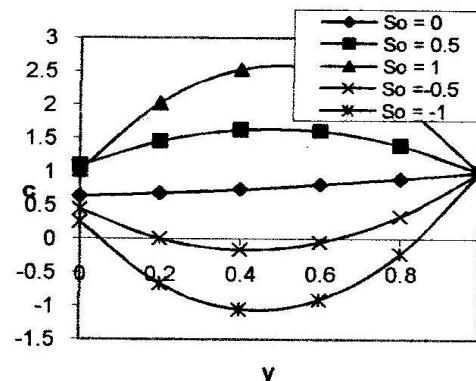


Fig. 5 Variation of C with So at  $Z = 1$  level.  
 $D^{-1} = 10^3$ ,  $G = 10^3$ ,  $N = 1$ ,  $Sc = 1.3$ ,  $S = 0.3$ ,  
 $Ec = 0.3$ ,  $\square \square \square \square \square \square \square$

Table - 1  
Sherwood Number(Sh)  
 $G = 3 \times 10^3$ ,  $D^{-1} = 3 \times 10^3$ ,  $N_1 = 0.5$ ,  $N_2 = 0.7$ ,  $N = 1$ ,  
 $S = 0.1$ ,  $Sc = 1.3$ ,  $So = 0.5$ ,  $\square = 2$

Z	I	II
0	-0.635075	-0.837208
0.5	-0.590987	-0.899634
1	-0.49493	-0.962936

III	IV	V	VI
-1.24148	-1.64574	-3.66708	-5.68841
-1.51693	-2.13422	-5.22069	-8.30716
-1.89895	-2.83496	-7.51502	-12.1951

	I	II	III	IV
Ec	0	0.1	0.3	0.5

V	VI
1.5	2.5

Table - 2  
Sherwood Number(Sh) at  $Z = 0$  Level  
 $S = 0.1$ ,  $N = 1$ ,  $Ec = 0.3$ ,  $Sc = 1.3$ ,  $So = 0.5$ ,  $\square = 2$ ,  
 $N_1 = 0.5$

$D^{-1}$	I	II
$5 \times 10^2$	-1.24148	-1.35847
$10^3$	-1.30407	-1.43399
$2 \times 10^3$	-1.36694	-1.50983
$3 \times 10^3$	-1.43008	-1.58594

III	IV	V
-1.47547	-1.52359	-0.191475
-1.56392	-1.62399	-0.254066
-1.65272	-1.72514	-0.316945
-1.74181	-1.82697	-0.380075

VI	VII	VIII
-0.308473	0.741527	0.858525
-0.383995	0.666005	0.795934
-0.459832	0.590168	0.733055
-0.535944	0.514056	0.669925

	I	II
G	$5 \times 10^2$	$10^3$
C <sub>w</sub>	1	1
N <sub>2</sub>	0.7	0.7

III	IV	V
$3 \times 10^3$	$5 \times 10^2$	$5 \times 10^2$
1	1	1 + N <sub>2</sub> /2
0.7	1.5	0.7

VI	VII	VIII
$10^3$	$10^3$	$5 \times 10^2$
1 + N <sub>2</sub> /2	1 + N <sub>2</sub>	1 + N <sub>2</sub>
0.7	0.7	0.7

Table - 3  
Sherwood Number(Sh) at Z = 0.5 Level  
 $S = 0.1, N = 1, Ec = 0.3, Sc = 1.3, So = 0.5, \square = 2, N_1 = 0.5$

D <sup>-1</sup>	I	II
$5 \times 10^2$	-1.51693	-1.69349
$10^3$	-1.57	-1.75718
$2 \times 10^3$	-1.62286	-1.82061
$3 \times 10^3$	-1.67552	-1.88381

III	IV	V
-1.87005	-1.94008	-0.466927
-1.94436	-2.0285	-0.519999
-2.01836	-2.11657	-0.572856
-2.0921	-2.20432	-0.62552

VI	VII	VIII
-0.643489	0.406511	0.583073
-0.707177	0.342823	0.530001
-0.77061	0.27939	0.477144
-0.833809	0.216191	0.42448

	I	II
G	$5 \times 10^2$	$10^3$
θ <sub>N</sub>	1	1
N <sub>2</sub>	0.7	0.7

III	IV	V
$3 \times 10^3$	$5 \times 10^2$	$5 \times 10^2$
1	1	1 + N <sub>2</sub> /2
0.7	1.5	0.7

VI	VII	VIII
$10^3$	$10^3$	$5 \times 10^2$
1 + N <sub>2</sub> /2	1 + N <sub>2</sub>	1 + N <sub>2</sub>
0.7	0.7	0.7

Table - 4  
Sherwood Number(Sh) at Z = 1 Level  
 $S = 0.1, N = 1, Ec = 0.3, Sc = 1.3, So = 0.5, \square = 2, N_1 = 0.7$

D <sup>-1</sup>	I	II
$5 \times 10^2$	-1.89895	-2.16109
$10^3$	-1.95071	-2.22277
$2 \times 10^3$	-2.0017	-2.28356
$3 \times 10^3$	-2.05201	-2.34355

III	IV	V
-2.42323	-2.48939	-0.848947
-2.49483	-2.5798	-0.900709
-2.56541	-2.66864	-0.951704
-2.63509	-2.75607	-1.00201

VI	VII	VIII
-1.11109	-0.0610885	0.201053
-1.17277	-0.122767	0.149291
-1.23356	-0.183558	0.0982957
-1.29355	-0.243552	0.0479873

	I	II
G	$5 \times 10^2$	$10^3$
θ <sub>N</sub>	1	1
N <sub>2</sub>	0.7	0.7

III	IV	V
$3 \times 10^3$	$5 \times 10^2$	$5 \times 10^2$
1	1	1 + N <sub>2</sub> /2
0.7	1.5	0.7

VI	VII	VIII
$10^3$	$10^3$	$5 \times 10^2$
1 + N <sub>2</sub> /2	1 + N <sub>2</sub>	1 + N <sub>2</sub>
0.7	0.7	0.7

The Sherwood number (Sh) which represents the rate of mass transfer at the boundary  $y = 1$  has been evaluated at three different axial positions  $x = 0, \frac{1}{2}$  & 1 of the channel. In view of the uniform concentration gradients the boundary concentration at these axial positions are 1,  $1 + N_2/2$  and  $1 + N_2$  respectively. The Sherwood number at the different positions  $z = 0, \frac{1}{2}$  & 1 across the boundary plane is shown in tables 1-4. The Sherwood number at these levels is negative for all variations. We find from table.1 that the rate of mass transfer enhances with Ec at all levels which indicates that the inclusion of the dissipation enhances the rate of mass transfer.

The rate of mass transfer at  $z = 0$  & 1 levels depreciates with axial distance  $x \leq \frac{1}{2}$  and enhances with  $x > \frac{1}{2}$  while at  $z = \frac{1}{2}$  level  $Sh$  decreases with  $x$ . An increase in  $G$  enhances the rate of mass transfer at all levels. Also lesser the permeability of the porous medium larger the magnitude of  $Sh$  at all levels. The variation of  $Sh$  with  $N_2$  shows that the rate of mass transfer experiences an enhancement with increase in  $N_2$ . The values of  $Sh$  at  $z = \frac{1}{2}$  level are greater than that at  $z = 0$  & 1 levels (Tables 2-4).

## VI. CONCLUSIONS

1. By using Mathematica 4.1 commands we executed and calculated Sherwood number values for different parameters.
2. At the vertical level  $z = 0$  the concentration  $C$  enhances with increase in  $S_o$  and reduces with  $|S_o| (<0)$ .
3. At the other vertical levels  $z = \frac{1}{2}$  & 1, we find an increment in  $C$  with  $|S_o| (>0)$ .
4. The rate of mass transfer enhances with  $E_c$  at all levels which indicates that the inclusion of the dissipation enhances the rate of mass transfer.
5. An increase in  $G$  enhances the rate of mass transfer at all levels

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