Comparitive Study on Naked Singularities Arising in Vaidya Space Time and Hussain Space Time

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Abstract: *Earlier many authors have shown the occurrence and nature of the singularities in Vaidya space time and Hussain space time. In the present work we explore the comparative study between both the space times. Implication of the same initial data to both the space times has been discussed.*

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I. INTRODUCTION

It is a well known fact that, in general relativity the important issue is end state of gravitational collapse whether it is a naked singularity or a black hole. From the last few years various models such as radiation, [2] dust, [1] perfect fluid [3] etc; have studied about this issue. The final state of all these models is either black hole or a visible singularity.

R.Penrose has given a conjecture which has no exact proof till date is Cosmic Censorship Conjecture [4]. This Conjecture gives an idea that, at the end state of gravitational collapse only black holes are formed which are invisible to any observer. Later on many counter examples were proved for this cosmic censorship conjecture. The one of the most important among these counter examples is Vaidya solution [5]. Since then this Vaidya solution is being used as scenario in general relativity for the case of gravitational collapse. The first Counter example was given by Papapetrou [6]. Openheimer and Synder [69] had studied about the dust collapse with static Schwarzschild exteriors, where as interior space time is represented by Friedman like solution. Later on, several scientists have extended their studies about the gravitational collapse.

The dissipation in source by allowing radial heat flow was given by Santos and Collaborations [11,12]. The static exterior for the perfect fluid in the interior was studied by Misner and Sharp [10] by using the idea of outgoing radiations of a collapsing body given by Vaidya [17]. According to Penrose and Hawking [7] the singularity need not be a black hole after its collapse due to the loss of its nuclear fuel.

Our investigation is focused on the nature of the singularity which may possibly form after the gravitational collapse. When the centre of the collapsing gets trapped before its boundary has entered the Schwarzschild radius, then there is a possibility of naked singularity. Einstein equation such as Hussain solution [13], Anti-de-sitter charged vaidya solution has generalized by Anzhong Wang [8]. Lately Hussain solution with P = Kρ null fluid has used as the formation of black hole with a short hair [9].

In this paper our aim is to investigate the nature of singularities formed in Hussain space time as well as Vaidya space time and their comparison. The comparative study of both the Hussain space time and Vaidya space time gives naked singularities in different aspects.

This paper shows the Vaidya space time in section 2 followed by Hussain space time in section 3. In section 4 we show the nature of the singularities as well as the comparative study between the two space times. We end the section 5 with concluding remarks.

II. VAIDYA SPACE TIME

Vaidya space time is given by [15]

$$ds^2 = -\left(1 - \frac{2m(w,r)}{r}\right)dw^2 + 2\,dw\,dr + r^2[d\theta^2 + \sin^2\theta\ d\varphi^2\ ] \quad (1)$$

Where w is the advanced Eddington time and m is a positive value and r is radial coordinate with the condition $0 < r < \infty$

And m (w,r) is gravitational mass which will present in the sphere of radius r and
The corresponding energy momentum tensor is given by [16]

$$T_{\delta\gamma} = T_{\delta\gamma}{}^{(n)} + T_{\delta\gamma}{}^{(m)} \quad (2)$$

$$T_{\delta\gamma}{}^{(n)} = \mu\, l_\delta\, l_\gamma \quad (3)$$

$$T_{\delta\gamma} = (P+\rho)\left(l_\delta\, n_\gamma + l_\gamma\, n_\delta\right) + P g_{\delta\gamma} \quad (4)$$

Here the above P and ρ represents the thermodynamic pressure and energy density, where as μ represents energy density of Vaidya null radiation.

Linearly independent two Eigen vectors of energy momentum tensor are l_γ, n_δ

These Eigen vectors are having the components

$$l_\gamma = \lambda_\gamma{}^0 , \quad (5)$$

$$n_\gamma = \frac{1}{2}\left[1 - \frac{m(w,r)}{r^{n-1}}\right]\lambda_\gamma{}^0 - \lambda_\gamma{}^1 , \quad (6)$$

$$l_\upsilon l^\upsilon = n_\upsilon n^\upsilon = 0, \qquad l_\upsilon n^\upsilon = -1 \quad (7)$$

Now consider the EMT of Eq.(6) as the general case.
The energy conditions for the above will be as follows:
The dominant energy conditions are

$$\mu \geq 0, \quad \rho \geq P \geq 0 \quad (8)$$

The weak and strong energy conditions are

$$\mu \geq 0, \quad P \geq 0 , \quad \rho \geq 0 \quad (9)$$

Einstein field equations is given by

$$G_{\delta\gamma} = KT_{\delta\gamma} \quad (10)$$

Where $G_{\delta\gamma}$ is Einstein tensor, K is Gravitational constant

For outgoing null radial geodesic $ds^2 = 0, d\theta = 0, d\varphi = 0$

$$2\,dw\,dr = \left(1 - \frac{2m(w,r)}{r}\right) dw^2 \quad (11)$$

The mass function is taken as

$$2m(w,r) = \frac{\delta}{3} w \quad (12)$$

$$2\,dw\,dr = \left(1 - \frac{\delta w}{3r}\right) dw^2 \quad (13)$$

The metric obtained for the outgoing radial null geodesics is given by

$$\frac{dr}{dw} = \frac{1}{2}\left[1 - \frac{\delta w}{3r}\right] \quad (14)$$

The radial null geodesics for the metric should satisfy the null condition

$$\frac{dw}{dr} = \frac{2}{1 - \frac{\delta w}{3r}}$$

The coordinate w is an advanced time coordinate. The above equation has a singularity at $r \to 0,\ w \to 0$

Here we are considering the limiting value as $X = \frac{w}{r}$ along the singular geodesic to approach a singularity

Hence, for the geodesic tangent to exist at this point uniquely we should have the following [23]

$$X_0 = \lim_{\substack{w\to 0 \\ r\to 0}} \frac{w}{r} = \lim_{\substack{w\to 0 \\ r\to 0}} \frac{dw}{dr}$$

$$X_0 = \lim_{\substack{w\to 0 \\ r\to 0}} \frac{2}{1-\frac{\delta u}{3r}}$$

$$X_0 = \frac{2}{1-\frac{\delta}{3}X_0} \tag{15}$$

$$X_0 - \frac{\delta}{3}X_0^2 = 2$$

$$X_0 - \frac{\delta}{3}X_0^2 - 2 = 0$$

$$\delta X_0^2 - 3X_0 + 6 = 0 \tag{16}$$

From this Eq. we decide the nature of the singularity. The singularity will be naked if there exists at least one real and positive root. If there is no positive root then it ends into a black hole.

To investigate, whether naked singularities will arise or not, we take some different values of δ. In section IVwe show the result.

III. HUSSAIN SPACE TIME

The Spherically symmetric space time is given as in equation 1 and where the non-vanishing components of Einstein tensor are given by [14]

$$G_0^0 = G_1^1 = -\frac{2m'}{r^2} \qquad G_0^1 = \frac{2\dot{m}}{r^2} \qquad G_2^2 = G_3^3 = -\frac{m''}{r^2}$$

Where $\dot{m} = \frac{\partial m}{\partial w}$ and $m' = \frac{\partial m}{\partial r}$ and Einstein equations are taken as above from Vaidya space time

$$P = \frac{-m''}{kr} \qquad \mu = \frac{n\dot{m}}{kr^2} \qquad \rho = \frac{2m'}{kr^2}$$

And by using the same energy conditions as in section 2 we take the choice of mass function

$$m(w,r) = f(w) - \frac{g(w)}{(2k-1)r^{2k-1}}, \; k \neq \frac{1}{2} \text{ and}$$

$$m(w,r) = f(w) + g(w)\ln r, \; k = \frac{1}{2}$$

From the above conditions, Hussain metric space time can be defined as

$$ds^2 = -\left[1 - \frac{2f(w)}{r} + \frac{2g(w)}{(2k-1)r^{2k}}\right]dw^2 + 2dwdr + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \tag{17}$$

The above metric is cosmological for k < 1 / 2 and asymptotically flat for k > 1/2
The radial null geodesic equation is given by

$$\frac{dr}{dw} = \frac{1}{2}\left[1 - \frac{2f(w)}{r} + \frac{2g(w)}{(2k-1)r^{2k}}\right] \tag{18}$$

For the above metric if k >1/2 that is for k = 1, it becomes Charged Vaidya solution.
This was already discussed in past papers. [18, 19]
Here we consider some different cases for 1/2 < k < 1.

Case (i)

Here we take k = 3/4

For the further simplifications let us choose $2f = \frac{\delta}{3}w$ and $g(w) = \mu w^{3/2}$ (19)

$$ds^2 = -\left[1 - \frac{\delta w}{3r} + \frac{2\mu w^{3/2}}{1/2\, r^{3/2}}\right]dw^2 + 2dwdr + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \tag{20}$$

$$\frac{dr}{dw} = \frac{1}{2}\left[1 - \frac{\delta w}{3r} + \frac{4\mu u^{3/2}}{r^{3/2}}\right] \tag{21}$$

$$X_0 = \lim_{\substack{w\to 0\\ r\to 0}} \frac{w}{r} = \lim_{\substack{w\to 0\\ r\to 0}} \frac{dw}{dr}$$

$$X_0 = \lim_{\substack{w\to 0\\ r\to 0}} \frac{2}{1 - \frac{\delta}{3}X_0 + 4\mu X_0^{3/2}} \tag{22}$$

$$X_0 - \frac{\delta}{3}X_0^2 + 4\mu X_0^{5/2} = 2$$

$$4\mu X_0^{5/2} - \frac{\delta}{3}X_0^2 + X_0 - 2 = 0 \qquad 12\mu X_0^{5/2} - \delta X_0^2 + 3X_0 - 6 = 0 \tag{23}$$

Let $X_0 = X^2$

Then the above equation becomes

$$12\mu X^5 - \delta X^4 + 3X^2 - 6 = 0 \tag{24}$$

Case (ii)

Here we take k = 3/5

For the further simplifications let us choose $2f = \frac{\delta}{3}w$ and $g(w) = \mu w^{6/5}$.

$$ds^2 = -\left[1 - \frac{\delta w}{3r} + \frac{2\mu w^{6/5}}{1/5\, r^{6/5}}\right]dw^2 + 2dwdr + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \tag{25}$$

$$\frac{dr}{dw} = \frac{1}{2}\left[1 - \frac{\delta w}{3r} + \frac{10\mu u^{6/5}}{r^{6/5}}\right]$$

$$X_0 = \lim_{\substack{w\to 0\\ r\to 0}} \frac{w}{r} = \lim_{\substack{w\to 0\\ r\to 0}} \frac{dw}{dr}$$

$$X_0 = \lim_{\substack{w\to 0\\ r\to 0}} \frac{2}{1 - \frac{\delta}{3}X_0 + 10\mu X_0^{6/5}} \tag{26}$$

$$X_0 - \frac{\delta}{3} X_0^{\,2} + 10\mu X_0^{\,11/5} = 2$$

$$10\mu X_0^{\,11/5} - \frac{\delta}{3} X_0^{\,2} + X_0 - 2 = 0$$

$$30\mu X_0^{\,11/5} - \delta X_0^{\,2} + 3X_0 - 6 = 0 \qquad (27)$$

Let $X_0 = X^5$

Then the above equation becomes

$$30\mu X^{11} - \delta X^{10} + 3X^5 - 6 = 0 \qquad (28)$$

Case (iii)

Here we take k = 4/5

For the further simplifications let us choose $2f = \frac{\delta}{3} w$ and $g(w) = \mu w^{8/5}$.

$$ds^2 = -\left[1 - \frac{\delta w}{3r} + \frac{2\mu w^{8/5}}{3/5\, r^{8/5}}\right] dw^2 + 2dwdr + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \qquad (29)$$

$$\frac{dr}{dw} = \frac{1}{2}\left[1 - \frac{\delta w}{3r} + \frac{10\mu u^{8/5}}{3r^{8/5}}\right]$$

$$X_0 = \lim_{\substack{w\to 0 \\ r\to 0}} \frac{w}{r} = \lim_{\substack{w\to 0 \\ r\to 0}} \frac{dw}{dr}$$

$$X_0 = \lim_{\substack{w\to 0 \\ r\to 0}} \frac{2}{1 - \frac{\delta}{3} X_0 + \frac{10}{3}\mu X_0^{\,13/5}} \qquad (30)$$

$$X_0 - \frac{\delta}{3} X_0^{\,2} + \frac{10}{3}\mu X_0^{\,13/5} = 2$$

$$10\mu X_0^{\,13/5} - \delta X_0^{\,2} + 3X_0 - 6 = 0 \qquad (31)$$

Let $X_0 = X^5$

Then the above equation becomes

$$10\mu X^{13} - \delta X^{10} + 3X^5 - 6 = 0 \qquad (32)$$

Case (iv)

Here we take k = 4/7

For the further simplifications let us choose $2f = \frac{\delta}{3} w$ and $g(w) = \mu w^{8/7}$.

$$ds^2 = -\left[1 - \frac{\delta w}{3r} + \frac{7\mu w^{8/7}}{1/2\, r^{8/7}}\right] dw^2 + 2dwdr + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \qquad (33)$$

$$\frac{dr}{dw} = \frac{1}{2}\left[1 - \frac{\delta w}{3r} + \frac{14\mu u^{8/7}}{r^{8/7}}\right]$$

$$X_0 = \lim_{\substack{w\to 0 \\ r\to 0}} \frac{w}{r} = \lim_{\substack{w\to 0 \\ r\to 0}} \frac{dw}{dr}$$

$$X_0 = \lim_{\substack{w\to 0 \\ r\to 0}} \frac{2}{1-\frac{\delta}{3}X_0 + 14\mu X_0^{8/7}} \quad (34)$$

$$X_0 - \frac{\delta}{3}X_0^2 + 14\mu X_0^{8/7} = 2$$

$$52\mu X_0^{15/7} - \delta X_0^2 + 3X_0 - 6 = 0 \quad (35)$$

Let $X_0 = X^5$

Then the above equation becomes

$$52\mu X^{15} - \delta X^{14} + 3X^7 - 6 = 0 \quad (36)$$

Case (v)

Here we take k = 2/3

For the further simplifications let us choose $2f = \frac{\delta}{3}w$ and $g(w) = \mu w^{4/3}$.

$$ds^2 = -\left[1 - \frac{\delta w}{3r} + \frac{3\mu w^{4/3}}{1/2\, r^{4/3}}\right]dw^2 + 2dwdr + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \quad (37)$$

$$\frac{dr}{dw} = \frac{1}{2}\left[1 - \frac{\delta w}{3r} + \frac{6\mu u^{4/3}}{r^{4/3}}\right]$$

$$X_0 = \lim_{\substack{w\to 0 \\ r\to 0}} \frac{w}{r} = \lim_{\substack{w\to 0 \\ r\to 0}} \frac{dw}{dr}$$

$$X_0 = \lim_{\substack{w\to 0 \\ r\to 0}} \frac{2}{1-\frac{\delta}{3}X_0 + 6\mu X_0^{4/3}} \quad (38)$$

$$X_0 - \frac{\delta}{3}X_0^2 + 6\mu X_0^{7/3} = 2$$

$$18\mu X_0^{7/3} - \delta X_0^2 + 3X_0 - 6 = 0 \quad (39)$$

Let $X_0 = X^3$

Then the above equation becomes

$$18\mu X^7 - \delta X^6 + 3X^3 - 6 = 0 \quad (40)$$

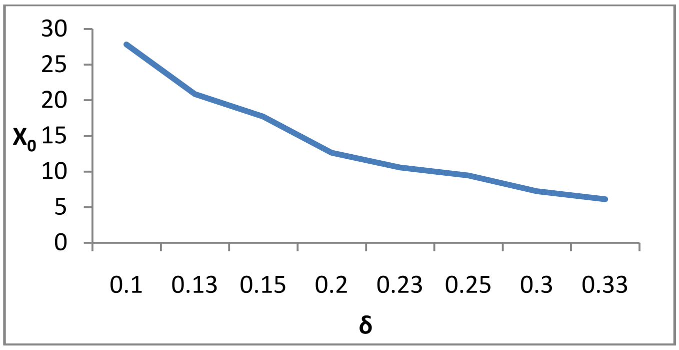
IV COMPARATIVE STUDY OF VAIDYA SPACE TIME AND HUSSAIN SPACE TIME

From equation 16 in Vaidya space time, we give different values for δ to get the values of X_0. If the roots of the equation give at least one positive root then we get a naked singularity.

So by taking different values for δ we verify whether the root will be positive or not.

Table.1. Values of X_0 for different values of δ

δ	X_0
0.1	27.845
0.13	20.865
0.15	17.746
0.2	12.623
0.23	10.577
0.25	9.464
0.3	7.236
0.33	6.12



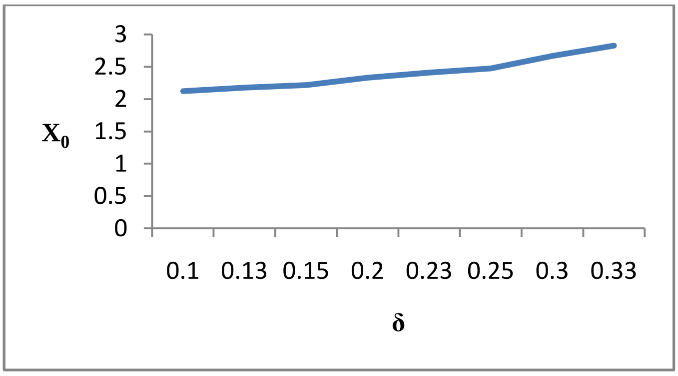
Graph.1. Graph of the Values of X_0 against the values of δ

Here in the above graph we observe that the value of X_0 decreases as the values of δ increases. The roots obtained are positive. Hence we get the naked singularity. As well as by increasing the values of δ the values of X_0 are decreasing.

Now from Hussain space time of equation 24 we get different values for X_0 for different values of δ. If the roots of the equation give at least one positive root then we get a naked singularity. The following table gives an idea of positive roots.

Table.2. Values of X_0 for different values of δ.

δ	X_0
0.1	2.122849
0.13	2.178576
0.15	2.217121
0.2	2.328676
0.23	2.408704
0.25	2.471184
0.3	2.663424
0.33	2.8224



Graph 2. Graph of the Values of X_0 against the values of δ

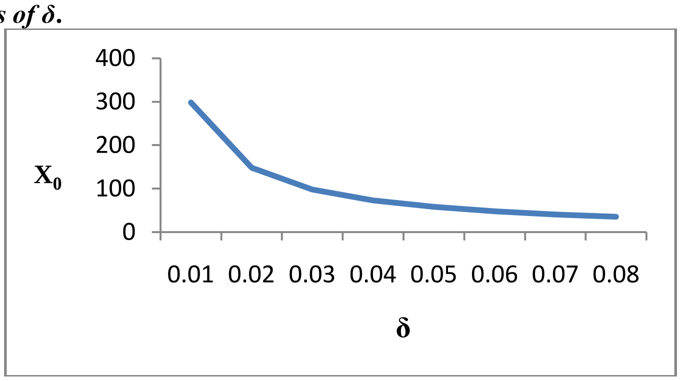
In the above graph we observe that the value of X_0 increases as the values of δ increases. Positive roots are obtained in the graph. Hence we observe the singularity is naked. As well as by increasing the values of δ the values of X_0 are increasing.

So by comparing the graph 1 of Vaidya space time, the values of X_0 are decreasing where as in graph 2 of Hussain space time the values of X_0 are increasing by the increase of values of δ in both space times.

From equation 16 in Vaidya space time, we analyze the following table.

Table.3. ***Values of X_0 for different values of δ.***

δ	X_0
0.01	297.986
0.02	147.973
0.03	97.958
0.04	72.944
0.05	57.928
0.06	47.913
0.07	40.754
0.08	35.38

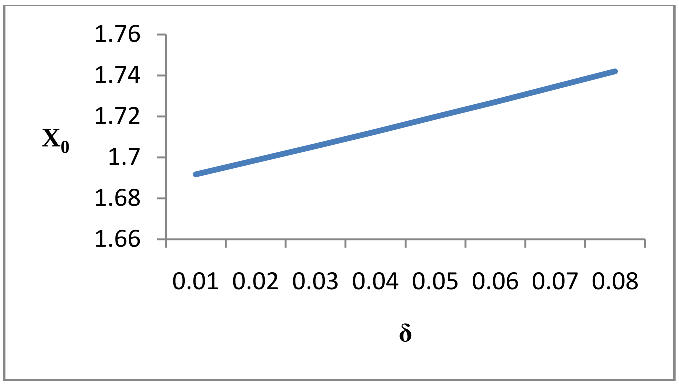


Graph 3. Graph of the Values of X_0 against the values of δ

Here in the above graph we observe that the value of X_0 decreases as the values of δ increases. The roots obtained are positive. Hence we get the naked singularity. As well as by increasing the values of δ the values of X_0 are decreasing.
Now from equation 28 of Hussain space time we get different values for X_0 for different values of δ. If the roots of the equation give at least one positive root then we get a naked singularity, shown in the table 4.

Table.4. Values of X_0 for different values of δ.

δ	X_0
0.01	1.691672
0.02	1.69853
0.03	1.70542
0.04	1.71248
0.05	1.71972
0.06	1.72698
0.07	1.73435
0.08	1.7419



Graph 4. Graph of the Values of X_0 against the values of δ

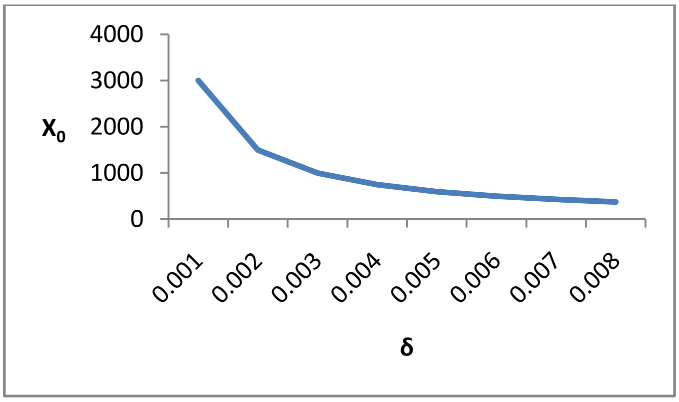
In the above graph we observe that the value of X_0 increases as the values of δ increases. Positive roots are obtained in the graph. Hence we observe the singularity is naked. As well as by increasing the values of δ the values of X_0 are increasing.

So by comparing the graph 3 of Vaidya space time, the values of X_0 are decreasing where as in graph 4 of Hussain space time the values of X_0 are increasing by the increase of values of δ in both space times.

From equation 16 in Vaidya space time, we get the following table.

Table.5. ***Values of X_0 for different values of δ.***

δ	X_0
0.001	2997.99
0.002	1497.997
0.003	997.996
0.004	747.995
0.005	597.993
0.006	497.992
0.007	426.562
0.008	372.989



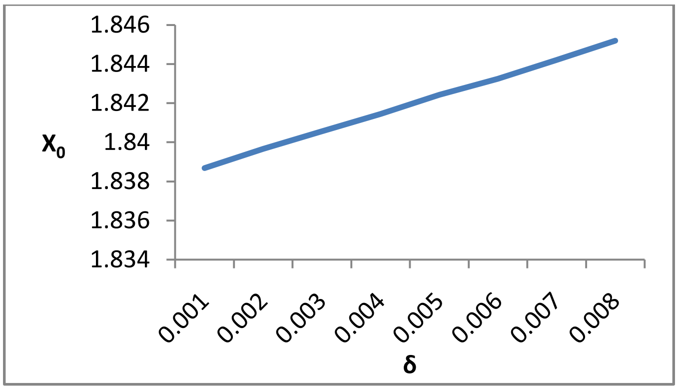
Graph 5. Graph of the Values of X_0 against the values of δ

Here in the above graph we observe that the value of X_0 decreases as the values of δ increases. The roots obtained are positive. Hence we get the naked singularity. As well as by increasing the values of δ the values of X_0 are decreasing.

Equation 32 of Hussain space time analyzes the following table with different values of X_0 for different values of δ

Table.6. ***Values of X_0 for different values of δ.***

δ	X_0
0.001	1.83868
0.002	1.83966
0.003	1.84056
0.004	1.84145
0.005	1.84243
0.006	1.84325
0.007	1.84422
0.008	1.8452



Graph 6. Graph of the Values of X_0 against the values of δ

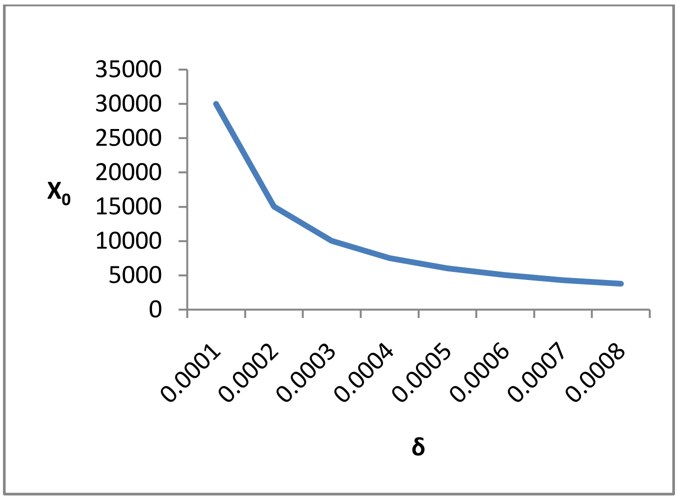
In the above graph we observe that the value of X_0 increases as the values of δ increases. Positive roots are obtained in the graph. Hence we observe the singularity is naked. As well as by increasing the values of δ the values of X_0 are increasing.

So by comparing the graph 5 of Vaidya space time, the values of X_0 are decreasing where as in graph 6 of Hussain space time the values of X_0 are increasing by the increase of values of δ in both space times.

From equation 16 in Vaidya space time, we calculate the following table.

Table 7. ***Values of X_0 for different values of δ.***

δ	X_0
0.0001	29998
0.0002	14998
0.0003	9998
0.0004	7497.99
0.0005	5997.99
0.0006	4997.99
0.0007	4283.73
0.0008	3747.99



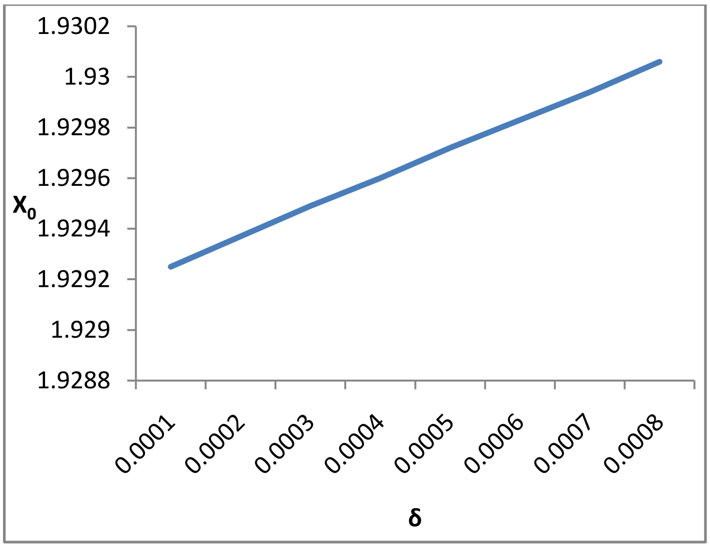
Graph 7. Graph of the Values of X_0 against the values of δ

Here in the above graph we observe that the value of X_0 decreases as the values of δ increases. The roots obtained are positive. Hence we get the naked singularity. As well as by increasing the values of δ the values of X_0 are decreasing.

Equation 36 of Hussain space time analyzes the following table with different values of X_0 for different values of δ

Table.8. ***Values of X_0 for different values of δ.***

δ	X_0
0.0001	1.92925
0.0002	1.92937
0.0003	1.92949
0.0004	1.9296
0.0005	1.92972
0.0006	1.92983
0.0007	1.92994
0.0008	1.93006



Graph 8. Graph of the Values of X_0 against the values of δ

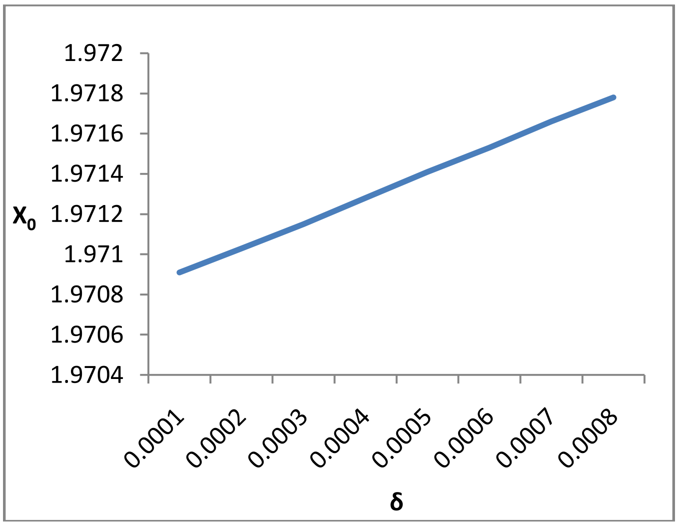
In the above graph we observe that the value of X_0 increases as the values of δ increases. Positive roots are obtained in the graph. Hence we observe the singularity is naked. As well as by increasing the values of δ the values of X_0 are increasing.

So by comparing the graph 7 of Vaidya space time, the values of X_0 are decreasing where as in graph 8 of Hussain space time the values of X_0 are increasing by the increase of values of δ in both space times.

From Equation 40 of Hussain space time analyzes the following table with different values of X_0 for different values of δ

Table.9 ***Values of X_0 for different values of δ..***

δ	X_0
0.0001	1.97091
0.0002	1.97103
0.0003	1.97115
0.0004	1.97128
0.0005	1.97141
0.0006	1.97153
0.0007	1.97166
0.0008	1.97178



Graph 9. Graph of the Values of X_0 against the values of δ

In the above graph we observe that the value of X_0 increases as the values of δ increases. Positive roots are obtained in the graph. Hence we observe the singularity is naked. As well as by increasing the values of δ the values of X_0 are increasing.

So by comparing the graph 7 of Vaidya space time, the values of X_0 are decreasing where as in graph 9 of Hussain space time the values of X_0 are increasing by the increase of values of δ in both space times

So, finally from all the above graphs we conclude that the values of X_0 are decreasing in Vaidya space time where as the values of X_0 are increasing in Hussain space time by increasing the values of δ.

V CONCLUSION

The most significant open problem in general relativity is cosmic censorship conjecture. Many researchers have given several attempts to prove this problem. But still it was unproven till date. Here in this paper we have studied the Vaidya space time as well as Hussain space time in which we admit the final result that the singularity is visible that is a naked singularity. Further we also tried to compare the naked singularities which were obtained in both the space times. From Vaidya space time we have shown by the graphical representation that the values are in decreasing order as well as in the Hussain space time it was found to be in reverse order. In other words we have successfully tried to compare the nature of singularities that were obtained in Vaidya space time and Hussain space time. The singularities that a raised are naked where as the values are in decreasing Vaidya space time and increasing order in Hussain space time.

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