A Review of two variables linearly related control chart for the enhancement of quality process

P.V.Ubale
Associate Professor in Statistics
G.S. Science, Arts & Commerce College Khamgaon

Abstract **-** *Today consumer as well as company is much desire of getting quality expectations. The quality of goods and services which are not up to the mark will suffer financially and loose customer's belief. Thus the interest of the company is to ensure that goods are of satisfactory level. The answer lies in developing the control charts that monitor the statistical process with the help of statistical measures. Traditional quality control charts are designed to monitor and control single quality characteristics. This paper reviews the idea of new control chart having two variables and the data exhibits linear trend between them. The study of charts with linear data and its construction is being studied during this paper. Also this paper reviews the work done by the various researchers on the regression control charts.*

Keywords: - *Shewhart control charts, variable statistical process, linearly related control charts, Self charting statistical control charts.*

I. Introduction

Control charts acquire established technique for rising productivity, preventing defects and estimating method capability. Shewhart control charts are useful in their simplicity and interpretability. In the conventional control charts we are controlling single quality characteristics. But it is less effective when trying to detect smaller shift in the process. Nowadays in business there are several things within which concurrent observance or dominant of two or an additional connected quality characteristic is important. Monitoring these quality characteristics independently can be very misleading. Process observations of issues during which many connected variables are of interest are studied together called as multivariate statistical process. In many manufacturing process and engineering applications the performance of the process can be monitor by identifying the errors and can be corrected on spot so that there will be reduction in the cost and discarding the nonconforming parts. The engineer wants to minimize the cost of production and monitor the process from its initial stage so that they can identify the assignable cause and removing it earlier. While constructing the conventional control charts we have not considered the data which exhibits linear trend. The data representing the relationship between the explanatory variables and response variables which shows linear trend is a common practice for the engineering sciences. Sometimes it is also possible that sufficient sampling for in controlling process is not available due to cost or time of collection. Therefore it becomes necessary to design another control charts which monitors in control and out of control situation.

In Quality Control Statistics regression control charts allows for observance a modification in process wherever two or more variables are related. Therefore it is necessary to study a control chart in which data exhibits a linear trend. Thus to monitor such a process instead of standard control charts regression control charts are used. In linearly related control charts it is designed to construct a variable average instead of constant average.

II. Literature Review

The basics of control chart have been designed by Walter A. Shewhart in the year 1931. He constructs control charts in such a way that the central line is the average quality characteristics and two control limits upper control limits and lower control limits are set at $\bar{x} \pm 3\sigma$ distances. If the points fall outside these limits then it is a cause of concern. In that situation the production has to be stopped and the error is identified and corrected before the further production process. This is a case of typical control charts given by Shewhart in the year 1931. In controlling of process statistically, statistical tools are often accustomed effectively monitor the producing method. As an example control charts will be used to predict significant deviations that will later lead to product nonconformity. Guh and O'Brien[1], Roberts[2] describes an effective chart that is typically used to monitor each the process mean and variation about this mean. Manuele[3] in his paper discuss the

modified regression control chart when the process exhibits tool wear.

DiPaola[4] described a chart for the simultaneous control of two variables and might better be labeled by correlation control chart. This type of control charts later treated more extensively by Jackson[5] and it is elaborately illustrated further by Weis[6]. Mansfield and Wein[7] implemented a regression control chart that uses the residuals from a regression of cost on output. Mendel [8] uses the combination of conventional control charts and regression analysis referred as the regression control charts. He uses the regression control chart to deal with the variety of postal management problems.

Zhang[9] developed a cause related chart that is analogous to a regression control chart and may be used to distinguish the incidence of quality problem across manufacturing methods. Ling et al[10] proposed a cumulative student t statistic based on regression residuals to control a photolithographic work cell process. Wade and Woodall[11] reviewed many cause related charts and located that quality characteristics should be controlled at the same time.

Hawkins[12],[13] Montgomery and Peck[14], Ryan[15], and Hawkins and Olwell[16] studied regression adjustment approach that is additionally useful to regulate charting. Since control charts when the data exhibits linear trend is based on regression model, it is necessary to use model based monitoring of the process. The basic assumptions of OLS regression are normality, constant variance and independence of the observations. In some cases these assumptions are not always to meet. Hence to overcome these assumptions a model building algorithm known as generalized linear method is used. Loredo *et al.*[17] use this model building approach in the regression adjustment framework when the data is related. The use of GLM for Poisson distributed counts in a regression adjustment procedure was explored in Skinner *et al.*[18],[19]. Shu et al.[20] mentioned in his paper regarding the run length performance of linearly related control charts on defined parameters for the given process. Shu et al.[21] investigated the errors occurring in parameters calculation in the cause related charts. After estimating parameter it indicates that the charted statistics are connected. Woodall[22] recently reviewed the potential applications of regression control charts in health-care and public-health surveillance in which the response variable is often non-normal.

May and Sulek[23] projected an alternate regression control charts based on least definite quantity regression for restricted method information and provided a series of control charting procedures used to identify the variables that are significant to an out of control condition. The residual chart which are the simplified version of the regression control is studied in the paper given by Utley and May[24] . O. Rama Mohan Rao et al.[25] use variable control charts to review hot metal production method in a steel industry. Quality of hot metal made is suffering from over one issue. Since the effect of one variable depends up on effect of more correlated variables it is not possible to monitor the quality by single variable control charts. Variable control charts monitor the link between the variables.

With the help of standard control charts we can monitor the process, whereas when the process output is affected by external covariate such as an incoming process variable and when it takes an extreme value the process output may appear in out of control status. In this case standard control charts are not used whereas the regression control charts proposed by Mendel is used. To construct control charts a least square regression programme must process the data.

Brian Matthew McClurg[26] in his thesis provides a self-charting control chart methodology for the data showing a linear trend.

III. Parameter estimation in regression control charts

We consider here a linear regression model with a single predictor which is commonly known as a simple linear regression. A pair (x_i,y_i) for i = 1,2,3,….,n values a regression model is given by

$$y_i = b_0 + b_1 x_i + e_i \qquad \ldots\ldots [1]$$

Where b_0 is the intercept parameter, b_1 represents slope parameter and e_i are the residuals.

The ultimate goal is to explain the relation between variable Y and variable X that expected the long run behavior of the variable Y.

By using least square estimation we have

$$\phi(b_0, b_1) = \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)^2 \qquad \ldots..[2]$$

After taking the derivatives w.r.to b_0 and b_1 and setting it as zero the estimators b_0 and b_1 can be estimated as

$$\hat{b_1} = \frac{(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \qquad \ldots..[3]$$

$$\hat{b_0} = \bar{y} - b_1 \bar{x} \qquad \ldots\ldots [4]$$

Mean sum of square $S_e^{\,2} = \dfrac{\phi(b_0, b_1)}{(n-2)}$ ….. [5]

The distribution of b_0 and b_1 is given as normally distributed as $\hat{b}_1 \sim N(b_1, \frac{\sigma^2}{S_{xx}})$,

$$\hat{b}_0 \sim N(b_0, \sigma^2\{\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\})$$

$$E(\hat{\mu}_0) = b_0 + b_1 x_0 = \mu_0 \qquad \ldots\ldots[6]$$

where $\hat{\mu}_0 = \hat{b}_0 + \hat{b}_1 x_0$

$$V(\hat{\mu}_0) = \sigma^2[\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}] \qquad \ldots\ldots[7]$$

where $S_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2$

IV. Construction of regression control chart

In regression control charts there is linear (casual) relation between dependent variable y and independent variable x. For given x values of variable, the y values are normally and independently distributed with the mean value estimated from the regression line and with a standard error which is independent values of x and is estimated from the deviation of the actual observations from the y values estimated from the regression line.

The regression control charts differ from conventional control charts in the following aspect.

- It is designed to control varying average rather than a constant average. The central line is that the curve y = a + bx.
- The control limits lines are parallel to the curve instead of horizontal axis.
- It is more complex and time consuming to compute as compare to the conventional control chart. The standard deviation is taken as standard error of estimate of the regression curve.

Using these regression lines and thrice the standard error of estimate $\pm 3S_e$ the control limit are set at 99% confidence.

As an initial step the data are plotted in the form of scatter diagram. The scatter diagram checks whether the relationship is linear and to detect a typical points. The points which are not showing linear pattern displayed by the graph are investigated. If they are found due to assignable causes they are excluded from the computations. When a point falls above the control limits indicates poor performance, poor scheduling of work. When a point falls below the lower control limits it could be due to real improvement in productivity. It is an indication of exceptionally good performance is cause for investigation.

V. Extension work in the study

Statistical process control is an important tool of monitoring the process in case of normal data. In several problem most of the variables are connected with one another and so it is very necessary to outline the connections between them. For this regression analysis is used to explore the relationship. When the data exhibits a linear trend it is not useful to use conventional control charts. We use a modified method regression control chart to deal with such data. After gathering the literatures from our research we realized several opportunities for future work. One such opportunity is extending simple linear regression case we presented to one in which there are two or more explanatory variables. In such case to deal with such data multiple linear regression is used. Another extension to this work in which monitoring of the process follows nonlinear trend in the data. Monitoring the process for nonlinear trend would certainly exhibit flexibility in the process The regression control chart is a model based monitoring procedure. The model fitting is done by using ordinary least square regression procedure. To overcome with the assumption of normality we can use generalized linear method. Modified regression control chart is also the further idea when autocorrelation occurs in the data. When the method exhibits tool wear it is better used modified regression control chart. Thus the process of tool wear is further new research idea to be used in this regard. The simplified version of the regression control chart is the residual control chart. Researchers can also utilize this controlling in their future related work.

VI. Conclusion

Statistical process control is important when the controlling of single quality characteristics is made. When the data exhibits a linear trend instead of using conventional control chart, a regression control chart is used. In this paper we throw a light on the construction of control charting in case of linear data. Since regression control chart is model building procedure parameters are estimated by using least square regression procedure. An overview of this charting is better explained further. Work done by various researchers is given elaborately. Further scopes in this regard are discussed in detail.

VII. References

[1] Guh, R.S. and O'Brien, C. *Economical statistical process control using quality cost simulation approach. International Journal of Industrial Engineering. 6: 48-60*, 1999.

[2] Roberts, L. *SPC for Right-Brain Thinkers: Process Control for Non-Statisticians. ML: Quality Press*, 2005.

[3] Manuele, J. *Control chart for determining tool wear. Industrial Quality Control 1: 7-10*, 1945

[4] Dipaola P. P. . *Use of correlations in quality control. Industrial quality control 2(1): 10-14*, 1945.

[5] Jackson J. E. *Quality control meyhods for two related variables, Industrial Quality Control, Vol. 12, No. 7, 4-8*, 1956.

[6] Weis P. E. *An application of a two way X bar Chart, Industrial Quality Control, Vol. 14, No. 6, 23-27*, 1957.

[7] Mansfield E. and Wein H.H . *A regression control chart for cost. Applied statistics 48-57*,1958.

[8] Mandel, B.J. *The regression control chart. Journal of Quality Technology 1969 1: 1-9*,1969.

[9] Zhang, G.X. *A new type of control charts and a theory of diagnosis with control charts. ASQC Annual Quality Congress Transactions 3: 175-185*,1984.

[10] Ling Z. M., Leang S. and Spanos C. J. *In line supervisory control in a photolithographic work cell, International Society for Optics and photonics, 660-669*, 1991..

[11] Wade, M.R. and Woodall, W.H. *A review and analysis of cause-selecting control charts. Journal of Quality Technology 25: 161-169*, 1993.

[12] Hawkins, D.L. *Multivariate quality control based on regression-adjusted variables. Technometrics 33: 61-75*, 1991.

[13] Hawkins, D.L. *Regression adjustment for variables in multivariate quality control. Journal of Quality Technology 25: 170-182*, 1993.

[14] Montgomery, D.C. and Peck E.A. *Introduction to linear regression analysis. NY: John Wiley and Sons*, 1992.

[15] Ryan, T.P. *Modern regression methods. NY: John Wiley and Sons*, 1997.

[16] Hawkins, D.L. and Olwell D.H. *Cumulative sum charts and charting for quality improvement. NY: Springer- Verlag*, 1998.

[17] Loredo, E., Jearkpaporn, D. and Borror, C. *Model based control chart for autoregressive and correlated data. Quality and Reliability Engineering International 18: 489–496*, 2002.

[18] Skinner, K., Montgomery, D. and Runger, G. *Process monitoring for multiple count data using generalized linear model-based control charts. International Journal of Production Research 41: 1167–1180*, 2003.

[19] Skinner, K., Montgomery, D. and Runger, G. *Generalized linear model-based control charts for discrete semiconductor process data. Quality and Reliability Engineering International 20: 777–786*, 2004.

[20] Shu, L., Tsung, F. and Tsui, K.L. *Effects of estimation errors on cause-selecting charts. Journal of Quality Technology 36: 280-292*, 2004.

[21] Shu, L., Tsung, F. and Tsui, K.L. *Run-Length performance of regression control charts with estimated parameters. IIE Transactions 37: 559–567*, 2005.

[22] Woodall, W. *The use of control charts in healthcare and public health surveillance. Journal of Quality Technology 38: 89–104*, 2006.

[23] May, J.G. and Sulek, J.M. *Regression control charts for limited process data. Southeast Decision Sciences Institute Conference Proceedings: 667-677*, 2008.

[24] Utley, J. S., and May, J. G. *Monitoring service quality with residual control Charts. Managing Service Quality 19 (2): 162-178*, 2009.

[25] O. Rama Mohan Rao, K. Venkata Subbaiah, K Narayana Rao, T. Srinivasa Rao *Application of multivariate control charts for improvement in Quality of hot metal – A case study, International journal for quality research 623-640*, 2013.

[26] Brian Matthew McClurg A Self charting statistical control chart methodology for data exhibiting linear trend, MS Thesis, University of IOWA, 1-78, 2016.