M/G/1 Queue with Restricted availability during Service interruption and Compulsory Vacation of Deterministic time

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Abstract— *This paper characterizes a non markovian queuing model in which arrival takes after a poisson procedure. All the arriving clients are rendered the administration following a general distribution. What's more, system may interrupt aimlessly because of different reasons. Immediately, the server gets into a repair procedure. Amid the season of breakdown, not all the arriving clients are permitted to join the framework. An idea of limited tolerability is connected over amid the service intrusion. After the consummation of the service, if there are no clients in the framework, the server experiences a mandatory vacation. At the season of get-away, support work are completed for the server which causes the system to keep running with superior along with insignificant intrusion. By the use of supplementary variable strategy, probability creating function of the queue size and the various execution measures are resolved. For the legitimization of the model, numerical delineation is completed.*

Keywords - *Non markovian queue, Restricted admissibility, compulsory vacation, deterministic time, probability generating function of the queue size.*

1. INTRODUCTION

Queues with server get-away have increased significant significance in the zone of research lately. It has risen as an essential zone of concentrate promotion scientists has been investigating distinctive ideas of lining models with get-aways.. Further, dominant part of creators who examined a lining framework in which server takes an vacation of irregular length. However in numerous genuine circumstances, the server may enjoy a reprieve or an excursion of deterministic length as it occurs in a factories, enterprises, bank part, railroad stations etc. In order to minimize uncertainty of availability of a server, a fixed length vacation is more realistic in many queuing situations. Choi.et.al [1] studied a non marovian queue with multiple type of vacations and gated vacations.Madan[5] contemplated lining framework with deterministic vacations. A Non Markoviaan line with optional administrations has been researched by Srinivasan and Maragathasundari [9].. In some lining frameworks with bunch landing there is a limitation to such an extent that not all groups are permitted to join the framework at untouched. This approach is named limited acceptability. Madan and choudhury [4] proposed a queuing framework with limited tolerability of arriving clumps and Bernoulli plan server excursions. Kavitha and Maragathasundari [3] looked into constrained mediocrity and discretionary sorts of repair in a Non Markovian line. Vignesh et.al [12] explored a Queuing procedure of confined admissibility. Madan and Abu-Rub [6,7] examined lining framework in light of thorough administration and deterministic excursion.

In this paper, we consider a non markovian line, in which the clients arrive one by one into the framework takes after a poisson dispersion. Administration is given by a solitary server with general distribution. After the fruition of the service , the server under goes a compulsory get-away of settled span. Furthermore, if the server breaks down, it goes into a repair procedure immediately. Amid the season of interference, if the client arrives the framework, not every one of the clients is permitted to enter the system. A confined acceptability is actualized in arriving clients amid the season of server separate. For this model, probability generating function of the queue estimate, system execution measures are inferred. Furthermore, numerical illustrations are exhibited to picturise the impact of parameters on framework execution measures.

2. MATHEMATICAL DESCRIPTION OF THE MODEL

We assume the following to describe the queuing model of our study.

Expecting that the enduring state exists, let $S_n(x)$ denote the steady state probability that are n($\geq$ 0) customers in the queue excluding one customer in Service and the elapsed service time of this customer is x. Next we define T_n as a steady state probability that there are n($\geq$ 0) customers in the queue and the server is on compulsory vacation. After the completion of the service, the server undergoes a compulsory vacation with probability $\delta > 0$. We further assume that M_j is the probability of j arrivals during the vacation period and therefore

$$M_j = \frac{e^{-\lambda f}(\lambda f)^j}{j!}$$

Finally let G denote the steady state probability that the system is empty.

Customers arrive at the system one by one in a compound poisson process with mean arrival rateλ. They are provided service on a first come first served basis. Service follows a general distribution. Let R(s) and r(s) be the distribution function and the density function of the service time S of a customer and let $\mu(x)$ be the conditional probability of completion of a service given that the elapsed time is x, so that $\mu(x) = \frac{r(x)}{1-R(x)}$ $, r(v) = \mu(v)e^{\left[-\int_0^v \mu(x)dx\right]}$

We assume interruptions occur at random while serving the customers follows a poisson process with mean time rate $\eta > 0$.Repair process is immediately implemented The repair time follows a general probability law with distribution function K(x) and the density function $k(x)$,.Let ψ (x) be the conditional probability of completion of repair and so

$\psi(\mathrm{x}) = \frac{k(v)}{1-K(v)}$ $, k(v) = \psi(v)e^{-\int_0^v \psi(x)dx}$

Furthermore, an idea of confined acceptability is presented amid the season of patch up. Let d > 0 be the probability that an arriving client will be permitted to join the framework amid the time of server revamp period.

3. STEADY STATE EQUATIONS GOVERNING THE SYSTEM

Connecting states of the system at time $t + dt$ with those at time t and then taking limit as t → ∞ , we obtain the following set of steady state equations governing the system.

$$\frac{d}{dx}S_n(x) + (\lambda + \eta + \mu(x))S_n(x) = \lambda S_{n-1}(x) \qquad (1)$$

$$\frac{d}{dx}S_0(x) + (\lambda + \eta + \mu(x))S_0(x) = 0 \qquad (2)$$

$$T_n = \delta \int_0^\infty S_n(x)\mu(x)dx \qquad (3)$$

$$\frac{d}{dx}K_n(x) + (\lambda + \psi(x))K_n(x) = \lambda(1-d)(x)K_n(x) + d\lambda K_{n-1}(x) \qquad (4)$$

$$\lambda G = \int_0^\infty S_0(x)\mu(x)dx + T_0 M_0 \qquad (5)$$

$$M_r = \frac{e^{-\lambda f}(\lambda f)^r}{r!} \qquad (6)$$

The above set of equations is to be solved under the following boundary conditions at $x = 0$ & for $n \geq 1$.

$$S_n(0) = \int_0^\infty S_{n+1}(x)\mu(x)dx + T_0 M_{n+1} + T_1 M_n + \cdots \ldots + T_n M_0 \qquad (7)$$

$$S_0(0) = \int_0^\infty S_1(x)\mu(x)dx + T_0 M_1 + T_1 M_0 + \lambda G \qquad (8)$$

$$K_n(0) = \eta \int_0^\infty S_n(x)dx \qquad (9)$$

The Steady state Probability Generating functions

We define the following probability generating function

$$\left.\begin{array}{ll} S_q(x,z) = \sum_{n=0}^\infty S_n(x)Z^n, & S_q(z) = \sum_{n=0}^\infty S_n Z^n, \\ K_q(x,z) = \sum_{n=0}^\infty K_n(x)Z^n, & K_q(z) = \sum_{n=0}^\infty K_n Z^n, \\ T(z) = \sum_{n=0}^\infty T_n Z^n, & |Z| \leq 1 \end{array}\right\} (10)$$

$$M(z) = \sum_{n=0}^\infty M_n(x)Z^n, \quad = \sum_{n=0}^\infty \frac{e^{-\lambda f}(\lambda f)}{n!} Z^n = e^{-\lambda f(1-Z)}$$

$$M'(z) = e^{-\lambda f(1-Z)}(\lambda f) \quad M''(z) = (\lambda f)^2$$

On multiplying equations 1 & 2 by Z^n sum over n & using (10), we have

$$\frac{d}{dx}S_q(x,z) + (\lambda - \lambda z + \eta + \mu(x))S_q(x,z) = 0 \qquad (11)$$

Similarly

$$\frac{d}{dx}K_q(x,z) + (\lambda d - \lambda zd + \eta + \psi(x))K_q(x,z) = 0 \qquad (12)$$

$$T_q(z) = \delta \int_0^\infty S_q(x,z)\mu(x)dx \qquad (13)$$

The same process has to be applied for the boundary conditions we get

$$ZS_q(0,z) = \int_0^\infty S_q(x,z)\mu(x)dx + T(z)M(z) + \lambda(Z-1)G + \int_0^\infty K_q(x,z)\psi(x)dx \qquad (14)$$

$$K_q(0,z) = \eta z S_q(z) \qquad (14a)$$

Next we integrate equations (11),(12),(13) between the limits 0 and x and the resultant equations to be integrated with respect to x.

We obtain

$$S_q(z) = S_q(0,z)\left[\frac{1-\bar{R}(\lambda - \lambda z + \eta)}{\lambda - \lambda z + \eta}\right] \qquad (15)$$

$$K_q(z) = K_q(0,z)\left[\frac{1-\bar{K}(\lambda d - \lambda zd + \eta)}{\lambda d - \lambda zd}\right] \qquad (16)$$

Where $\bar{S}(\lambda - \lambda z + \eta)$ and $\bar{K}(\lambda d - \lambda zd)$ are Laplace Stieltjes transform of the service time and repair time respectively.

To find $\int_0^\infty S_q(x,z)\mu(x)dx$

For this purpose, we multiply (15),(16) by $\mu(x), \psi(x)$ respectively and integrate them with respect to x and use (b), we get

$$\int_0^\infty S_q(x,z)\mu(x)dx = S_q(0,z)\bar{S}(\lambda - \lambda z + \eta) \qquad (17)$$

$$\int_0^\infty K_q(x,z)\psi(x)dx = K_q(0,z)\bar{K}(\lambda d - \lambda zd) \qquad (18)$$

Using (17) & (18) in (14) and (14a) we get

$S_q(0,z) = \frac{[\lambda(z-1)]G}{(\lambda-\lambda z+\eta)[z-\bar{S}(\lambda-\lambda z+\eta)(1+\delta M(z))]-\eta z\bar{K}(\lambda d-\lambda zd)[1-\bar{S}(\lambda-\lambda z+\eta)]}$ (19)

Hence,

$S_q(z) = \frac{[\lambda(z-1)]G}{(\lambda-\lambda z+\eta)[z-\bar{S}(\lambda-\lambda z+\eta)(1+\delta M(z))]-\eta z\bar{K}(\lambda d-\lambda zd)[1-\bar{S}(\lambda-\lambda z+\eta)]} \times \left[\frac{1-\bar{S}(\lambda-\lambda z+\eta)}{\lambda-\lambda z+\eta}\right]\left[\frac{1-\bar{K}(\lambda d-\lambda zd+\eta)}{\lambda d-\lambda zd}\right]$ (20)

$K_q(z) = \eta z\frac{[\lambda(z-1)]G\left[\frac{1-\bar{S}(\lambda-\lambda z+\eta)}{\lambda-\lambda z+\eta}\right]}{(\lambda-\lambda z+\eta)[z-\bar{S}(\lambda-\lambda z+\eta)(1+\delta M(z))]-\eta z\bar{K}(\lambda d-\lambda zd)[1-\bar{S}(\lambda-\lambda z+\eta)]}$ (21)

$T_q(z) = \frac{\delta[\lambda(z-1)]G\bar{S}(\lambda-\lambda z+\eta)}{(\lambda-\lambda z+\eta)[z-\bar{S}(\lambda-\lambda z+\eta)(1+\delta M(z))]-\eta z\bar{K}(\lambda d-\lambda zd)[1-\bar{S}(\lambda-\lambda z+\eta)]}$ (22)

4. IDLE TIME ρ AND UTILIZATION FACTOR ρ

In order to determine Q, we proceed as follows

$S_q(1) = \lim_{z\to 1} S_q(z) = \frac{0}{0}$ form

Hence we apply L'Hopital's rule

$S_q(1) = \frac{[1-\bar{R}(\eta)]G\lambda}{-\lambda(1-\bar{S}(\eta))(1+\delta)+\eta\left[1+\lambda\bar{R}'(\eta)(1+\delta)-\bar{R}(\eta)\lambda\delta d\right] -\eta\bar{K}(\eta)(1-\bar{R}(\eta))+\eta\bar{K}'(\eta)\lambda d(1-\bar{R}(\eta))}$ (23)

Similarly

$K_q(1) = \frac{\eta G(1-\bar{R}(\eta))}{-\lambda(1-\bar{R}(\eta))(1+\delta)+\eta\left[1+\lambda\bar{R}'(\eta)(1+\delta)-\bar{R}(\eta)\lambda\delta d\right] -\eta\bar{K}(\eta)(1-\bar{R}(\eta))+\eta\bar{K}'(\eta)\lambda d(1-\bar{R}(\eta))}$ (24)

$T_q(1) = \frac{\eta\delta G\bar{R}(\eta)}{-\lambda(1-\bar{R}(\eta))(1+\delta)+\eta\left[1+\lambda\bar{R}'(\eta)(1+\delta)-\bar{R}(\eta)\lambda\delta d\right] -\eta\bar{K}(\eta)(1-\bar{R}(\eta))+\eta\bar{K}'(\eta)\lambda d(1-\bar{R}(\eta))}$ (25)

IDLE TIME AND UTILIZATION FACTOR

Now we use the normalizing condition to obtain G

$S_q(1) + K_q(1) + T_q(1) + G = 1$ (26)

Therefore Substituting (23,(24),(25) in (26) we get

$G = \frac{-\lambda(1-\bar{R}(\eta))(1+\delta)+\eta\left[1+\lambda\bar{R}'(\eta)(1+\delta)-\bar{R}(\eta)\lambda\delta d\right] -\eta\bar{K}(\eta)(1-\bar{R}(\eta))+\eta\bar{K}'(\eta)\lambda d(1-\bar{R}(\eta))}{-\lambda(1-\bar{R}(\eta))(1+\delta)+\eta\left[1+\lambda\bar{R}'(\eta)(1+\delta)-\bar{R}(\eta)\lambda\delta d\right] -\eta\bar{K}(\eta)(1-\bar{R}(\eta))+\eta\bar{K}'(\eta)\lambda d(1-\bar{R}(\eta))+([1-\bar{R}(\eta)]G\lambda+\eta G(1-\bar{R}(\eta))+\eta\delta G\bar{R}(\eta)}$

The Utilization factor ρ is obtained from $\rho = 1 - G < 1$ which is the stability condition under which the steady state exists.

Next we substitute the value of G in equations (20)(21) and (22). Thus $K_q(z), T_q(z)$ and $S_q(z)$ are completely determined.

5. STEADY STATE AVERAGE NUMBER OF CUSTOMERS AND AVERAGE WAITING TIME IN THE QUEUE AS WELL AS IN THE SYSTEM

We define $D_q(z)$ as the steady state probability generating function for the number of customers in the queue. So we have

$D_q(z) = S_q(z) + K_q(z) + T_q(z) = \frac{N(z)}{D(z)}$

Let L_q denote the mean number of customers in the queue under the following steady state.

$L_q = \frac{d}{dz}D_q(z)_{z=1}$

But $D_q(z) = \frac{0}{0}$ form at $z = 1$

Therefore,

$L_q = \lim_{z\to 1}\frac{d}{dz}P_q(z) = \lim_{z\to 1}\frac{D'(z)N''(z)-N'(z)D''(z)}{2[D'(z)]^2}$ (28)

$= \frac{D'(1)N''(1)-N'(1)D''(1)}{2[D'(1)]^2}$

Where primes and double primes indicates single and double differentiation with respect to z respectively.

Here,

$N'(1) = \lambda[1-\bar{S}(\eta)(1+\delta\eta)]$ (29)

$N''(1) = 2\lambda[\bar{S}(\eta)(1-\delta\eta)+\lambda^2 S'(\eta)]$ (30)

$D'(1) = (-\lambda)(1+\eta)(1-\bar{S}(\eta)) + \eta\begin{bmatrix}1+\lambda\bar{S}'(\eta)(1+\delta)\\ -\bar{S}(\eta)\delta\lambda f\end{bmatrix} -\eta(1-\bar{S}(\eta)) - \eta E(K)\lambda d (1-\bar{S}(\eta))$ (31)

$D''(1) = -2\lambda[1+\bar{S}'(\eta)\lambda(1+\eta)-\bar{S}(\eta)\delta\lambda d]+\eta[-S''(\eta)\lambda^2(1+\delta)+2\bar{S}'(\eta)\lambda^2\delta f-\bar{S}(\eta)\delta\lambda f^2] - 2\eta\lambda S'(\eta)+2\eta E(k)\lambda d(1-\bar{S}(\eta)) - \eta E(K^2)(\lambda d^2)(1-\bar{S}(\eta)) + 2\eta E(K)\lambda^2 d\bar{S}'(\eta)$ (32)

Substituting (29) in (28), we get L_q in closed form

Using L_q obtained in (29) into Little's formulae we can obtain the following

The steady state average number of customer in the system $L = L_q + \rho$ (27)

The Steady state average waiting time in the queue

$W_q = \frac{L_q}{\lambda}$

The Steady state average waiting time in the system

$W = \frac{L}{\lambda}$

6. NUMERICAL ILLUSTRATION

For the purpose of numerical illustration we assume that the service time and vacation time are exponentially distributed. We choose the following arbitrary values $\mu = 5$, $\lambda = 3$, $\psi = 4$, $d = 2$

δ	η	G	ρ	L_q	L	W_q	W
0.5	3	0.6800	0.3200	2.800	3.120	0.9333	1.040
	4	0.5231	0.4769	5.400	5.876	1.800	1.959
	5	0.3680	0.6320	12.270	12.902	4.090	4.300
0.6	3	0.6735	0.3265	2.469	2.796	0.823	0.932
	4	0.6083	0.3917	3.889	4281	1.296	1.427
	5	0.5385	0.4615	6.679	7.141	2.226	2.380
0.7	3	0.6694	0.3306	2.150	2.481	0.7167	0.827
	4	0.6181	0.3819	3.159	3.541	1.053	1.180
	5	0.5677	0.4323	3.910	4.342	1.303	1.447

All the values are chosen such that the steady state condition is satisfied.The above table gives the computed values of various states of the server, the proportion of idle time, the utilization factor,number of customers in the queue, number of customers in the system, waiting time in the queue and waiting time in the system.The values clearly show that as long as we increase δ for a constant arrival of break down, the utilization factor increases while the idle time of the server gets decreases.In addition the other performance measures also gets decreased.More over, for a constant value of δ,if the arrival of break down increases, then idle time reduces and hence the utilization factor increases.Also the length of the queue and all the other execution measures gets increased. All the values are as expected.

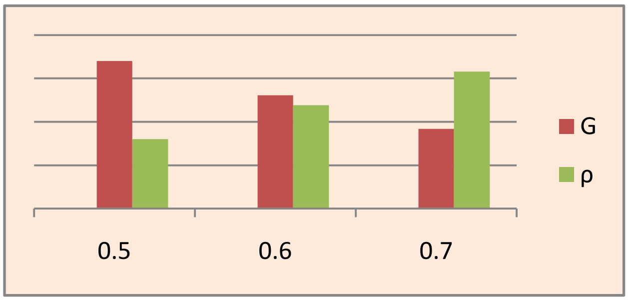

Figure 1 : Effect of fixed value of break down rate and increase in probability vaction on utilization factor and idle time

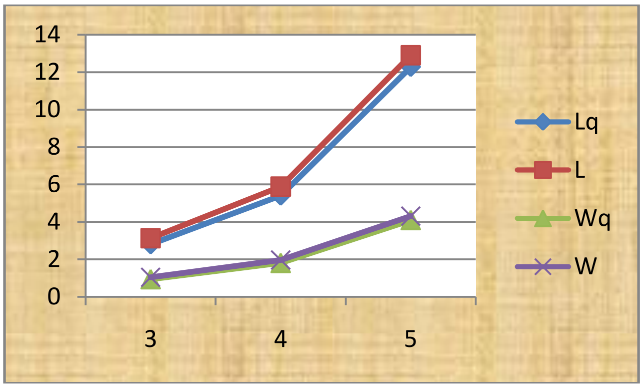

Figure 2: Effect of constant probability vacation and increase in break down rate on various performance measures

7. CONCLUSION

In this examination, we have contemplated a non markovian queue with mandatory vacation of deterministic time and restricted suitability amid benefit interference. The expository outcomes approved with the assistance of numerical representations might be helpful in different genuine circumstances to outline the outputs.The probability generating function of the queue estimate is resolved utilizing supplementary variable method.The other queuing execution measures are additionally obtained.As a future work, stand by server, phases of administration, various vacations can be actualized in this examination.

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