Finite double integrals involving multivariable I-function and a class of multivariable polynomials

Frédéric Ayant

*Teacher in High School , France

Vinod Gill
Department of Mathematics Amity University, Rajasthan, Jaipur-303002, India

ABSTRACT
In this paper we establish two finite double integrals involving the multivariable I-function defined by Prasad and a class of multivariable polynomials with general arguments. Our integrals are quite general in character and a number of new integrals can be deduced as particular cases We will study the particular cases concerning the multivariable H-function and the Srivastava-Daoust polynomials.

KEYWORDS : I-function of several variables, finite double integral, H-function of several variables , Class of multivariable polynomials, Srivastava-Daoust polynomial.

2010 Mathematics Subject Classification. 33C60, 82C31

1.Introduction

The multivariable I-function defined by Prasad [4] is a extension of the multivariable H-function defined by Srivastava et al [7]. We will use the contracted form.
The I-function of r-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_r) = I^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)}}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r \end{array}\middle| \begin{array}{c} (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \\ (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}_{rj};\alpha^{(1)}_{rj},\cdots,\alpha^{(r)}_{rj})_{1,p_r}:(a^{(1)}_j,\alpha^{(1)}_j)_{1,p^{(1)}};\cdots;(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj};\beta^{(1)}_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r}:(b^{(1)}_j,\beta^{(1)}_j)_{1,q^{(1)}};\cdots;(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{array}\right) \tag{1.1}$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_r \tag{1.2}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [4]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega_i\pi$, where

$$\Omega_i=\sum_{k=1}^{n^{(i)}}\alpha_k^{(i)}-\sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha_k^{(i)}+\sum_{k=1}^{m^{(i)}}\beta_k^{(i)}-\sum_{k=m^{(i)}+1}^{q^{(i)}}\beta_k^{(i)}+\left(\sum_{k=1}^{n_2}\alpha_{2k}^{(i)}-\sum_{k=n_2+1}^{p_2}\alpha_{2k}^{(i)}\right)+\cdots+$$

$$\left(\sum_{k=1}^{n_s}\alpha_{sk}^{(i)}-\sum_{k=n_s+1}^{p_s}\alpha_{sk}^{(i)}\right)-\left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)}+\sum_{k=1}^{q_3}\beta_{3k}^{(i)}+\cdots+\sum_{k=1}^{q_s}\beta_{sk}^{(i)}\right) \tag{1.3}$$

where $i=1,\cdots,r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,)\,,max(\,|z_1|,\cdots,|z_r|\,)\rightarrow 0$$

$$I(z_1,\cdots,z_r)\;=0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,)\,,min(\,|z_1|,\cdots,|z_r|\,)\rightarrow\infty$$

where $k=1,\cdots,r:\alpha'_k=min[Re(b_j^{(k)}/\beta_j^{(k)})],j=1,\cdots,m^{(k)}$ and

$$\beta'_k=max[Re((a_j^{(k)}-1)/\alpha_j^{(k)})],j=1,\cdots,n^{(k)}$$

We will use these following notations in this section :

$$U_r=p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1};V_r=0,n_2;0,n_3;\cdots;0,n_{r-1} \tag{1.4}$$

$$W_r=(p^{(1)},q^{(1)});\cdots;(p^{(r)},q^{(r)});X_r=(m^{(1)},n^{(1)});\cdots;(m^{(r)},n^{(r)}) \tag{1.5}$$

$$A=(a_{2k};\alpha_{2k}^{(1)},\alpha_{2k}^{(2)});\cdots;(a_{(r-1)k};\alpha_{(r-1)k}^{(1)},\alpha_{(r-1)k}^{(2)},\cdots,\alpha_{(r-1)k}^{(r-1)}) \tag{1.6}$$

$$B=(b_{2k};\beta_{2k}^{(1)},\beta_{2k}^{(2)});\cdots;(b_{(r-1)k};\beta_{(r-1)k}^{(1)},\beta_{(r-1)k}^{(2)},\cdots,\beta_{(r-1)k}^{(r-1)}) \tag{1.7}$$

$$\mathfrak{A}=(a_{rk};\alpha_{rk}^{(1)},\alpha_{rk}^{(2)},\cdots,\alpha_{rk}^{(r)}):\mathfrak{B}=(b_{rk};\beta_{rk}^{(1)},\beta_{rk}^{(2)},\cdots,\beta_{rk}^{(r)}) \tag{1.8}$$

$$A'=(a_k^{(1)},\alpha_k^{(1)})_{1,p^{(1)}};\cdots;(a_k^{(r)},\alpha_k^{(r)})_{1,p^{(r)}}\;;B'=(b_k^{(1)},\beta_k^{(1)})_{1,q^{(1)}};\cdots;(b_k^{(r)},\beta_k^{(r)})_{1,q^{(r)}} \tag{1.9}$$

;

The multivariable I-function of r-variables write :

$$I(z_1,\cdots,z_r)=\;I_{U_r:p_r,q_r;W_r}^{V_r;0,n_r;X_r}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A}\;;\mathfrak{A};\mathrm{A'} \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \mathrm{B};\;\mathfrak{B};\;\mathrm{B'} \end{array}\right) \tag{1.10}$$

Srivastava and Garg [6] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_s}[z_1,\cdots,z_s]=\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)\frac{z_1^{R_1}\cdots z_s^{R_s}}{R_1!\cdots R_s!} \tag{1.11}$$

The coefficients $B(L;R_1,\cdots,R_s)$ are arbitrary constants, real or complex.

We will note :$B_s=\dfrac{(-L)_{h_1R_1+\cdots+h_sR_s}B(L;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$ (1.12)

2. Required integral

The following known integrals ([2], page 450, Eq.(4)), ([3], page 71, Eq. (3.1.8)) and ([1], page 192, Eq. (46)) will be required during the evaluation of our main results :

$$\int_0^{\pi/2} e^{i(\alpha+\beta)\theta} sin^{\alpha-1}\theta cos^{\beta-1}\theta \mathrm{d}\theta = e^{(i\pi\alpha)/2}\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, (Re(\alpha)>0, Re(\beta)>0) \tag{2.1}$$

$$\int_0^{\pi/2} e^{i(\alpha+\beta)\theta} sin^{\alpha-1}\theta cos^{\beta-1}\theta {}_2F_1[a,b;\beta;e^{i\theta}cos\theta]\mathrm{d}\theta = e^{(i\pi\alpha)/2}\frac{\Gamma(\alpha)\Gamma(\beta)\Gamma(\alpha+\beta-a-b)}{\Gamma(\alpha+\beta-a)\Gamma(\alpha+\beta-b)}$$

$$(Re(\alpha)>0, Re(\beta)>0, Re(\alpha+\beta-a-b)>0) \tag{2.2}$$

$$\int_0^1 x^{\lambda-1}(1-x)^{\mu-1}P_\upsilon^{(\alpha,\beta)}(1-zx)\mathrm{d}x = \frac{(\alpha+1)_\upsilon\Gamma(\mu)}{\upsilon!}\sum_{k=0}^{\upsilon}\frac{(-\upsilon)_k(1+\alpha+\beta+\upsilon)_k}{k!(\alpha+1)_k\Gamma(\lambda+\mu+k)}\left(\frac{z}{2}\right)^k$$

$$(Re(\lambda)>0, Re(\mu)>0) \tag{2.3}$$

3. Main integrals

First integral

$$\int_0^1\int_0^1 x^{\lambda-1}y^{\rho-1}z^{\rho+2\sigma}(1-x)^{\mu-1}(1-y^2)^{\sigma-1}P_\upsilon^{(\alpha,\beta)}(1-zx)$$

$$S_L^{h_1,\cdots,h_s}\begin{pmatrix} \mathrm{y}_1x^{a_1}(1-x)^{b_1}y^{c_1}(1-y^2)^{d_1}z^{c_1+2d_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{y}_sx^{a_s}(1-x)^{b_s}y^{c_s}(1-y^2)^{d_s}z^{c_s+2d_s} \end{pmatrix} I\begin{pmatrix} \mathrm{z}_1x^{\rho_1}(1-x)^{\sigma_1}y^{\mu_1}(1-y^2)^{\delta_1}z^{\mu_1+2\delta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_rx^{\rho_r}(1-x)^{\sigma_r}y^{\mu_r}(1-y^2)^{\delta_r}z^{\mu_r+2\delta_r} \end{pmatrix}$$

$$\mathrm{d}x\mathrm{d}y = \frac{(\alpha+1)_\upsilon}{\upsilon!}e^{i\pi\rho/2}\sum_{k=0}^{\upsilon}\frac{(-\upsilon)_k(1+\alpha+\beta+\upsilon)_k}{k!(\alpha+1)_k}\left(\frac{z}{2}\right)^k\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}B_sI^{V_r;0,n_r+4;X_r}_{U_r:p_r+4,q_r+2;W_r}\left(\begin{array}{c} \mathrm{z}_1e^{i\pi\mu_1/2} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_re^{i\pi\mu_r/2} \end{array}\right|$$

A; (1-$\mu-\sum_{i=1}^s b_iR_i;\sigma_1,\cdots,\sigma_r),(1-2\sigma-2\sum_{i=1}^s R_id_i;2\delta_1,\cdots,2\delta_r)$,

$\cdots$

$\cdots$

C; (1-$\lambda-\mu-k-\sum_{i=1}^s R_i(a_i+b_i);\sigma_1+\rho_1,\cdots,\sigma_r+\rho_r)$,

$$\left.\begin{array}{c} (1\text{-}\lambda - k - \sum_{i=1}^{s} R_i a_i; \rho_1, \cdots, \rho_r), (1-\rho - \sum_{i=1}^{s} R_i c_i; \mu_1, \cdots, \mu_r), \mathfrak{A} : A' \\ \cdots \\ \cdots \\ (1\text{-}\rho - 2\sigma - \sum_{i=1}^{r} R_i(c_i + 2d_i); \mu_1 + 2\delta_1, \cdots, \mu_r + 2\delta_r), \mathfrak{B} : B' \end{array}\right) y_1^{R_1} \cdots y_s^{R_s} \quad (3.1)$$

where $z = \sqrt{1-y^2} + iy$

The integral (3.1) is valid if the following sets of sufficient conditions are satisfied :

a) $\min\{a_i, b_i, c_i, d_i, \mu_j, \delta_j, \rho_j, \sigma_j\} > 0, i = 1, \cdots, s; j = 1, \cdots, r; Re(\alpha) > -1, Re(\beta) > -1$

b) $Re(\rho) + \sum_{i=1}^{r} \mu_i \min_{1 \leqslant j \leqslant m^{(i)}} Re\left(\frac{b_j^{(i)}}{\beta_j^{(i)}}\right) > 0 \,; Re(\lambda) + \sum_{i=1}^{r} \rho_i \min_{1 \leqslant j \leqslant m^{(i)}} Re\left(\frac{b_j^{(i)}}{\beta_j^{(i)}}\right) > 0$

$Re(\mu) + \sum_{i=1}^{r} \sigma_i \min_{1 \leqslant j \leqslant m^{(i)}} Re\left(\frac{b_j^{(i)}}{\beta_j^{(i)}}\right) > 0$ and $Re(\sigma) + \sum_{i=1}^{r} \delta_i \min_{1 \leqslant j \leqslant m^{(i)}} Re\left(\frac{b_j^{(i)}}{\beta_j^{(i)}}\right) > 0$

c) $|argz_i| < \frac{1}{2}\Omega_i \pi$, where Ω_i is defined by (1.3)

Second integral

$$\int_0^1 \int_0^1 x^{\lambda-1} y^{\gamma-1} z^{\gamma+\delta} (1-x)^{\mu-1} (1-y^2)^{\delta/2-1} P_\upsilon^{(\alpha,\beta)}(1-zx) {}_2F_1[a,b;\delta; z\sqrt{1-y^2}]$$

$$S_L^{h_1,\cdots,h_s}\left(\begin{array}{c} \mathrm{y}_1 x^{a_1}(1-x)^{b_1} y^{c_1} z^{c_1} \\ . \\ . \\ . \\ \mathrm{y}_s x^{a_s}(1-x)^{b_s} y^{c_s} z^{c_s} \end{array}\right) I \left(\begin{array}{c} \mathrm{z}_1 x^{\rho_1}(1-x)^{\sigma_1} y^{\mu_1} z^{\mu_1} \\ . \\ . \\ . \\ \mathrm{z}_r x^{\rho_r}(1-x)^{\sigma_r} y^{\mu_r} z^{\mu_r} \end{array}\right) \mathrm{d}x\mathrm{d}y = \frac{(\alpha+1)_\upsilon \Gamma(\delta)}{\upsilon!} e^{i\pi\gamma/2}$$

$$\sum_{k=0}^{\upsilon} \frac{(-\upsilon)_k (1+\alpha+\beta+\upsilon)_k}{k!(\alpha+1)_k} \left(\frac{z}{2}\right)^k \sum_{R_1,\cdots,R_s=0}^{h_1 R_1 + \cdots h_s R_s \leqslant L} B_s\, y_1^{R_1} \cdots y_s^{R_s} I_{U_r:p_r+3,q_r+3;W_r}^{V_r;0,n_r+3;X_r} \left(\begin{array}{c} \mathrm{z}_1 e^{i\pi\mu_1/2} \\ . \\ . \\ . \\ \mathrm{z}_r e^{i\pi\mu_r/2} \end{array}\right|$$

$$\begin{array}{c} \mathrm{A};\ (1\text{-}\gamma - \sum_{i=1}^{s} R_i c_i; \mu_1, \cdots, \mu_r), (1-\gamma-\delta - \sum_{i=1}^{s} R_i c_i; \mu_1, \cdots, \mu_r), \\ \cdots \\ \cdots \\ \mathrm{B};\ (1\text{-}\gamma - \delta + a - \sum_{i=1}^{s} R_i c_i; \mu_1, \cdots, \mu_r), (1-\gamma-\delta+b - \sum_{i=1}^{s} R_i c_i; \mu_1, \cdots, \mu_r), \end{array}$$

$$\left.\begin{array}{c} (1\text{-}\lambda - k - \sum_{i=1}^{s} R_i a_i; \rho_1, \cdots, \rho_r), \mathfrak{A} : A' \\ \cdots \\ \cdots \\ (1\text{-}\lambda - \mu - k - \sum_{i=1}^{s} R_i(a_i + b_i); \rho_1 + \sigma_1, \cdots, \rho_r + \sigma_r), \mathfrak{B} : B' \end{array}\right) \quad (3.2)$$

where $z = \sqrt{1-y^2} + iy$

The integral (3.3) is valid if the following sets of sufficient conditions are satisfied :

a) $\min\{a_i, b_i, c_i, \mu_j, \rho_j, \sigma_j\} > 0, i = 1, \cdots, s; j = 1, \cdots, r$ and

$Re(\alpha) > -1, Re(\beta) > -1, Re(\delta) > 0, Re(\gamma + \delta - a - b) > 0$

b) $Re(\gamma) + \sum_{i=1}^{r} \mu_i \min_{1 \leqslant j \leqslant m^{(i)}} Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right) > 0\,; Re(\lambda) + \sum_{i=1}^{r} \rho_i \min_{1 \leqslant j \leqslant m^{(i)}} Re\left(\frac{b_j^{(i)}}{\beta_j^{(i)}}\right) > 0$

and $Re(\mu) + \sum_{i=1}^{r} \sigma_i \min_{1 \leqslant j \leqslant m^{(i)}} Re\left(\frac{b_j^{(i)}}{\beta_j^{(i)}}\right) > 0$

c) $|argz_i| < \frac{1}{2}\Omega_i \pi$, where Ω_i is defined by (1.3)

Proof

To establish the integral (3.1), first expressing the class of multivariable polynomials $S_L^{h_1, \cdots, h_s}[.]$ in multiple serie with the help of (1.11), we use the Mellin-Barnes type contour integral with the help of (1.1) for the multivariable I-function defined by Prasad [4] occurring on the left hand side of (3.1) and changing the order of integration and summation (which is justified under the conditions given with (3.1)) and using (2.1) and (2.3) to evaluate the resulting y-integral and x-integral, respectively. Finally interpreting the resulting contour integral as the multivariable I-function, defined by Prasad [4], we get the desired formula (3.1).

The proof of the integral formula (3.2) is similar to that of the first integral with the only difference that here we use the integral (2.2) instead of (2.1).

4. Multivariable H-function

If $U_r = V_r = A = B = 0$, the multivariable I-function reduces to the multivariable H-function defined by Srivastava et al [7] and we obtain the following results.

$$\int_0^1 \int_0^1 x^{\lambda-1} y^{\rho-1} z^{\rho+2\sigma} (1-x)^{\mu-1} (1-y^2)^{\sigma-1} P_{\upsilon}^{(\alpha,\beta)}(1-zx)$$

$$S_L^{h_1,\cdots,h_s}\begin{pmatrix} \mathrm{y}_1 x^{a_1}(1-x)^{b_1} y^{c_1}(1-y^2)^{d_1} z^{c_1+2d_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{y}_s x^{a_s}(1-x)^{b_s} y^{c_s}(1-y^2)^{d_s} z^{c_s+2d_s} \end{pmatrix} H \begin{pmatrix} \mathrm{z}_1 x^{\rho_1}(1-x)^{\sigma_1} y^{\mu_1}(1-y^2)^{\delta_1} z^{\mu_1+2\delta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r x^{\rho_r}(1-x)^{\sigma_r} y^{\mu_r}(1-y^2)^{\delta_r} z^{\mu_r+2\delta_r} \end{pmatrix}$$

$$\mathrm{d}x\mathrm{d}y = \frac{(\alpha+1)_\upsilon}{\upsilon!} e^{i\pi\rho/2} \sum_{k=0}^{\upsilon} \frac{(-\upsilon)_k (1+\alpha+\beta+\upsilon)_k}{k!(\alpha+1)_k} \left(\frac{z}{2}\right)^k \sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s \leqslant L} B_s H_{p_r+4,q_r+2;W_r}^{0,n_r+4;X_r}\left(\begin{array}{c|} \mathrm{z}_1 e^{i\pi\mu_1/2} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r e^{i\pi\mu_r/2} \end{array}\right.$$

$$(1\text{-}\mu-\sum_{i=1}^s b_iR_i;\sigma_1,\cdots,\sigma_r),(1-2\sigma-2\sum_{i=1}^s R_id_i;2\delta_1,\cdots,2\delta_r),$$
$$\cdots$$
$$\cdots$$
$$(1\text{-}\lambda-\mu-k-\sum_{i=1}^s R_i(a_i+b_i);\sigma_1+\rho_1,\cdots,\sigma_r+\rho_r),$$

$$\left.\begin{matrix}(1\text{-}\lambda-k-\sum_{i=1}^s R_ia_i;\rho_1,\cdots,\rho_r),(1-\rho-\sum_{i=1}^s R_ic_i;\mu_1,\cdots,\mu_r),\mathfrak{A}:A'\\ \cdots\\ \cdots\\ (1\text{-}\rho-2\sigma-\sum_{i=1}^r R_i(c_i+2d_i);\mu_1+2\delta_1,\cdots,\mu_r+2\delta_r),\mathfrak{B}:B'\end{matrix}\right)y_1^{R_1}\cdots y_s^{R_s} \quad (4.1)$$

under the same notations and conditions that (3.1) with $U_r=V_r=A=B=0$

Second integral

$$\int_0^1\int_0^1 x^{\lambda-1}y^{\gamma-1}z^{\gamma+\delta}(1-x)^{\mu-1}(1-y^2)^{\delta/2-1}P_\upsilon^{(\alpha,\beta)}(1-zx)\,{}_2F_1[a,b;\delta;z\sqrt{1-y^2}]$$

$$S_L^{h_1,\cdots,h_s}\begin{pmatrix}\mathrm{y}_1x^{a_1}(1-x)^{b_1}y^{c_1}z^{c_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{y}_sx^{a_s}(1-x)^{b_s}y^{c_s}z^{c_s}\end{pmatrix}H\begin{pmatrix}\mathrm{z}_1x^{\rho_1}(1-x)^{\sigma_1}y^{\mu_1}z^{\mu_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}(1-x)^{\sigma_r}y^{\mu_r}z^{\mu_r}\end{pmatrix}\mathrm{d}x\mathrm{d}y=\frac{(\alpha+1)_\upsilon\Gamma(\delta)}{\upsilon!}e^{i\pi\gamma/2}$$

$$\sum_{k=0}^{\upsilon}\frac{(-\upsilon)_k(1+\alpha+\beta+\upsilon)_k}{k!(\alpha+1)_k}\left(\frac{z}{2}\right)^k\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}B_s\,y_1^{R_1}\cdots y_s^{R_s}H_{p_r+3,q_r+3;W_r}^{0,n_r+3;X_r}\left(\begin{matrix}\mathrm{z}_1e^{i\pi\mu_1/2}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_re^{i\pi\mu_r/2}\end{matrix}\right|$$

$$(1\text{-}\gamma-\sum_{i=1}^s R_ic_i;\mu_1,\cdots,\mu_r),(1-\gamma-\delta-\sum_{i=1}^s R_ic_i;\mu_1,\cdots,\mu_r),$$
$$\cdots$$
$$\cdots$$
$$(1\text{-}\gamma-\delta+a-\sum_{i=1}^s R_ic_i;\mu_1,\cdots,\mu_r),(1-\gamma-\delta+b-\sum_{i=1}^s R_ic_i;\mu_1,\cdots,\mu_r),$$

$$\left.\begin{matrix}(1\text{-}\lambda-k-\sum_{i=1}^s R_ia_i;\rho_1,\cdots,\rho_r),\mathfrak{A}:A'\\ \cdots\\ \cdots\\ (1\text{-}\lambda-\mu-k-\sum_{i=1}^s R_i(a_i+b_i);\rho_1+\sigma_1,\cdots,\rho_r+\sigma_r),\mathfrak{B}:B'\end{matrix}\right) \quad (4.2)$$

under the same notations and conditions that (3.2) with $U_r=V_r=A=B=0$

5. Srivastava-Daoust polynomial

If $B(L;R_1,\cdots,R_s)=\dfrac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta'_j+\cdots+R_s\theta_j^{(s)}}\prod_{j=1}^{B'}(b'_j)_{R_1\phi'_j}\cdots\prod_{j=1}^{B^{(s)}}(b_j^{(s)})_{R_s\phi_j^{(s)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{m_1\psi'_j+\cdots+m_s\psi_j^{(s)}}\prod_{j=1}^{D'}(d'_j)_{R_1\delta'_j}\cdots\prod_{j=1}^{D^{(s)}}(d_j^{(s)})_{R_s\delta_j^{(s)}}}$ **(5.1)**

then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_s}[z_1,\cdots,z_s]$ reduces to generalized Srivastava-Daoust polynomial defined by Srivastava et al [5].

$$F^{1+\bar{A}:B';\cdots;B^{(s)}}_{\bar{C}:D';\cdots;D^{(s)}}\left(\begin{array}{c|c} \mathrm{z}_1 & [(\text{-L});\mathrm{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}] \\ \cdots & \\ \cdots & [(\mathrm{c});\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}] \\ \mathrm{z}_s & \end{array}\right) \quad (5.2)$$

and we have the following results

First integral

$$\int_0^1\int_0^1 x^{\lambda-1}y^{\rho-1}z^{\rho+2\sigma}(1-x)^{\mu-1}(1-y^2)^{\sigma-1}P_{\upsilon}^{(\alpha,\beta)}(1-zx)$$

$$F^{1+\bar{A}:B';\cdots;B^{(s)}}_{\bar{C}:D';\cdots;D^{(s)}}\left(\begin{array}{c|} \mathrm{y}_1x^{a_1}(1-x)^{b_1}y^{c_1}(1-y^2)^{d_1}z^{c_1+2d_1} \\ . \\ . \\ . \\ \mathrm{y}_sx^{a_s}(1-x)^{b_s}y^{c_s}(1-y^2)^{d_s}z^{c_s+2d_s} \end{array}\right.$$

$$\left.\begin{array}{c} [(\text{-L});\mathrm{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}] \\ [(\mathrm{c});\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}] \end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z}_1x^{\rho_1}(1-x)^{\sigma_1}y^{\mu_1}(1-y^2)^{\delta_1}z^{\mu_1+2\delta_1} \\ . \\ . \\ . \\ \mathrm{z}_rx^{\rho_r}(1-x)^{\sigma_r}y^{\mu_r}(1-y^2)^{\delta_r}z^{\mu_r+2\delta_r} \end{array}\right)$$

$$\mathrm{d}x\mathrm{d}y=\frac{(\alpha+1)_\upsilon}{\upsilon!}e^{i\pi\rho/2}\sum_{k=0}^{\upsilon}\frac{(-\upsilon)_k(1+\alpha+\beta+\upsilon)_k}{k!(\alpha+1)_k}\left(\frac{z}{2}\right)^k\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}B'_sI^{V_r;0,n_r+4;X_r}_{U_r:p_r+4,q_r+2;W_r}\left(\begin{array}{c|} \mathrm{z}_1e^{i\pi\mu_1/2} \\ . \\ . \\ . \\ \mathrm{z}_re^{i\pi\mu_r/2} \end{array}\right.$$

$$A; (1\text{-}\mu-\sum_{i=1}^s b_iR_i;\sigma_1,\cdots,\sigma_r),(1-2\sigma-2\sum_{i=1}^s R_id_i;2\delta_1,\cdots,2\delta_r),$$
$$\cdots$$
$$\cdots$$
$$C; (1\text{-}\lambda-\mu-k-\sum_{i=1}^s R_i(a_i+b_i);\sigma_1+\rho_1,\cdots,\sigma_r+\rho_r),$$

$$\left.\begin{matrix}(1\text{-}\lambda-k-\sum_{i=1}^s R_ia_i;\rho_1,\cdots,\rho_r),(1-\rho-\sum_{i=1}^s R_ic_i;\mu_1,\cdots,\mu_r),\mathfrak{A}:A'\\ \cdots \\ \cdots \\ (1\text{-}\rho-2\sigma-\sum_{i=1}^r R_i(c_i+2d_i);\mu_1+2\delta_1,\cdots,\mu_r+2\delta_r),\mathfrak{B}:B'\end{matrix}\right)y_1^{R_1}\cdots y_s^{R_s} \quad (5.3)$$

under the same notations and conditions that (3.1) with

$B'_s=\dfrac{(-L)_{h_1R_1+\cdots+h_sR_s}B(L;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$ where $B(L;R_1,\cdots,R_s)$ **is defined by (5.1)**

Second integral

$$\int_0^1\int_0^1 x^{\lambda-1}y^{\gamma-1}z^{\gamma+\delta}(1-x)^{\mu-1}(1-y^2)^{\delta/2-1}P_\upsilon^{(\alpha,\beta)}(1-zx)\,{}_2F_1[a,b;\delta;z\sqrt{1-y^2}]$$

$$F_{\bar{C}:D';\cdots;D^{(s)}}^{1+\bar{A}:B';\cdots;B^{(s)}}\left(\begin{matrix}\mathrm{y}_1x^{a_1}(1-x)^{b_1}y^{c_1}z^{c_1}\\ .\\ .\\ .\\ \mathrm{y}_sx^{a_s}(1-x)^{b_s}y^{c_s}z^{c_s}\end{matrix}\right|$$

$$\left.\begin{matrix}[(\text{-L});\mathrm{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}]\\ [(\mathrm{c});\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}]\end{matrix}\right)$$

$$I\left(\begin{matrix}\mathrm{z}_1x^{\rho_1}(1-x)^{\sigma_1}y^{\mu_1}z^{\mu_1}\\ .\\ .\\ .\\ \mathrm{z}_rx^{\rho_r}(1-x)^{\sigma_r}y^{\mu_r}z^{\mu_r}\end{matrix}\right)\mathrm{d}x\mathrm{d}y=\frac{(\alpha+1)_\upsilon\Gamma(\delta)}{\upsilon!}e^{i\pi\gamma/2}\sum_{k=0}^{\upsilon}\frac{(-\upsilon)_k(1+\alpha+\beta+\upsilon)_k}{k!(\alpha+1)_k}\left(\frac{z}{2}\right)^k$$

$$\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}B'_sy_1^{R_1}\cdots y_s^{R_s}I_{U_r:p_r+3,q_r+3;W_r}^{V_r;0,n_r+3;X_r}\left(\begin{matrix}\mathrm{z}_1e^{i\pi\mu_1/2}\\ .\\ .\\ .\\ \mathrm{z}_re^{i\pi\mu_r/2}\end{matrix}\right|$$

$$\text{A; } (1\text{-}\gamma - \textstyle\sum_{i=1}^{s} R_i c_i; \mu_1, \cdots, \mu_r), (1 - \gamma - \delta - \sum_{i=1}^{s} R_i c_i; \mu_1, \cdots, \mu_r),$$

$$\cdots$$

$$\cdots$$

$$\text{B; } (1\text{-}\gamma - \delta + a - \textstyle\sum_{i=1}^{s} R_i c_i; \mu_1, \cdots, \mu_r), (1 - \gamma - \delta + b - \sum_{i=1}^{s} R_i c_i; \mu_1, \cdots, \mu_r),$$

$$\left. \begin{array}{c} (1\text{-}\lambda - k - \sum_{i=1}^{s} R_i a_i; \rho_1, \cdots, \rho_r), \mathfrak{A} : A' \\ \cdots \\ \cdots \\ (1\text{-}\lambda - \mu - k - \sum_{i=1}^{s} R_i (a_i + b_i); \rho_1 + \sigma_1, \cdots, \rho_r + \sigma_r), \mathfrak{B} : B' \end{array} \right) \tag{5.4}$$

under the same notations and conditions that (3.2) with

$$B'_s = \frac{(-L)_{h_1 R_1 + \cdots + h_s R_s} B(L; R_1, \cdots, R_s)}{R_1! \cdots R_s!}$$ where $B(L; R_1, \cdots, R_s)$ **is defined by (5.1)**

6. Conclusion

The I-function of several variables defined by Prasad [4] presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions o several variables such as multivariable I-function ,multivariable Fox's H-function, Fox's H-function , Meijer's G-function, Wright's generalized Bessel function, Wright's generalized hypergeometric function, MacRobert's E-function, generalized hypergeometric function, Bessel function of first kind, modied Bessel function, Whittaker function, exponential function , binomial function etc. as its special cases, and therefore, various unified integral presentations can be obtained as special cases of our results

References

[1] Erdelyi A et al. Tables of integral transforms. Vol II. McGraw-Hill, New-York (1954).

[2] Mac-Robert T.M. Beta function formulae and integrals involving E-function. Math.Ann. 142, page 450-452 (1961).

[3] Mathai A.M. And Saxena R.K. Generalized hypergeometric functions with applications in statistics and physical sciences. Lecture Notes in Mathematics. Vol 348. Springer-Verlag. New York (1973).

[4] Y.N. Prasad , Multivariable I-function , Vijnana Parishad Anusandhan Patrika 29 (1986) , page 231-237.

[5] Srivastava H.M. and Daoust M.C. Certain generalized Neumann expansions associated with Kampé de Fériet function. Nederl. Akad. Wetensch. Proc. Ser A72 = Indag Math 31(1969) page 449-457.

[6] Srivastava H.M. And Garg M. Some integral involving a general class of polynomials and multivariable H-function.Rev. Roumaine Phys. 32(1987), page 685-692.

[7] H.M. Srivastava And R.Panda. Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24(1975), p.119-137.