Ordering L-R type Generalized Trapezoidal Fuzzy Numbers

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Abstract - *Methods for ordering fuzzy numbers play a vital role as decision criteria. In fuzzy literature there are many techniques for ordering fuzzy numbers. In practice, some special L-R fuzzy numbers, like triangular fuzzy number, the Gaussian fuzzy number, Cauchy fuzzy number and Trapezoidal fuzzy number are widely used various areas to deal with many vague information. Based on the recent developments in research of fuzzy number ranking,the paper extends thenew ranking approach to rank and orderGeneralized L-R Fuzzy Numbers. The purpose of this paper is to introduce a general frame work for comparing fuzzy sets with respect to fuzzy orderings in a gradual way. The approach proposed herein is relatively simple in terms of computational efforts and is efficient when ranking a large quantity of fuzzy numbers.*

I. INTRODUCTION

Fuzzy set theory [19] has been applied to many areas in decision making like, approximate reasoning, optimization controland data mining etc., which need to manage uncertain and vague information(demand, time,distance, etc.) in the real life situation.In order to rank fuzzy numbers, one fuzzy number needs to be evaluated and compared with the others, which is not so easy. An efficient approach for ordering the fuzzy numbers is by the use of a ranking function $R: F(R) \rightarrow R,$ where $F(R)$ is a set of fuzzy numbers defined on real line, which maps each fuzzy number into the real line, where a natural order exists. In the fuzzy number literature, there are many proposals for ordering *L-R*(left and right) type fuzzy numbers. Proposals are based on the strategy of characterization of a fuzzy number by a real number in order to rank fuzzy numbers, which is classified into four main groups [11]. Namely, the first group is based on geometric procedures such as centroid, area, mode, expansions and weights of fuzzy numbers [4, 5, 8, 9, 15]. Second group includes distance between a fuzzy number and the origin (0, 0, 0), using metrics like Euclidean distance, Hamming distance, and Tchebychev distance, etc. [1, 3]. Third group of methods uses probability / possibility measures defined over fuzzy events to generate a real number [10, 12]. Finally the fourth group involves ordering fuzzy numbers by generating a sequence of finite / infinite real numbers [17].

Ordering and ranking of fuzzy numbers is an important criterion in decision making, it should have the following properties: [11]

(1) The method should be consistent with the ranking of real numbers when a real number is considered as a particular situation of a fuzzy number.
(2) The method should avoid evaluation expressions such as a / 0 or 0 / 0, where 'a' is a real number in the calculus of ordering.
(3) The method should avoid inconsistencies, such as two different fuzzy numbers with the same ranking.
(4) The method should be based on easy calculation of mathematical expressions.
(5) The method should show consistency with other ordering methods proposed in literature.

In literature, there are many methods for the ordering of fuzzy numbers. However, many of these methods do not consider the above points. The methods proposed by [7, 8, 13] can generate situations where expressions such as those mentioned in point (2) take place [14, 18], the methods proposed by [1, 3, 4,9, 15], present the problem pointed out in (3), the issue pointed out in (1) is not discussed in works of [14, 17] and finally [17] does not present a discussion about point (5).

The aim of this paper is to propose a method for ordering of *L-R* type fuzzy numbers, considering the five aspects mentioned above. Our method is based on the new ranking techniqueof fuzzy numbers which is simple, efficient and consistent. The paper is organized as follows: Section 2 represents a brief review of basic definition of fuzzy theory. Arithmetic operations of generalized *L-R* fuzzy numbers is discussed in section 3. In section 4, our proposal and ranking technique for ordering fuzzy numbers is provided. Section 5, presents an illustration of the proposed method with the comparison table. In section 6, the conclusion and future discussions are revealed.

II. BASIC DEFINITIONS

Definition 2.1: Fuzzy Set

A Fuzzy set$\tilde{A}$ is characterized by a membership function, mapping elements of a domain, space, or universe of discourse X to the unit interval*[0,* *1]*.(i.e.) $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \epsilon X\}$, here $\mu_{\tilde{A}}: X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ called the membership function value of $x \in X$ in the fuzzy set $\tilde{A}$. These membership grades are often represented by real numbers ranging from *[0, 1]*.

Definition 2.2: FuzzyNumber

A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}}: R \to [0,1]$is continuous.
2. $\mu_{\tilde{A}}(x) = 0 \, for \, all \, x \in (-\infty, a] \cup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$strictly increasing on *[a, b]* and strictly decreasing on *[c, d]*.
4. $\mu_{\tilde{A}}(x) = 1$for all $x \in [b, c]$,where *a<b<c<d.*

Definition 2.3: Trapezoidal Fuzzy Number

A fuzzy number $\tilde{A} = (a, b, c, d) is$ said to be atrapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \le x \le b \\ 1, & b \le x \le c \\ \frac{(x-d)}{(c-d)}, & c \le x \le d \end{cases}$$

Definition 2.4:Generalized Fuzzy Number

A fuzzy set$\tilde{A}$, defined on the universal set of real numbers R, is said to be generalized fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}}: R \to [0, \omega]$*is continuous.*
2. $\mu_{\tilde{A}}(x) = 0 \, for \, all \, x \in (-\infty, a] \cup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$*strictly increasing on [a, b] and strictly decreasing on [c, d].*
4. $\mu_{\tilde{A}}(x) = \omega$, *for all* $x \in [b, c]$, *where* $0 < \omega \le 1$.

Definition 2.5: (Dubois and Prade [10])

A shape function *L (or R)* is a decreasing function from $R^+ \to [0, 1]$ such that

(1) $L(0) = 1$;
(2) $L(x) < 1, \forall \, x > 0$;
(3) $L(x) > 0, \forall \, x < 1$;
(4) $L(1) = 0 \, [\, or \, L(x) > 0, \forall x \, and \, L(+\infty) = 0]$.

Definition 2.6: L-R type Generalized Trapezoidal Fuzzy Number

A fuzzy number $\tilde{A}$is said to be L-R type if there exists two decreasing functions $L, R: [0, +\infty) \to [0, 1]$with
$L(0) = R(0) = 1, lim_{\omega \to +\infty} L(\omega) = lim_{\omega \to +\infty} R(\omega) = 0$ and positive real numbers $a_m \ge 0, \alpha > 0, \beta > 0$ such that

$$\mu_{\tilde{A}}(\omega) = \begin{cases} L\left(\frac{a_m - \omega}{\alpha}\right), & for \, \omega \le a_m, \\ R\left(\frac{\omega - a_m}{\beta}\right), & for \, \omega \ge a_m, \end{cases}$$

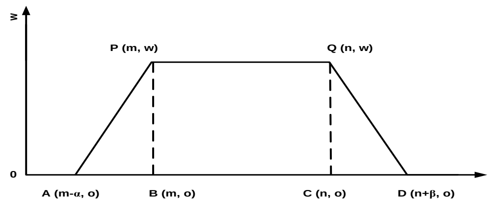
wherea_m is called the centre of $\tilde{A}$and $\alpha = a_m - a_l$ and $\beta = a_r - a_m$ are called the left and right propagations, respectively.
If $\alpha = \beta, \tilde{A}$ is called a symmetric fuzzy number; it is important to stress that for a symmetric membership function, the equality $L\left(\frac{a_m - \omega}{\alpha}\right) = R\left(\frac{\omega - a_m}{\beta}\right)$holds for $\omega \epsilon R$. If $L$and $R$are segments that starts at points $(a_l, 0)$and $(a_r, 0)$, respectively, and end at $(a_m, 1)$, then we say that $\tilde{A}$is a triangular fuzzy number.

(OR)

A fuzzy number $\tilde{A} = (m, n, \alpha, \beta; \omega)_{LR}$ is said to be the*L-R* type generalizedtrapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \omega L\left(\frac{m - x}{\alpha}\right), & x \le m, \alpha > 0, \\ \omega R\left(\frac{x - n}{\beta}\right), & x \ge n, \beta > 0, \\ \omega, & otherwise. \end{cases}$$

where L and R are reference functions.



L-R type Generalized Trapezoidal Fuzzy Number

III. ARITHMETIC OPERATIONS ONL-R TYPE GENERALIZED TRAPEZOIDAL FUZZY NUMBERS

In this section, the formulas for the elementary operations (addition, subtraction, multiplication) between *L-R* fuzzy type generalized trapezoidal fuzzy numbers will be presented.

Let $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1; \omega_1)_{LR}$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2; \omega_2)_{LR}$ be any two *L-R* type generalized fuzzy numbers then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (m_1 + m_2,\ n_1 + n_2,\ \alpha_1 + \alpha_2,\ \beta_1 + \beta_2; minimum(\omega_1, \omega_2))_{LR}$

(ii) $\tilde{A}_1 \ominus \tilde{A}_2 = (m_1 - \beta_2,\ n_1 - \alpha_2,\ \alpha_1 - n_2,\ \beta_1 - m_2; minimum(\omega_1, \omega_2))_{LR}$

(iii) $\lambda\tilde{A}_1 = \begin{cases} (\lambda m_1, \lambda n_1, \lambda\alpha_1, \lambda\beta_1;\ \omega_1)_{LR} & \lambda > 0 \\ (\lambda\beta_1, \lambda\alpha_1, \lambda n_1, \lambda m_1;\ \omega_1)_{RL} & \lambda < 0. \end{cases}$

IV. PROPOSED APPROACH

An efficient approach for comparing the fuzzy numbers is by the use of a ranking function.

(i) $\tilde{A} > \tilde{B}\ iff\ R(\tilde{A}) > R(\tilde{B})$

(ii) $\tilde{A} < \tilde{B}\ iff\ R(\tilde{A}) < R(\tilde{B})$

(iii) $\tilde{A} \sim \tilde{B}\ iff\ R(\tilde{A}) = R(\tilde{B})$,then

$$\begin{cases} \tilde{A} < \tilde{B}, & if\ \omega_1 < \omega_2 \\ \tilde{A} > \tilde{B}, & if\ \omega_1 > \omega_2. \\ \tilde{A} = \tilde{B}, & if\ \omega_1 = \omega_2 \end{cases}$$

Remark 1. [2]

(i) $\tilde{A} > \tilde{B} \Rightarrow \tilde{A} \oplus \tilde{C} > \tilde{B} \oplus \tilde{C}$

(ii) $\tilde{A} > \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{C} > \tilde{B} \ominus \tilde{C}$

(iii) $\tilde{A} \sim \tilde{B} \Rightarrow \tilde{A} \oplus \tilde{C} \sim \tilde{B} \oplus \tilde{C}$

(iv) $\tilde{A} > \tilde{B}, \tilde{C} > \tilde{D} \Rightarrow \tilde{A} \oplus \tilde{C} > \tilde{B} \oplus \tilde{D}$

4.1 Ranking Function

Let $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1; \omega_1)_{LR}$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2; \omega_2)_{LR}$ be any two *L-R* type generalized trapezoidal fuzzy numbers then we define the ranking function to order the fuzzy numbers as follow:

$$R(\tilde{A}) = \frac{w(2m_1 + 2n_1 - \alpha_1 + \beta_1)}{12} and$$

$$R(\tilde{B}) = \frac{w(2m_2 + 2n_2 - \alpha_2 + \beta_2)}{12}$$

$where\ \omega = minimum\ (\omega_1, \omega_2).$

V. NUMERICAL ILLUSTRATIONS

Considering four sets of *L-R* type generalized trapezoidalfuzzy numbers as discussed in [2], our ranking is applied for ordering these fuzzy numbers.

Set 1. $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 1), \tilde{B} = (0.2, 0.3, 0.3, 0.4; 1)$ $R(\tilde{A}) = 0.3 = R(\tilde{B}) = 0.3$, and $\omega_1 = \omega_2$ so $\tilde{A} = \tilde{B}$.

Set 2. $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 0.8), \tilde{B} = (0.1, 0.3, 0.3, 0.5; 1)$ $R(\tilde{A}) = 0.024 = R(\tilde{B}) = 0.024$, and $\omega_1 < \omega_2$ so $\tilde{A} < \tilde{B}$.

Set3. $\tilde{A} = (-0.8, -0.6, -0.4, -0.2; 0.35), \tilde{B} = (-0.4, -0.3, -0.2, -0.1; 0.7)$ $R(\tilde{A}) = -0.175 < R(\tilde{B}) = -0.0875$, so $\tilde{A} < \tilde{B}$.

Set 4. $\tilde{A} = (0.2, 0.4, 0.6, 0.8; 0.35), \tilde{B} = (0.1, 0.2, 0.3, 0.4; 0.7)$ $R(\tilde{A}) = 0.0175 < R(\tilde{B}) = 0.0875$, so $\tilde{A} < \tilde{B}$.

Table 1. Comparison of existing methods and proposed method

Methods	Set 1	Set 2	Set 3	Set 4
Chen and Chen [6]	$\tilde{B} > \tilde{A}$ but $\tilde{B} \ominus \tilde{A} < \tilde{A} \ominus \tilde{A}$, i.e., $\tilde{B} > \tilde{A} \nRightarrow \tilde{B} \ominus \tilde{A} > \tilde{A} \ominus \tilde{A}$	$\tilde{B} > \tilde{A}$ but $\tilde{B} \ominus \tilde{A} < \tilde{A} \ominus \tilde{A}$, i.e., $\tilde{B} > \tilde{A} \nRightarrow \tilde{B} \ominus \tilde{A} > \tilde{A} \ominus \tilde{A}$	$\tilde{A} > \tilde{B}$ but $\tilde{A} \ominus \tilde{B} < \tilde{B} \ominus \tilde{B}$, i.e., $\tilde{A} > \tilde{B} \nRightarrow \tilde{A} \ominus \tilde{B} > \tilde{B} \ominus \tilde{B}$	$\tilde{B} > \tilde{A}$ but $\tilde{B} \ominus \tilde{A} < \tilde{A} \ominus \tilde{A}$, i.e., $\tilde{B} > \tilde{A} \nRightarrow \tilde{B} \ominus \tilde{A} > \tilde{A} \ominus \tilde{A}$
Amit Kumar et.al [2]	$\tilde{A} > \tilde{B}$ with $\alpha = 0, \tilde{A} < \tilde{B}$ with $\alpha = 1, \tilde{A} \sim \tilde{B}$, with $\alpha = 0.5$	$\tilde{A} \sim \tilde{B}\ \forall\ \alpha$	$\tilde{A} < \tilde{B}$ with $\alpha = 0, \tilde{A} < \tilde{B}$ with $\alpha = 1, \tilde{A} < \tilde{B}$, with $\alpha = 0.5$	$\tilde{A} > \tilde{B}$ with $\alpha = 0, \tilde{A} > \tilde{B}$ with $\alpha = 1, \tilde{A} > \tilde{B}$, with $\alpha = 0.5$
Y.L.P Thorani and N. Ravishankar [16]	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}\ by\ \omega_1 < \omega_2$	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$
Proposed method	$\tilde{A} \sim \tilde{B}$	$\tilde{A} < \tilde{B}\ by\ \omega_1 < \omega_2$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$

VI. CONCLUSION

This paper proposes a new approach for ranking and ordering*L-R*type generalized trapezoidal fuzzy numbers based on the average. This proposed method is simple and easy in calculations. Its efficiency gives satisfactory results and overcome the pitfalls in ordering the fuzzy numbers. For future research, the proposed technique can be applied in practical problems involving decision making, optimization, transportation, assignment problems, etc. and can be extended to problems involving *L-R* type generalized intuitionistic fuzzy numbers.

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