Square Difference Prime Labeling of Some Star Related Graphs

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Abstract — *Square difference prime labeling of a graph is the labeling of the vertices with {0,1,2-----,p-1} and the edges with absolute difference of the squares of the labels of the incident vertices .The greatest common incidence number of a vertex (**gcin**) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the **gcin** of each vertex of degree greater than one is one, then the graph admits square difference prime labeling. Here we investigate some star related graphs for square difference prime labeling.*

Keywords — *Graph labeling, square difference, prime labeling, prime graphs, star graph.*

I. INTRODUCTION

All graphs in this paper are finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1], [2],[3], [4] and [5] . Some basic concepts are taken from Frank Harary [1]. In [6], we introduced the concept, square difference prime labeling and proved that some snake graphs admit this kind of labeling. In [7], [8] , [9], [10] and [11] ,we extended our study and proved that the result is true for some path related graphs, some planar graphs, some tree graphs, some cycle related graphs, fan graph, helm graph, umbrella graph, gear graph, friendship and wheel graph. Here we investigate some star related graphs for square difference prime labeling.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

II. MAIN RESULTS

Definition 2.1 Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection
f : V(G) → {0,1,2,---, p-1} by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping
f_{sdp}^* : E(G) → set of natural numbers N by $f_{sdp}^*(uv) = | f(u)^2 - f(v)^2 |$. The induced function f_{sdp}^* is said to be a square difference prime labeling, if for each vertex of degree at least 2, the ***gcin*** of the labels of the incident edges is 1.

Definition 2.2 A graph which admits square difference prime labeling is called a square difference prime graph.

Definition 2.3 The Lilly graph L_n , n ≥ 2 can be constructed by two star graphs $K_{1,n}$ and two path graphs P_n sharing a common vertex.

Theorem 2.1 Splitting graph of star graph $K_{1,n}$ (n is a natural number greater than 3) admits square difference prime labeling.

Proof: Let G =S'($K_{1,n}$) and let a,b,v_1,v_2,----,v_n, u_1,u_2,---,u_n are the vertices of G.
Here |V(G)| = 2n+2 and |E(G)| = 3n
Define a function f : V → {0,1,2,---,2n+1 } by

$f(v_i) = i+1$, i = 1,2,---,n
$f(u_i) = n+i+1$, i = 1,2,---,n
f(a) = 0, f(b) = 1.

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^*(a\, v_i) = (i+1)^2$, i = 1,2,---,n.
$f_{sdp}^*(a\, u_i) = (n+i+1)^2$, i = 1,2,---,n.
$f_{sdp}^*(b\, v_i) = (i+1)^2 - 1$, i = 1,2,---,n.

Clearly f^*_{sdp} is an injection.

gcin of (a) $= \gcd$ of $\{f^*_{sdp}(a\,v_1),\ f^*_{sdp}(a\,v_2)\}$
= gcd of {4, 9}
= 1.

gcin of (v_i) = gcd of $\{f^*_{sdp}(a\,v_i),\ f^*_{sdp}(b\,v_i)\}$
= gcd of { $(i+1)^2$, $(i+1)^2 - 1$}
= 1, i = 1,2,----,n.

gcin of (b) = gcd of $\{f^*_{sdp}(b\,v_1),\ f^*_{sdp}(b\,v_2)\}$
= gcd of {3, 8}
= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence S'($K_{1,n}$) , admits square difference prime labeling.

Example 2.1 G = S'($K_{1,4}$)

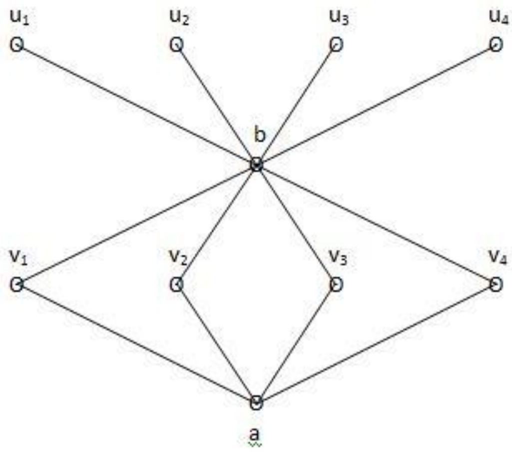

Fig – 2.1

Theorem 2.2 Double graph of star graph $K_{1,n}$ (n is a natural number greater than 3) admits square difference prime labeling.

Proof: Let G =D($K_{1,n}$) and let a,b,v_1,v_2,---,v_n, u_1,u_2,---,u_n are the vertices of G.
Here |V(G)| = 2n+2 and |E(G)| = 4n
Define a function f : V → {0,1,2,---,2n+1 } by

f(v_i) = i+1 , i = 1,2,---,n
f(u_i) = n+i+1 , i = 1,2,---,n
f(a) = 0, f(b) = 1.

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows

$f^*_{sdp}(a\,v_i)$ = $(i+1)^2$, i = 1,2,---,n.
$f^*_{sdp}(a\,u_i)$ = $(n+i+1)^2$, i = 1,2,---,n.
$f^*_{sdp}(b\,v_i)$ = $(i+1)^2 - 1$, i = 1,2,---,n.
$f^*_{sdp}(b\,u_i)$ = $(n+i+1)^2 - 1$, i = 1,2,---,n.

Clearly f^*_{sdp} is an injection.

gcin of (a) = gcd of $\{f^*_{sdp}(a\,v_1),\ f^*_{sdp}(a\,v_2)\}$
= gcd of {4, 9}
= 1.

gcin of (v_i) = gcd of $\{f^*_{sdp}(a\,v_i),\ f^*_{sdp}(b\,v_i)\}$
= gcd of { $(i+1)^2$, $(i+1)^2 - 1$}
= 1, i = 1,2,---,n.

gcin of (b) = gcd of $\{f^*_{sdp}(b\,v_1),\ f^*_{sdp}(b\,v_2)\}$
= gcd of {3, 8}
= 1.

gcin of (u_i) = gcd of $\{f^*_{sdp}(a\,u_i),\ f^*_{sdp}(b\,u_i)\}$
= gcd of { $(n+i+1)^2$, $(n+i+1)^2 - 1$}
= 1, i = 1,2,---,n.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence D($K_{1,n}$) , admits square difference prime labeling.

Example 2.2 Let G = D($K_{1,4}$)

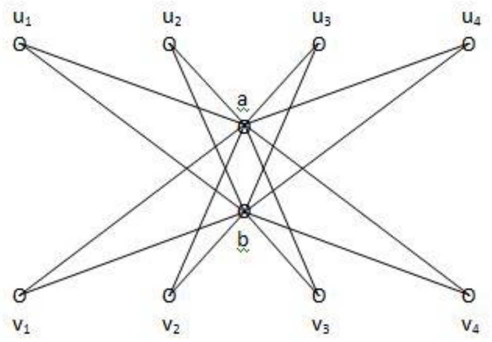

Fig-2.2

Theorem 2.3 Let G be the graph obtained by duplicating each pendant vertex of star $K_{1,n}$ by an edge. G admits square difference prime labeling.

Proof: Let G be the graph and let $v_1, v_2, ---, v_{3n+1}$ are the vertices of G.

Here |V(G)| = 3n+1 and |E(G)| = 4n

Define a function f : V → {0,1,2,---,3n } by

$f(v_i) = i-1$, i = 1,2,---,3n+1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^* (v_{3i-2}\ v_{3i-1})$ = 6i-5, i = 1,2,---,n.

$f_{sdp}^* (v_{3i-1}\ v_{3i})$ = 6i-3, i = 1,2,---,n.

$f_{sdp}^* (v_{3i-2}\ v_{3i})$ = 12i-8, i = 1,2,---,n.

$f_{sdp}^* (v_{3i-1}\ v_{3n+1})$= $9n^2 - (3i-2)^2$, i = 1,2,---,n.

Clearly f_{sdp}^* is an injection.

gcin of (v_{3i-2}) = gcd of $\{f_{sdp}^* (v_{3i-2}\ v_{3i-1}),\ f_{sdp}^* (v_{3i-2}\ v_{3i})\}$

= gcd of {6i-5, 12i-8}

= gcd of {2,6i-5}= 1, i = 1,2,---,n.

gcin of (v_{3i-1}) = gcd of $\{f_{sdp}^* (v_{3i-2}\ v_{3i-1}), f_{sdp}^* (v_{3i-1}\ v_{3i})\}$

= gcd of {6i-5, 6i-3}

= gcd of {2,6i-5}= 1, i = 1,2,---,n.

gcin of (v_{3i}) = gcd of $\{f_{sdp}^* (v_{3i-2}\ v_{3i}), f_{sdp}^* (v_{3i-1}\ v_{3i})\}$

= gcd of {12i-8, 6i-3}

= gcd of {6i-3,6i-5}=1, i = 1,2,---,n.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits square difference prime labeling.

Example 2.3 Let G be the duplication of each pendant vertex of $K_{1,4}$

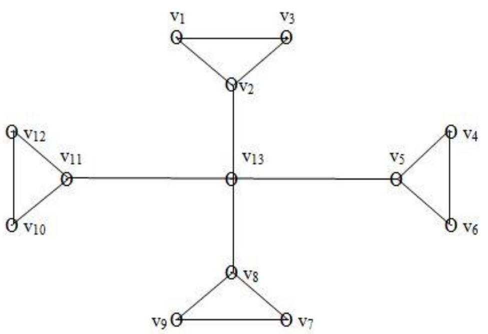

Fig – 2.3

Theorem 2.4 Let G be the graph obtained by duplicating the central vertex of star $K_{1,n}$ by an edge. G admits square difference prime labeling.

Proof: Let G be the graph and let $v_1, v_2, ---, v_{n+3}$ are the vertices of G.

Here |V(G)| = n+3 and |E(G)| = n+3

Define a function f : V → {0,1,2,---,n+2 } by

$f(v_i) = i-1$, i = 1,2,---,n+3

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^* (v_i\ v_{i+1})$ = 2i-1, i = 1,2,3.

$f_{sdp}^* (v_1\ v_3)$ = 4.

$f_{sdp}^* (v_3\ v_{i+4})$ = i^2+6i+5, i = 1,2,---,n-1.

Clearly f_{sdp}^* is an injection.

gcin of (v_1) = gcd of $\{f_{sdp}^* (v_1\ v_2), f_{sdp}^* (v_1\ v_3)\}$
= gcd of {1, 4}= 1.

gcin of (v_2) = gcd of $\{f_{sdp}^* (v_1\ v_2), f_{sdp}^* (v_2\ v_3)\}$
= gcd of {1, 3}= 1.

gcin of (v_3) = gcd of $\{f_{sdp}^* (v_2\ v_3), f_{sdp}^* (v_3\ v_4)\}$
= gcd of {3, 5}= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits square difference prime labeling.

Theorem 2.5 Lilly graph admits square difference prime labeling.

Proof: Let G =L_n and let $v_1,v_2,---,v_{4n-1}$ are the vertices of G.

Here |V(G)| = 4n-1 and |E(G)| = 4n-2

Define a function f : V → {0,1,2,---,4n-2 } by

$f(v_i)$ = i-1 , i = 1,2,---,4n-1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^* (v_i\ v_{i+1})$ = 2i-1, i = 1,2,---,2n-1.

$f_{sdp}^* (v_n\ v_{3n+i-1})$ = $(3n+i-2)^2 – (n-1)^2$, i = 1,2,---,n.

$f_{sdp}^* (v_n\ v_{2n+i-1})$ = $(2n+i-2)^2 – (n-1)^2$, i = 1,2,---,n.

Clearly f_{sdp}^* is an injection.

gcin of (v_{i+1}) = gcd of $\{f_{sdp}^* (v_i\ v_{i+1}), f_{sdp}^* (v_{i+1}\ v_{i+2})\}$
= gcd of {2i-1, 2i+1}
=1, i = 1,2,---,2n-3.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence L_n, admits square difference prime labeling.

Theorem 2.6 Let G_1 be the first copy of star $K_{1,n}$ and G_2 be the second copy of star $K_{1,n}$. Let a be the central vertex and let $v_1,v_2,---,v_n$ are the pendant vertices of G_1. Let b be the central vertex and let $u_1, u_2,---,u_n$ are the pendant vertices of G_2. Let G be the graph obtained by replacing the vertices u_i, v_i by w_i and joining a to w_i and b to w_i , for every i. G admits square difference prime labeling.

Proof: Let G be the graph and let $a,b,w_1,w_2,---,w_n$ are the vertices of G.

Here |V(G)| = n+2 and |E(G)| = 2n

Define a function f : V → {0,1,2,---,n+1 } by

$f(w_i)$ = i+1 , i = 1,2,---,n

f(a) = 0, f(b) = 1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^* (a\ w_i)$ = $(i+1)^2$, i = 1,2,---,n.

$f_{sdp}^* (b\ w_i)$ = $(i+1)^2$-1, i = 1,2,---,n.

Clearly f_{sdp}^* is an injection.

gcin of (w_i) = gcd of $\{f_{sdp}^* (a\ w_i), f_{sdp}^* (b\ w_i)\}$
= gcd of $\{(i+1)^2, (i+1)^2-1\}$
= 1, i = 1,2,---,n.

gcin of (a) = gcd of $\{f_{sdp}^* (a\ w_1), f_{sdp}^* (a\ w_2)\}$
= gcd of {4, 9}= 1.

gcin of (b) = gcd of $\{f_{sdp}^* (b\ w_1), f_{sdp}^* (b\ w_2)\}$
= gcd of {3, 8}=1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits square difference prime labeling.

Example 2.4 G be the graph obtained by joining two copies of star $K_{1,5}$.

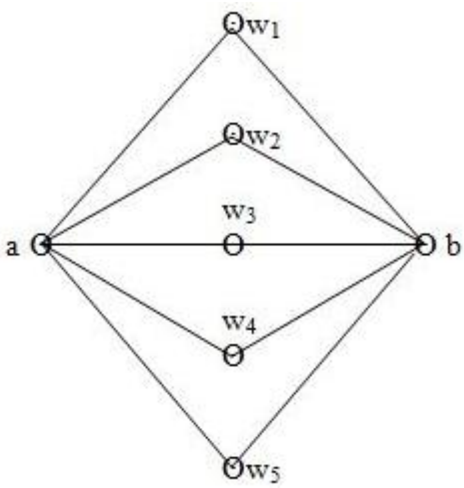

Fig – 2.4

Theorem 2.7 Strong Double graph of star graph $K_{1,n}$ (n is a natural number greater than 3) admits square difference prime labeling.

Proof: Let G =S{D($K_{1,n}$)} and let a,b,v_1,v_2,----,v_n, u_1,u_2,---,u_n are the vertices of G.

Here |V(G)| = 2n+2 and |E(G)| = 5n+1

Define a function f : V → {0,1,2,---,2n-1 } by

$f(v_i)$ = i+1 , i = 1,2,---,n

$f(u_i)$ = n+i+1 , i = 1,2,---,n

f(a) = 0, f(b) = 1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^* (a\, v_i)$ = $(i+1)^2$, i = 1,2,---,n.

$f_{sdp}^* (a\, u_i)$ = $(n+i+1)^2$, i = 1,2,---,n.

$f_{sdp}^* (b\, v_i)$ = $(i+1)^2 - 1$, i = 1,2,---,n.

$f_{sdp}^* (b\, u_i)$ = $(n+i+1)^2 - 1$, i = 1,2,---,n.

$f_{sdp}^* (u_i\, v_i)$ = $(n+i+1)^2 - (i+1)^2$, i = 1,2,---,n.

$f_{sdp}^* (a\, b)$ = 1.

Clearly f_{sdp}^* is an injection.

gcin of (a) = gcd of {$f_{sdp}^* (a\, v_1)$, $f_{sdp}^* (a\, v_2)$ }
= gcd of {4, 9}= 1.

gcin of (v_i) = gcd of {$f_{sdp}^* (a\, v_i)$, $f_{sdp}^* (b\, v_i)$ }
= gcd of { $(i+1)^2$, $(i+1)^2 - 1$}
= 1, i = 1,2,---,n.

gcin of (b) = gcd of {$f_{sdp}^* (b\, v_1)$, $f_{sdp}^* (b\, v_2)$ }
= gcd of {3, 8}= 1.

gcin of (u_i) = gcd of {$f_{sdp}^* (a\, u_i)$, $f_{sdp}^* (b\, u_i)$ }
= gcd of { $(n+i+1)^2$, $(n+i+1)^2 - 1$}
= 1, i = 1,2,---,n.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence S{D($K_{1,n}$)} , admits square difference prime labeling.

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