W-semi-I-open and w-semi-I-closed sets

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Abstract: *In this paper we will give various properties of w-semi-I-open and w-semi-I-closed sets. Also Examples are given throughout the paper.*

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1. Introduction

In [3], Janković and Hamlett introduced the concept of $\mathfrak{I}$-open sets in topological spaces. In [1], Dontchev introduced the concept of pre- $\mathfrak{I}$-open sets and in [3] Hatir and Noiri introduced the notion of semi-$\mathfrak{I}$-open sets, α- $\mathfrak{I}$-open sets and β- $\mathfrak{I}$-open sets. The subject of ideals in topological spaces were introduced by Kuratowski[4] and further studied by Vaidyanathaswamy[5]. Corresponding to an ideal a new topology $\tau^*(\mathfrak{I}, \tau)$ called the *-topology was given which is generally finer than the original topology having the kuratowski closure operator $cl^*(A) = A \cup A^*(\mathfrak{I},\tau)$[6], where $A^*(\mathfrak{I},\tau) = \{x \in X : U \cap A \notin \mathfrak{I}$ for every open subset U of x in X called a local function of A with respect to $\mathfrak{I}$ and τ. We will write τ^* for $\tau^*(\mathfrak{I}, \tau)$.

The following section contains some definitions and results that will be used in our further sections.

Definition 1.1.[4]: Let (X, τ) be a topological space. An ideal $\mathfrak{I}$ on X is a collection of non-empty subsets of X such that (a) $\phi \in \mathfrak{I}$ (b) $A \in \mathfrak{I}$ and $B \in \mathfrak{I}$ implies $A \cup B \in \mathfrak{I}$ (c) $B \in \mathfrak{I}$ and $A \subset B$ implies $A \in \mathfrak{I}$.

Definition 1.2 : Let$(X,\tau,\mathfrak{I})$ be an ideal space and A be any subset of X. Then A is said to be

a.) $\mathfrak{I}$-open[3] if $A \subset int(A^*)$.
b.) semi-$\mathfrak{I}$-open[2] if $A \subset cl^*(int(A))$.
c.) pre-$\mathfrak{I}$-open[1] if $A \subset int(cl^*(A))$.
d.) α- $\mathfrak{I}$-open[2] if $A \subset int(cl^*(int(A)))$.
e.) β-$\mathfrak{I}$-open[2] if $A \subset cl(int(cl^*(A)))$.

2. Results

Definition 2.1: Let $(X,\tau,\mathfrak{I})$ be an ideal space and A be any subset of X. Then A is said to be

a.) w- α- $\mathfrak{I}$- open if $A \subset int(cl(int^*(A)))$.
b.) w-semi-$\mathfrak{I}$-open if $A \subset cl(int^*(A))$.
c.) w-pre-$\mathfrak{I}$-open if $A \subset int^*(cl(A)$.
d.) w-β-$\mathfrak{I}$-open if $A \subset cl(int^*(cl(A)))$

Lemma 2.2: Let (X,τ) be any topological space and U and V be two open subsets of X. Then prove that

$cl(U) \cap V \subset cl(U \cap V)$.

Proof: Let $x \in cl(U) \cap V$. To prove $x \in cl(U \cap V)$. Let W be any open set containing x. Then $x \in V$ and v is open set implies that $V \cap W$ is also open set containing x. Now $x \in cl(U)$ implies that $V \cap W \cap U \neq \phi$ and so $W \cap (U \cap V) \neq \phi$ implies that $x \in cl(U \cap V)$.

Hence $cl(U) \cap V \subset cl(U \cap V)$.

Theorem 2.3: Let $(X,\tau, \mathfrak{I})$ be an ideal space and A be any subset of X. Then prove that A is w-semi-$\mathfrak{I}$-open iff $cl(A) = cl(int^*(A))$.

Proof: Firstly, let A be w-semi-$\mathfrak{I}$-open subset of X. Then $A \subset cl(int^*(A)$ and so $cl(A) \subset cl(cl(int^*(A)))$.

But we know that cl(cl(A)) = cl(A). This implies that cl(A) ⊂ cl(int*(A)). Also int*(A) ⊂ A implies that cl(int*(A)) ⊂ cl(A). Hence cl(A) = cl(int*(A)).

Conversely, let cl(A) = cl(int*(A)). We have to prove that A is w-semi-$\mathfrak{I}$-open.

Now, A ⊂ cl(A) implies that A ⊂ cl(int*(A)).

Hence A is w-semi-$\mathfrak{I}$-open.

Theorem 2.4: Let (X,τ, $\mathfrak{I}$) be an ideal space. Then a subset A of X is w-semi-$\mathfrak{I}$-open iff there exist τ*-open subset G of X such that G ⊂ A ⊂ cl(G).

Proof: Firstly, let A be w-semi-$\mathfrak{I}$-open subset of X. Then A ⊂ cl(int*(A)). Let G = int*(A). Since we know that int*(A) is open so G is τ*-open subset of X such that G ⊂ A ⊂ cl(G).

Conversely, let there exist τ*-open subset G of X such that G ⊂ A ⊂ cl(G). Now G ⊂ A implies that int*(G) ⊂ int*(A) and so G ⊂ int*(A). Therefore, A ⊂ cl(G) implies that A ⊂ cl(int*(A)).

Hence A is w-semi-$\mathfrak{I}$-open.

Theorem 2.5: If A is w-semi-$\mathfrak{I}$-open subset of an ideal space (X,τ,$\mathfrak{I}$) and be any subset of X such that

A ⊂ B ⊂ cl(A) then prove that B is also w-semi-$\mathfrak{I}$-open.

Proof: Let A be any w-semi-$\mathfrak{I}$-open subset of X and B be any subset of X such that A ⊂ B ⊂ cl(A). Now A is w-semi-$\mathfrak{I}$-open subset of X so by the above Theorem 2.4 there exist τ*-open subset G of X such that G ⊂ A ⊂ cl(G) and so G ⊂ A ⊂ B ⊂ cl(A) ⊂ cl(cl(G)). Therefore, G ⊂ B ⊂ cl(G). Hence B is w-semi-$\mathfrak{I}$-open.

Theorem 2.6: Let (X,τ, $\mathfrak{I}$) be an ideal space. Then prove the following:

(a) If $\{U_\alpha\}_{\alpha\in\Delta}$ be a family of w-semi-$\mathfrak{I}$-open subsets of X. Then prove that $\bigcup_\alpha U_\alpha$ is also a w-semi-$\mathfrak{I}$-open set.
(b) If U is w-semi-$\mathfrak{I}$-open subset of X and V is τ-open subset of X then prove that U⋂V is also a w-semi-$\mathfrak{I}$-open set.

Proof: (a) Since ∀ α ϵ Δ, U_α is w-semi-$\mathfrak{I}$-open subset of X. So $U_\alpha \subset \text{cl}(\text{int*}(U_\alpha))$.

Now $\bigcup_\alpha U_\alpha \subset \bigcup_\alpha \text{cl}(\text{int}*(U_\alpha))$ and so $\bigcup_\alpha U_\alpha \subset \text{cl}(\bigcup_\alpha \text{int}*(U_\alpha))$ since $\bigcup_\alpha cl(A_\alpha) \subset \text{cl}(\bigcup_\alpha A_\alpha)$. Further, $\bigcup_\alpha int(A_\alpha) \subset \text{int}(\bigcup_\alpha A_\alpha)$ implies that $\bigcup_\alpha U_\alpha \subset \text{cl}(\text{int*}(\bigcup_\alpha U_\alpha))$. Hence $\bigcup_\alpha U_\alpha$ is w-semi-$\mathfrak{I}$-open subset of X.

(b)Let U be w-semi-$\mathfrak{I}$-open subset of X and V be τ*-open subset of X. Then U ⊂ cl(int*(U)). Now U⋂V ⊂ cl(int*(U)) ⋂ V = cl(int*(U)) ⋂ int(V) since V is τ-open subset of X and so U⋂V ⊂ cl(int*(U)⋂ int*(V)) using Lemma 2.2 .But int*(A) ⋂ int*(B) = int*(A ⋂ B). Therefore, U⋂V ⊂ cl(int*(U⋂V)).

Hence U⋂V is w-semi-$\mathfrak{I}$-open.

Next we introduce w-semi-$\mathfrak{I}$-closed sets.

Definition 2.7: Let (X,τ,$\mathfrak{I}$) be an ideal space. Then a subset C is called w-semi-$\mathfrak{I}$-closed if its complement X-C is w-semi-$\mathfrak{I}$-open.

Theorem 2.8: If a subset C of an ideal space (X,τ,$\mathfrak{I}$) is w-semi-$\mathfrak{I}$-closed then prove that int(cl*(C) ⊂ C.

Proof: Let C be any w-semi-$\mathfrak{I}$-closed subset of X. Then X-C is w-semi-$\mathfrak{I}$-open subset of X. Therefore,

X-C ⊂ cl(int*(X-C)) = cl(X-cl*(C)) using cl(X-A) = X-int(A) or X-cl(A) = int(X-A) for any subset A of X and so X-C ⊂ X-int(cl*(C)) and so int(cl*(C)) ⊂ C.

Hence int(cl*(C)) ⊂ C.

The following Example shows that in an ideal space (X,τ,$\mathfrak{I}$) for a subset A of X, the following result need not be true.

X- int*(cl(A)) = cl(int*(X-A)).

Example 2.9: Let X = {a,b,c} with τ ={ ϕ,{a},{b},{a,b},X}. And 𝔗= {ϕ, {a}, {b}, {a,b}}. Then

τ* = {ϕ ,{a},{b},{c}{a,b},{b,c},{a,c},X}.

Now consider the subset A = {a}. So int*{X-A} = int*{b,c} = {b,c} and cl{a} = {a,c}.

Then X-int*(cl{a}) = X-int*{a,c} = X-{a,c} = {b}.

And cl(int*(X-{a})) = cl(int*{b,c}) = cl{b,c} ={b,c}.

Hence X- int*(cl(A)) ≠ cl(int*(X-A)).

Theorem 2.10: Let (X,τ,𝔗) be an ideal space and let A be any subset of X such that

X- int*(cl(A)) = cl(int*(X-A)). Then prove that A is w-semi-𝔗-closed subset of X if and only if int*(cl(A)) ⊂ A.

Proof: Firstly, let A be w-semi-𝔗-closed subset of X. Then X-A is w-semi-𝔗-open subset of X. So,

X-A ⊂ cl(int*(X-A)) and so X-A ⊂ X-int*(cl(A)) and so int*(cl(A)) ⊂ A.

Hence int*(cl(A)) ⊂ A.

Conversely, let int*(cl(A)) ⊂ A. We have to prove that A is w-semi-𝔗-closed. We will prove that X-A is w-semi-𝔗-open and so A is w-semi-𝔗-closed.

Now, int*(cl(A)) ⊂ A implies that X-A ⊂ X – int*(cl(A)) and so X-A ⊂ cl(int*(X-A)).

Therefore, X-A is w-semi-𝔗-open.

Hence A is w-semi-𝔗-closed.

References

[1]J.Dontchev, *On pre-I-open sets and a decomposition of I-continuity*, Banyan Math. J., 2(1996).
[2]E. Hatir and T.Noiri, *On decompositions of continuity via idealization*, Acta Math. Hunger., 96 (2002), 341-349.
[3]D.Jankovic and T.R. Hamlett, *New topologies from old via ideals*, The American Mathematical Monthly, 97, No. 4 (1990), 295-310.
[4] K. Kuratowski, *Topology*, volume I, Academic Press,New York, 1966.
[5] R. Vaidyanathaswamy, *The localisation Theory in Set Topology*, Proc. Indian Acad. Sci.,20(1945),51-61.
[6] -------, *Set Topology*, Chelsea Publishing Company, New York,1946.