

# New results of anti intuitionistic subtraction Fuzzy Soft A-Ideal

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## Abstract

The notion of an anti intuitionistic fuzzy a-ideal is introduced. Conditions for an intuitionistic fuzzy a-ideal to be an anti intuitionistic fuzzy a-ideal are provided. New results of anti intuitionistic subtraction Fuzzy Soft A-Ideal theorems and also examples are established.

**Key words:** Fuzzy, Fuzzy soft set, a-ideal, Fuzzy Soft A-Ideal.

## I. INTRODUCTION

Fuzzy set theory was introduced by L-Zadeh since 1965. Immediately it became a useful method to study in the problems of imprecision and uncertainty. Since, a lot of new theories treating imprecision and uncertainty have been introduced. For instance intuitionistic fuzzy sets were introduced in 1986 by K.Atanassov which is a generalization of the notion of a fuzzy set. Liu and Zhang discussed the fuzzification of h-ideals, gave relations between fuzzy ideals, fuzzy h-ideals and fuzzy p-ideals.

The important concept that addresses uncertain information is the soft set theory originated by Molodtsov. This concept is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Molodtsov has successfully applied the soft set theory in many different fields such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, and probability. In recent years, soft set theory has been received much attention since its appearance. There are many papers devoted to fuzzify the concept of soft set theory which leads to a series of mathematical models such as fuzzy soft set generalized fuzzy soft set possibility fuzzy soft set and so on. Thereafter, P.K.Maji and his coauthor introduced the notion of intuitionistic fuzzy soft set which is based on a combination of the fuzzy soft set which is based on a combination of the intuitionistic fuzzy sets and soft set models and they studied the properties of intuitionistic fuzzy soft set.

## II. Preliminaries

**Definition 2.1:** A fuzzy set A on the universe U is an object of the form

$$A = \{x, \mu_A(x) : x \in U\}$$

Where  $\mu_A(x)$  ( $X \in [0,1]$ ) is called the degree of membership of x in A.

**Definition 2.2:** An Intuitionistic fuzzy (IF) set A on the universe U is an object of the form  $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in U\}$  where  $\mu_A(x)$  ( $\in [0,1]$ ) is called the “degree of membership of x in A”,  $\lambda_A(x)$  ( $X \in [0,1]$ ) is called the “degree of non- membership of x in A” and where  $\mu_A$  and  $\lambda_A$  satisfy the following condition  $\mu_A(x) + \lambda_A(x) \leq 1, (\forall x \in U)$ .

**Definition 2.3:** An intuitionistic fuzzy set  $A = \langle X, \mu_A, \gamma_A \rangle$  in X is called an intuitionistic fuzzy ideal of X, if it satisfies the following axioms

$$(I) \lambda_A(x) \leq \max \{ \lambda_A(x-y), \lambda_A(y) \}, \forall x, y \in X.$$

And  $(\forall x, y, z \in X)$

$$(\mu_A(y-x) \geq \min \{ \mu_A((x-z) - (0 - y)), \mu_A(z) \}),$$

$(\forall x, y, z \in X)$

$$(\lambda_A(y-x) \leq \max \{ \lambda_A((x-z) - (0 - y)), \lambda_A(z) \}).$$

**Definition 2.4:** A pair  $(F,A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F: A \rightarrow P(U)$ . In other words, a soft set over  $U$  is a parameterized family of subsets of the universe  $U$ . For  $\epsilon \in A$ ,  $F(\epsilon)$  may be considered as the set of  $\epsilon$ -approximate elements of the soft set  $(F,A)$ .

**Definition 2.5:** Let  $U$  be an initial universe set and  $E$  be the set of parameters. Let  $IF^U$  denote the collection of all intuitionistic fuzzy subsets of  $U$ . A  $\subseteq E$  pair  $(F,A)$  is called an intuitionistic fuzzy soft set over  $U$  where  $F$  is a mapping given by  $F: A \rightarrow IF^U$ .

**Definition 2.6:** Let  $F: A \rightarrow IF^U$  then  $F$  is a function defined as

$F(\epsilon) = \{x, \mu_{F(\epsilon)}(x), \gamma_{F(\epsilon)}(x) : x \in U, \epsilon \in E\}$  where  $\mu, \gamma$  denote the degree of membership and degree of non-membership respectively.

**Definition 2.7:** For two intuitionistic fuzzy soft sets  $(F,A)$  and  $(G,B)$  over a common universe  $U$ , we say that  $(F,A)$  is an intuitionistic fuzzy soft subset of  $(G,B)$  if (1)  $A \subseteq B$  and (2)  $F(\epsilon) \subseteq G(\epsilon)$  for all  $\epsilon \in A$ . i.e  $\mu_{F(\epsilon)}(x) \leq \mu_{G(\epsilon)}(x), \gamma_{F(\epsilon)}(x) \geq \gamma_{G(\epsilon)}(x)$  for all  $\epsilon \in E$  and we write  $(F,A) \subseteq (G,B)$ .

**Definition 2.8:** Two intuitionistic fuzzy soft sets  $(F,A)$  and  $(G,B)$  over a common universe  $U$  are said to be soft equal if  $(F,A)$  is a soft subset of  $(G,B)$  and  $(G,B)$  is a soft subset of  $(F,A)$ .

**Definition 2.9:** Let  $U$  be an initial universe,  $E$  be the set of parameters, and  $A \subseteq E$ . (a)  $(F, A)$  is called a null intuitionistic fuzzy soft set (with respect to the parameter set  $A$ ), denoted by  $\varphi_A$ , if  $F(a) = \varphi$  for all  $a \in A$ .

(b)  $(G, A)$  is called an absolute intuitionistic fuzzy soft set (with respect to the parameter set  $A$ ), denoted by  $U_A$ , if  $G(e) = U$  for all  $e \in A$ .

**Definition 2.10:** Let  $(F, A)$  and  $(G, B)$  be two intuitionistic fuzzy soft sets over the same common universe  $U$ . Then the union of  $(F,A)$  and  $(G,B)$  is denoted by ' $(F,A) \cup (G,B)$ ' and is defined by  $(F,A) \cup (G,B) = (H,C)$ , where  $C = A \cup B$  and the truth-membership, falsity- membership of  $(H,C)$  are as follows:

$$H(\epsilon) = \begin{cases} \{(\mu_{F(\epsilon)}(x), \gamma_{F(\epsilon)}(x)) : x \in U\}, & \text{if } \epsilon \in A - B, \\ \{(\mu_{G(\epsilon)}(x), \gamma_{G(\epsilon)}(x)) : x \in U\}, & \text{if } \epsilon \in B - A \\ \{\max(\mu_{F(\epsilon)}(x), \mu_{G(\epsilon)}(x)), \min(\gamma_{F(\epsilon)}(x), \gamma_{G(\epsilon)}(x)) : x \in U\}, & \text{if } \epsilon \in A \cup B \end{cases}$$

Where  $\mu_{H(\epsilon)}(x) = \max(\mu_{F(\epsilon)}(x), \mu_{G(\epsilon)}(x))$  and  $\gamma_{H(\epsilon)}(x) = \min(\gamma_{F(\epsilon)}(x), \gamma_{G(\epsilon)}(x))$ .

**Definition 2.11:** Let  $(F,A)$  and  $(G,B)$  be two intuitionistic fuzzy soft sets over the same common universe  $U$  such that  $A \cap B \neq \emptyset$ . Then the intersection of  $(F,A)$  and  $(G,B)$  is denoted by ' $(F,A) \cap (G,B)$ ' and is defined by  $(F,A) \cap (G,B) = (K,C)$ , where  $C = A \cap B$  and the truth-membership, falsity- membership of  $(K,C)$  are related to those of  $(F,A)$  and  $(G,B)$  as follows:

$$K(\epsilon) = \begin{cases} \{(\mu_{F(\epsilon)}(x), \gamma_{F(\epsilon)}(x)) : x \in U\}, & \text{if } \epsilon \in A - B, \\ \{(\mu_{G(\epsilon)}(x), \gamma_{G(\epsilon)}(x)) : x \in U\}, & \text{if } \epsilon \in B - A \\ \{\min(\mu_{F(\epsilon)}(x), \mu_{G(\epsilon)}(x)), \max(\gamma_{F(\epsilon)}(x), \gamma_{G(\epsilon)}(x)) : x \in U\}, & \text{if } \epsilon \in A \cap B \end{cases}$$

Where  $\mu_{K(\epsilon)}(x) = \min(\mu_{F(\epsilon)}(x), \mu_{G(\epsilon)}(x))$  and

$\gamma_{K(\epsilon)}(x) = \max(\gamma_{F(\epsilon)}(x), \gamma_{G(\epsilon)}(x))$ .

### 3. New results of anti intuitionistic subtraction Fuzzy Soft A-Ideal

**Theorem 3.1:** Let  $(F,A)$  and  $(G,B)$  be two anti intuitionistic subtraction fuzzy soft a-ideal over the common universe  $X$ . Then the union of  $(F,A)$  and  $(G,B)$  is also anti intuitionistic subtraction fuzzy soft a-ideal.

**Proof:**

An AIFSS  $A = \langle X, \mu_{F(\epsilon)}, \gamma_{F(\epsilon)} \rangle$  in  $X$  is called an Anti intuitionistic fuzzy soft a-ideal of  $X$  if  $\mu_{F(\epsilon)}(0) \leq \mu_{F(\epsilon)}(x), \mu_{G(\epsilon)}(0) \leq \mu_{G(\epsilon)}(x)$  and

$$I) \max\{\mu_{F(\epsilon)}(0), \mu_{G(\epsilon)}(0)\} \leq \max\{\mu_{F(\epsilon)}(x), \mu_{G(\epsilon)}(x)\}, \mu_{H(\epsilon)}(0) \leq \max\{\mu_{F(\epsilon)}(x), \mu_{G(\epsilon)}(x)\},$$

$\forall x \in X$  if  $\epsilon \in A \cup B$ .

$$\gamma_{F(\epsilon)}(0) \geq \gamma_{F(\epsilon)}(x), \gamma_{G(\epsilon)}(0) \geq \gamma_{G(\epsilon)}(x) \text{ and}$$

$$\min\{\gamma_{F(\epsilon)}(0), \gamma_{G(\epsilon)}(0)\} \geq \min\{\gamma_{F(\epsilon)}(x), \gamma_{G(\epsilon)}(x)\}, \gamma_{H(\epsilon)}(0) \geq \min\{\gamma_{F(\epsilon)}(x), \gamma_{G(\epsilon)}(x)\}, \forall x \in X \text{ if } \epsilon \in A \cup B.$$

$$II) \text{ If } \mu_{F(\epsilon)}(y-x) \leq \max\{\mu_{F(\epsilon)}((x-z) - (0-y)),$$

$$\mu_{F(\epsilon)}(z)\} \text{ and } \mu_{G(\epsilon)}(y-x) \leq \max\{\mu_{G(\epsilon)}((x-z) - (0-y)), \mu_{G(\epsilon)}(z)\}$$

$$\max\{\mu_{F(\epsilon)}(y-x), \mu_{G(\epsilon)}(y-x)\} \leq \max\{\max\{\mu_{F(\epsilon)}((x-z) - (0-y)), \mu_{F(\epsilon)}(z)\}, \max\{\mu_{G(\epsilon)}((x-z) - (0-y)), \mu_{G(\epsilon)}(z)\}\}$$

$$\mu_{H(\epsilon)}(y-x) \leq \max\{\max\{\mu_{F(\epsilon)}((x-z) - (0-y)), \mu_{G(\epsilon)}((x-z) - (0-y))\}, \max\{\mu_{F(\epsilon)}(z), \mu_{G(\epsilon)}(z)\}\}$$

$$\mu_{H(\epsilon)}(y-x) \leq \max\{\mu_{H(\epsilon)}((x-z) - (0-y)), \mu_{H(\epsilon)}(z)\}.$$

$$\text{If } \gamma_{F(\epsilon)}(y-x) \geq \min\{\gamma_{F(\epsilon)}((x-z) - (0-y)), \gamma_{F(\epsilon)}(z)\} \text{ and}$$

$$\gamma_{G(\epsilon)}(y-x) \geq \min\{\gamma_{G(\epsilon)}((x-z) - (0-y)), \gamma_{G(\epsilon)}(z)\}$$

$$\min\{\gamma_{F(\epsilon)}(y-x), \gamma_{G(\epsilon)}(y-x)\} \geq \min\{\min\{\gamma_{F(\epsilon)}((x-z) - (0-y)), \gamma_{F(\epsilon)}(z)\}, \min\{\gamma_{G(\epsilon)}((x-z) - (0-y)), \gamma_{G(\epsilon)}(z)\}\}$$

$$\gamma_{H(\epsilon)}(y-x) \geq \min\{\min\{\gamma_{F(\epsilon)}((x-z) - (0-y)), \gamma_{G(\epsilon)}((x-z) - (0-y))\}, \min\{\gamma_{F(\epsilon)}(z), \gamma_{G(\epsilon)}(z)\}\}$$

$$\gamma_{H(\epsilon)}(y-x) \geq \min\{\gamma_{H(\epsilon)}((x-z) - (0-y)), \gamma_{H(\epsilon)}(z)\}.$$

Hence  $(F,A) \cup (G,B)$  is also anti intuitionistic subtraction fuzzy soft a-ideal.

**Example 3.1:** Consider a BCI-algebra  $X = \{0, a, b\}$  with the following Cayley table:

-	0	a	B
0	0	b	A
A	A	0	B
B	B	a	0

X	0	A	B
$\mu_{F(\epsilon)}$	0.2	0.7	0.2
$\gamma_{F(\epsilon)}$	0.5	0.2	0.5

X	0	A	B
$\mu_{G(\epsilon)}$	0.3	0.8	0.3
$\gamma_{G(\epsilon)}$	0.7	0.4	0.7

I) Let  $x=a$ ,

$$\max\{\mu_{F(\epsilon)}(0), \mu_{G(\epsilon)}(0)\} \leq \max\{\mu_{F(\epsilon)}(x), \mu_{G(\epsilon)}(x)\}$$

$$\max\{\mu_{F(\epsilon)}(0), \mu_{G(\epsilon)}(0)\} \leq \max\{\mu_{F(\epsilon)}(a), \mu_{G(\epsilon)}(a)\},$$

$$\max\{0.2, 0.3\} \leq \max\{0.7, 0.8\}$$

$$0.3 \leq 0.8$$

Let  $x=b$ ,

$$\min\{\gamma_{F(\epsilon)}(0), \gamma_{G(\epsilon)}(0)\} \geq \min\{\gamma_{F(\epsilon)}(x), \gamma_{G(\epsilon)}(x)\}$$

$$\min\{0.5, 0.7\} \geq \min\{\gamma_{F(\epsilon)}(b), \gamma_{G(\epsilon)}(b)\},$$

$$0.5 \geq \min\{0.5, 0.7\}$$

$0.5 \geq 0.5$  is satisfied.

II) If  $x=b, y=a, z=0$

$$\mu_{F(\epsilon)}(y-x) \leq \max\{\mu_{F(\epsilon)}((x-z) - (0 - y)), \mu_{F(\epsilon)}(z)\} \text{ and}$$

$$\mu_{G(\epsilon)}(y-x) \leq \max\{\mu_{G(\epsilon)}((x-z) - (0 - y)), \mu_{G(\epsilon)}(z)\}$$

$$\max\{\mu_{F(\epsilon)}(y-x), \mu_{G(\epsilon)}(y-x)\} \leq \max\{\max\{\mu_{F(\epsilon)}((x-z) - (0 - y)), \mu_{F(\epsilon)}(z)\}, \max\{\mu_{G(\epsilon)}((x-z) - (0 - y)), \mu_{G(\epsilon)}(z)\}\}.$$

$$\max\{\mu_{F(\epsilon)}(a - b), \mu_{G(\epsilon)}(a - b)\} \leq \max\{\max\{\mu_{F(\epsilon)}((b-0) - (0 - a)), \mu_{F(\epsilon)}(0)\}, \max\{\mu_{G(\epsilon)}((b-0) - (0 - a)), \mu_{G(\epsilon)}(0)\}\}.$$

$$\max\{\mu_{F(\epsilon)}(b), \mu_{G(\epsilon)}(b)\} \leq \max\{\max\{\mu_{F(\epsilon)}(b-b), \mu_{F(\epsilon)}(0)\}, \max\{\mu_{G(\epsilon)}(b-b), \mu_{G(\epsilon)}(0)\}\}$$

$$\max\{\mu_{F(\epsilon)}(b), \mu_{G(\epsilon)}(b)\} \leq \max\{\max\{\mu_{F(\epsilon)}(0), \mu_{F(\epsilon)}(0)\}, \max\{\mu_{G(\epsilon)}(0), \mu_{G(\epsilon)}(0)\}\}$$

$$\max\{0.2, 0.3\} \leq \max\{\max\{0.2, 0.2\}, \max\{0.3, 0.3\}\}$$

$$\max\{0.2, 0.3\} \leq \max\{0.2, 0.3\}$$

$0.3 \leq 0.3$  is satisfied.

If  $x=b, y=a, z=0$ ,

$$\gamma_{F(\epsilon)}(y-x) \geq \min\{\gamma_{F(\epsilon)}((x-z) - (0 - y)), \gamma_{F(\epsilon)}(z)\} \text{ and } \gamma_{G(\epsilon)}(y-x) \geq \min\{\gamma_{G(\epsilon)}((x-z) - (0 - y)), \gamma_{G(\epsilon)}(z)\}$$

$$\min\{\gamma_{F(\epsilon)}(y-x), \gamma_{G(\epsilon)}(y-x)\} \geq \min\{\min\{\gamma_{F(\epsilon)}((x-z) - (0 - y)), \gamma_{F(\epsilon)}(z)\}, \min\{\gamma_{G(\epsilon)}((x-z) - (0 - y)), \gamma_{G(\epsilon)}(z)\}\}$$

$$\min\{\gamma_{F(\epsilon)}(a - b), \gamma_{G(\epsilon)}(a - b)\} \geq \min\{\min\{\gamma_{F(\epsilon)}((b-0) - (0 - a)), \gamma_{F(\epsilon)}(0)\}, \min\{\gamma_{G(\epsilon)}((b-0) - (0 - a)), \gamma_{G(\epsilon)}(0)\}\}$$

$$\min\{\gamma_{F(\epsilon)}(b), \gamma_{G(\epsilon)}(b)\} \geq \min\{\min\{\gamma_{F(\epsilon)}(b-b), \gamma_{F(\epsilon)}(0)\},$$

$$\min\{\gamma_{G(\epsilon)}(b-b), \gamma_{G(\epsilon)}(0)\}\}$$

$$\min\{0.5, 0.7\} \geq \min\{\min\{\gamma_{F(\epsilon)}(0), \gamma_{F(\epsilon)}(0)\}, \min\{\gamma_{G(\epsilon)}(0), \gamma_{G(\epsilon)}(0)\}\}$$

$$\min\{0.5, 0.7\} \geq \min\{\min\{0.5, 0.5\}, \min\{0.7, 0.7\}\}$$

$$0.7 \geq \min\{0.5, 0.7\},$$

$$0.7 \geq 0.7 \text{ is satisfied.}$$

This completes the proof.

**Theorem 3.2:** Let (F,A) and (G,B) be two anti intuitionistic subtraction fuzzy soft a-ideal over the common universe X. Then the intersection of (F,A) and (G,B) is also anti intuitionistic subtraction fuzzy soft a-ideal.

**Proof:**

An anti intuitionistic subtraction fuzzy soft a-ideal  $A = \langle X, \mu_{F(\epsilon)}, \gamma_{F(\epsilon)} \rangle$  in X is called an Anti intuitionistic fuzzy a-ideal of X if

$$\mu_{F(\epsilon)}(0) \leq \mu_{F(\epsilon)}(x), \mu_{G(\epsilon)}(0) \leq \mu_{G(\epsilon)}(x) \text{ and}$$

$$I) \quad \min\{\mu_{F(\epsilon)}(0), \mu_{G(\epsilon)}(0)\} \leq \min\{\mu_{F(\epsilon)}(x), \mu_{G(\epsilon)}(x)\},$$

$$\mu_{H(\epsilon)}(0) \leq \mu_{H(\epsilon)}(x), \forall x \in X.$$

$$\gamma_{F(\epsilon)}(0) \geq \gamma_{F(\epsilon)}(x), \gamma_{G(\epsilon)}(0) \geq \gamma_{G(\epsilon)}(x) \text{ and}$$

$$\max\{\gamma_{F(\epsilon)}(0), \gamma_{G(\epsilon)}(0)\} \geq \max\{\gamma_{F(\epsilon)}(x), \gamma_{G(\epsilon)}(x)\},$$

$$\gamma_{H(\epsilon)}(0) \geq \gamma_{H(\epsilon)}(x), \forall x \in X.$$

$$II) \quad \mu_{F(\epsilon)}(y-x) \leq \max\{\mu_{F(\epsilon)}((x-z) - (0-y)), \mu_{F(\epsilon)}(z)\} \text{ and}$$

$$\mu_{G(\epsilon)}(y-x) \leq \max\{\mu_{G(\epsilon)}((x-z) - (0-y)), \mu_{G(\epsilon)}(z)\}$$

$$\min\{\mu_{F(\epsilon)}(y-x), \mu_{G(\epsilon)}(y-x)\} \leq \min\{\max\{\mu_{F(\epsilon)}((x-z) - (0-y)), \mu_{F(\epsilon)}(z)\}, \max\{\mu_{G(\epsilon)}((x-z) - (0-y)), \mu_{G(\epsilon)}(z)\}\}$$

$$\mu_{H(\epsilon)}(y-x) \leq \max\{\min\{\mu_{F(\epsilon)}((x-z) - (0-y)), \mu_{G(\epsilon)}((x-z) - (0-y))\}, \min\{\mu_{F(\epsilon)}(z), \mu_{G(\epsilon)}(z)\}\}$$

$$\mu_{H(\epsilon)}(y-x) \leq \max\{\mu_{H(\epsilon)}((x-z) - (0-y)), \mu_{H(\epsilon)}(z)\}.$$

$$\text{If } \gamma_{F(\epsilon)}(y-x) \geq \min\{\gamma_{F(\epsilon)}((x-z) - (0-y)), \gamma_{F(\epsilon)}(z)\} \text{ and}$$

$$\gamma_{G(\epsilon)}(y-x) \geq \min\{\gamma_{G(\epsilon)}((x-z) - (0-y)), \gamma_{G(\epsilon)}(z)\}$$

$$\max\{\gamma_{F(\epsilon)}(y-x), \gamma_{G(\epsilon)}(y-x)\} \geq \max\{\min\{\gamma_{F(\epsilon)}((x-z) - (0-y)), \gamma_{F(\epsilon)}(z)\}, \min\{\gamma_{G(\epsilon)}((x-z) - (0-y)), \gamma_{G(\epsilon)}(z)\}\}$$

If B is a subset of A and A is a subset of B,

$$\gamma_{H(\epsilon)}(y-x) \geq \min\{\max\{\gamma_{F(\epsilon)}((x-z) - (0-y)), \gamma_{G(\epsilon)}((x-z) - (0-y))\}, \max\{\gamma_{F(\epsilon)}(z), \gamma_{G(\epsilon)}(z)\}\}$$

$$\gamma_{H(\epsilon)}(y-x) \geq \max\{\gamma_{H(\epsilon)}((x-z) - (0 - y)), \gamma_{H(\epsilon)}(z)\}.$$

Hence (F,A)  $\cap$  (G,B) is also anti intuitionistic subtraction fuzzy soft a-ideal.

**Example 3.2:** Consider a BCI-algebra X = {0, a, b} with the following Cayley table:

–	0	a	B
0	0	b	A
A	A	0	B
B	B	a	0

X	0	A	B
$\mu_{F(\epsilon)}$	0.2	0.7	0.2
$\lambda_{F(\epsilon)}$	0.5	0.2	0.5

X	0	A	B
$\mu_{G(\epsilon)}$	0.3	0.8	0.3
$\lambda_{G(\epsilon)}$	0.7	0.4	0.7

I) Let x=a,

$$\min\{\mu_{F(\epsilon)}(0), \mu_{G(\epsilon)}(0)\} \leq \min\{\mu_{F(\epsilon)}(x), \mu_{G(\epsilon)}(x)\},$$

$$\min\{(0.2), (0.3)\} \leq \min\{(0.7), (0.8)\}$$

$$0.2 \leq 0.7$$

Let x=b,

$$\max\{\gamma_{F(\epsilon)}(0), \gamma_{G(\epsilon)}(0)\} \geq \max\{\gamma_{F(\epsilon)}(x), \gamma_{G(\epsilon)}(x)\},$$

$$\max\{0.5, 0.7\} \geq \max\{\gamma_{F(\epsilon)}(b), \gamma_{G(\epsilon)}(b)\}$$

$$0.7 \geq \max\{0.5, 0.7\},$$

$$0.7 \geq 0.7$$

II) If x=b, y=a, z=0 Now,

$$\mu_{F(\epsilon)}(y-x) \leq \min\{\mu_{F(\epsilon)}((x-z) - (0 - y)), \mu_{F(\epsilon)}(z)\} \text{ and}$$

$$\mu_{G(\epsilon)}(y-x) \leq \min\{\mu_{G(\epsilon)}((x-z) - (0 - y)), \mu_{G(\epsilon)}(z)\}$$

$$\min\{\mu_{F(\epsilon)}(y-x), \mu_{G(\epsilon)}(y-x)\} \leq \min\{\max\{\mu_{F(\epsilon)}((x-z) - (0 - y)), \mu_{F(\epsilon)}(z)\}, \max\{\mu_{G(\epsilon)}((x-z) - (0 - y)), \mu_{G(\epsilon)}(z)\}\}$$

$$\min\{\mu_{F(\epsilon)}(a-b), \mu_{G(\epsilon)}(a-b)\} \leq \max\{\min\{\mu_{F(\epsilon)}((b-0) - (0 - a)), \mu_{F(\epsilon)}(0)\}, \min\{\mu_{G(\epsilon)}((b-0) - (0 - a)), \mu_{G(\epsilon)}(0)\}\}$$

$$\min\{\mu_{F(\epsilon)}(b), \mu_{G(\epsilon)}(b)\} \leq \max\{\min\{\mu_{F(\epsilon)}(b-b), \mu_{F(\epsilon)}(0)\}, \min\{\mu_{G(\epsilon)}(b-b), \mu_{G(\epsilon)}(0)\}\}$$

$$\min\{\mu_{F(\epsilon)}(b), \mu_{G(\epsilon)}(b)\} \leq \max\{\min\{\mu_{F(\epsilon)}(0), \mu_{F(\epsilon)}(0)\}, \min\{\mu_{G(\epsilon)}(0), \mu_{G(\epsilon)}(0)\}\}$$

$$\min\{0.2, 0.3\} \leq \max\{\min\{0.2, 0.2\}, \min\{0.3, 0.3\}\}$$

$$\min\{0.2, 0.3\} \leq \max\{0.2, 0.3\},$$

$0.2 \leq 0.3$  is satisfied.

If  $x=b, y=a, z=0$

Now,

$$\gamma_{F(\epsilon)}(y-x) \geq \min\{ \gamma_{F(\epsilon)}((x-z) - (0 - y)), \gamma_{F(\epsilon)}(z) \} \text{ and}$$

$$\gamma_{G(\epsilon)}(y-x) \geq \min\{ \gamma_{G(\epsilon)}((x-z) - (0 - y)), \gamma_{G(\epsilon)}(z) \}$$

$$\max\{ \gamma_{F(\epsilon)}(y-x), \gamma_{G(\epsilon)}(y-x) \} \geq \max\{ \min\{ \gamma_{F(\epsilon)}((x-z) - (0 - y)), \gamma_{F(\epsilon)}(z) \},$$

$$\min\{ \gamma_{G(\epsilon)}((x-z) - (0 - y)), \gamma_{G(\epsilon)}(z) \} \}$$

If B is a subset of A and A is a subset of B,

$$\max\{ \gamma_{F(\epsilon)}(a-b), \gamma_{F(\epsilon)}(a - b) \} \geq \min\{ \max\{ \gamma_{F(\epsilon)}((b-0) - (0 - a)), \gamma_{F(\epsilon)}(0) \},$$

$$\max\{ \gamma_{G(\epsilon)}((b-0) - (0 - a)), \gamma_{G(\epsilon)}(0) \} \}$$

$$\max\{ \gamma_{F(\epsilon)}(b), \gamma_{G(\epsilon)}(b) \} \geq \min\{ \max\{ \gamma_{F(\epsilon)}(b-b), \gamma_{F(\epsilon)}(0) \},$$

$$\max\{ \gamma_{G(\epsilon)}(b-b), \gamma_{G(\epsilon)}(0) \} \}$$

$$\max\{ 0.5, 0.7 \} \geq \min\{ \max\{ \gamma_{F(\epsilon)}(0), \gamma_{F(\epsilon)}(0) \},$$

$$\max\{ \gamma_{G(\epsilon)}(0), \gamma_{G(\epsilon)}(0) \} \}$$

$$\max\{ 0.5, 0.7 \} \geq \min\{ \max\{ 0.5, 0.5 \}, \max\{ 0.7, 0.7 \} \}$$

$$0.7 \geq \min\{ 0.5, 0.7 \},$$

$$0.7 \geq 0.5 \text{ is satisfied.}$$

This completes the proof.

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