k-Harmonic Mean Labeling of Some Graphs

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Abstract - *Somasundaram and Ponraj [12] have introduced the notion of mean labeling of graphs. R. Ponraj and D. Ramya introduced Super mean labeling of graphs in [5]. S. Somasundaram and S.S. Sandhya introduced the concept of Harmonic mean labeling in [6] and studied their behavior in [7,8,9]. In this paper, we introduce the concept of k-harmonic mean labeling and we investigate k-harmonic mean labeling of some graphs.*

Key words - *Harmonic mean labeling, harmonic mean graph, k-harmonic mean labeling, k-harmonic mean graph.*

1. INTRODUCTION

By a graph $G = (V(G), E(G))$ with p vertices and q edges we mean a simple, connected and undirected graph. In this paper a brief summary of definitions and other information is given in order to maintain compactness. The term not defined here are used in the sense of Harary [3].

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. A useful survey on graph labeling by J.A. Gallian (2016) can be found in [1]. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).

Somasundaram and Ponraj [12] have introduced the notion of mean labeling of graphs. R. Ponraj and D. Ramya introduced Super mean labeling of graphs in [5]. S. Somasundaram and S.S. Sandhya introduced the concept of Harmonic mean labeling in [6] and studied their behavior in [7,8,9].

In this paper, we introduce the concept of k-harmonic mean labeling and we investigate k-harmonic mean labeling of some graphs.

Definition 1.1

Let G be a (p, q) graph. A function f is called a harmonic mean labeling of a graph G if $f : V(G) \rightarrow \{1, 2, 3, \ldots, q + 1\}$ is injection and the induced edge function $f^* : E(G) \rightarrow \{1, 2, \ldots, q\}$ defined as $f^*(e = uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ is bijective. The graph which admits harmonic mean labeling is called harmonic mean graph.

Definition 1.2

Let G be a (p, q) graph. A function f is called a k-harmonic mean labeling of a graph G if $f : V(G) \rightarrow \{k, k+1, k+2, \ldots, k + q\}$ is injection and the induced edge function $f^* : E(G) \rightarrow \{k, k+1, k+2, \ldots, k+q-1\}$ defined as $f^*(e = uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ is bijective. The graph which admits k-harmonic mean labeling is called k-harmonic mean graph.

Definition 1.3

The product $P_2 \times P_n$ is called a Ladder and it is denoted by L_n.

Definition 1.4

A Twig graph is a tree obtained from a path by attaching exactly two pendent edges to each internal vertex of the path.

Definition 1.5

A Triangular ladder TL_n, $n \geq 2$ is a graph obtained from a ladder L_n by adding the edges u_iv_{i+1} for $1 \leq i \leq n-1$, where u_i and v_i $1 \leq i \leq n$, are the vertices of L_n such that $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ are two paths of length n in L_n.

Definition 1.6

If G has order n, the Corona of G with H, G Θ H is the graph obtained by taking one copy of G and n copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H.

Definition 1.7

Let P_n be a path on n vertices denoted by (1, 1), (1, 2), ... (1, n) and with n – 1 edges denoted by $e_1, e_2, \ldots, e_{n-1}$ where e_i is the edge joining the vertices (1, i) and (1, i+1). On each edge e_i, $1 \le i \le n-1$, we errect a ladder with $n - (i - 1)$ steps including the edge e_i. The graph obtained is called a step ladder graph and denoted by $S(T_n)$ where n denotes the number of vertices in the base.

2. MAIN RESULTS

Theorem 2.1

The path P_n is a k-harmonic mean graph for all k and n ≥ 2.

Proof

Let $V(P_n) = \{v_i ; 1 \le i \le n\}$ and

$E(P_n) = \{e_i = (v_i, v_{i+1}); 1 \le i \le n-1\}$

Define a function $f: V(P_n) \to \{k, k+1, k+2, \ldots, k+q\}$ by

$f(v_i) = k + i - 1 \qquad 1 \le i \le n$

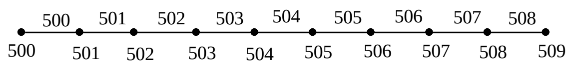
Then the induced edge labels are

$f^*(e_i) = k + i - 1 \qquad 1 \le i \le n-1$

The above defined function f provides k-harmonic mean labeling of the graph.

Hence P_n is a k-harmonic mean graph.

Example 2.1



500-harmonic mean labeling of P_{10}

Theorem 2.2

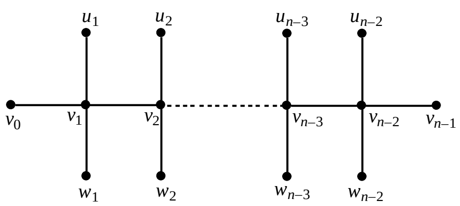
The Twig graph T_n is k-harmonic mean graph for all k and n ≥ 3.

Proof

Let $V(T_n) = \{v_i ; 0 \le i \le n-1, u_i, w_i ; 1 \le i \le n-2\}$ and

$E(T_n) = \{v_iu_i, v_iw_i; 1 \le i \le n-2, v_iv_{i+1}; 0 \le i \le n-2\}$

The ordinary labeling is



First we label the vertices as follows

Define a function $f : V(T_n) \to \{k, k+2, k+2, \ldots k+q\}$ by

$f(v_0) = k$

$f(v_i) = k + 3i - 2, \qquad$ for $1 \le i \le n-1$

$f(w_i) = k + 3i - 1, \qquad$ for $1 \le i \le n-2$

$f(u_i) = k + 3i, \qquad$ for $1 \le i \le n-2$

Then the induced edge labels are

$f^*(v_iv_{i+1}) = k + 3i, \qquad$ for $0 \le i \le n-2$

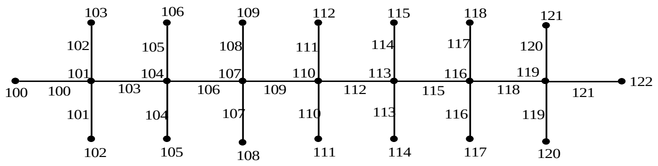
$f^*(v_iu_i) = k + 3i - 1, \qquad$ for $1 \le i \le n-2$

$f^*(v_iw_i) = k + 3i - 2, \qquad$ for $1 \le i \le n-2$

The above defined function *f* provides k-harmonic mean labeling of the graph.

Hence T_n is a k-harmonic mean graph.

Example 2.2



100-harmonic mean labeling of T_9

Theorem 2.3

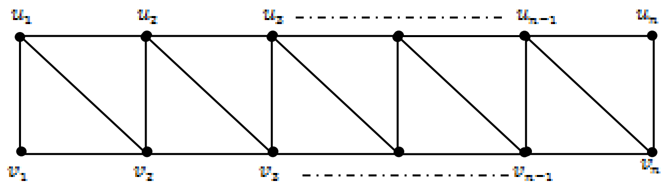
The Triangular ladder TL_n is k-harmonic mean graph for all k and n ≥ 2.

Proof

Let $V(TL_n) = \{u_i, v_i \,;\, 1 \le i \le n\}$ and

$E(TL_n) = \{u_iu_{i+1}, v_iv_{i+1}, u_iv_{i+1} \,;\, 1 \le i \le n-1, u_iv_i \,;\, 1 \le i \le n\}$

The ordinary labeling is



First we label the vertices as follows

Define a function $f : V(TL_n) \to \{k, k+1, k+2,\ldots k+q\}$ by

$f(u_i) = k + 4i - 3$, for $1 \le i \le n$

$f(v_1) = k$

$f(v_i) = k + 4i - 5$, for $2 \le i \le n$.

Then the induced edge labels are

$f^*(u_iu_{i+1}) = k + 4i - 1$, for $1 \le i \le n-1$

$f^*(v_iv_{i+1}) = k + 4i - 3$, for $1 \le i \le n-1$

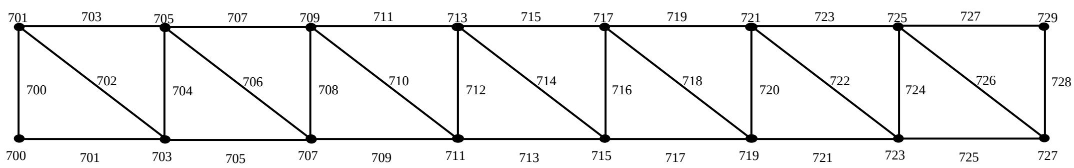
$f^*(u_iv_i) = k + 4i - 4$, for $1 \le i \le n$

$f^*(u_iv_{i+1}) = k + 4i - 2$, for $1 \le i \le n-1$

The above defined function f provides k-harmonic mean labeling of the graph.

Hence TL_n is a k-harmonic mean graph.

Example 2.3



700-harmonic mean labeling of TL_8

Theorem 2.4

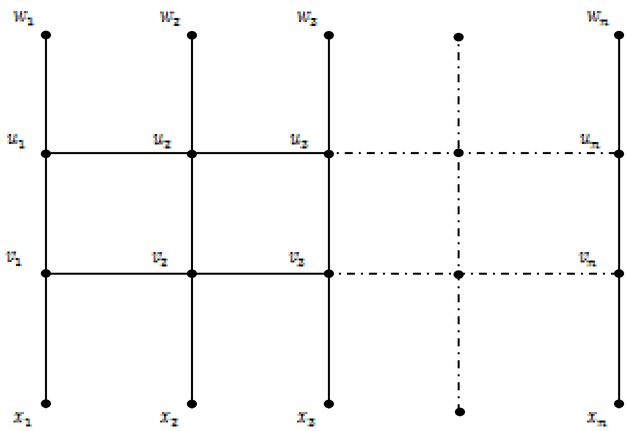
$L_n \odot k_1$ is k-harmonic mean graph for all k and n ≥ 2.

Proof

Let $V(L_n \odot k_1) = \{u_i, v_i, w_i, x_i \,;\, 1 \le i \le n\}$ and

$E(L_n \odot k_1)=\{u_iv_i, u_iw_i, v_ix_i; 1\le i\le n,\ u_iu_{i+1},\ v_iv_{i+1},\ 1 \le i \le n-1\}$

The ordinary labeling is



First we label the vertices as follows

Define a function $f : V(L_n \odot k_1) \to \{k, k+1, k+2, \dots k+q\}$ by

$f(u_i) = k + 5i - 3$, for $1 \le i \le n$

$f(v_i) = k + 5i - 4$, for $1 \le i \le n$

$f(w_i) = k + 5i - 2$, for $1 \le i \le n$

$f(x_i) = k + 5i - 5$, for $1 \le i \le n$

Then the induced edge labels are

$f^*(u_iu_{i+1}) = k + 5i - 1$, for $1 \le i \le n-1$

$f^*(v_iv_{i+1}) = k + 5i - 2$, for $1 \le i \le n-1$

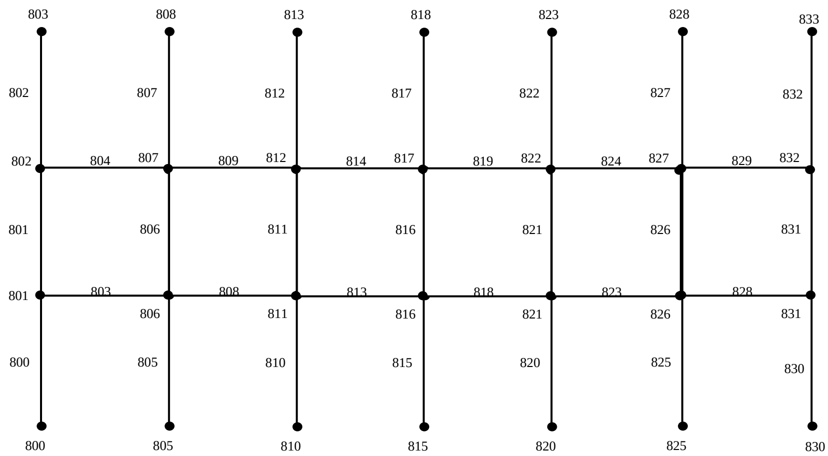
$f^*(u_iv_i) = k + 5i - 4$, for $1 \le i \le n$

$f^*(u_iw_i) = k + 5i - 3$, for $1 \le i \le n$

$f^*(v_ix_i) = k + 5i$, for $1 \le i \le n$

The above defined function f provides k-harmonic mean labeling of the graph. Hence $L_n \odot k_1$ is a k-harmonic mean graph.

Example 2.4



800-harmonic mean labeling of $L_7 \odot k_1$

Theorem 2.5

A graph obtained by attaching a triangle at each pendent vertex of a comb is k-harmonic mean graph for all k.

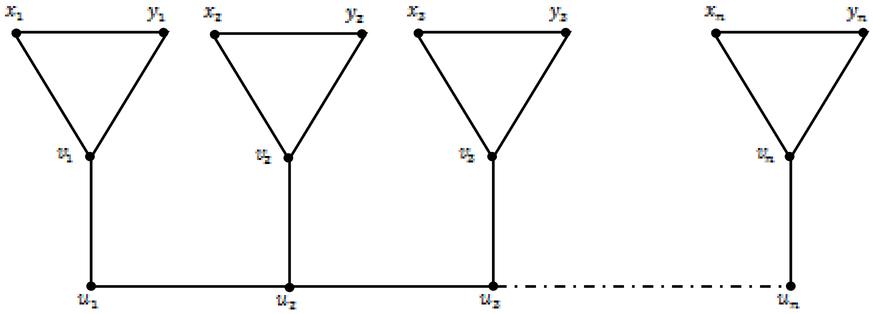
Proof

Let G be a graph obtained by attaching a triangle K_3 at each pendent vertex of $P_n \odot k_1$.

Let u_i, v_i be the vertices of the comb $P_n \odot k_1$ in which v_i is joined with the vertex u_i of P_n and let x_i, y_i, z_i be the vertices of i^{th} copy of K_3. Identify z_i with v_i, $1 \le i \le n$. The resultant graph is G whose edge set is

$E = \{u_iu_{i+1} \,;\, 1 \le i \le n-1\} \cup \{u_iv_i, v_ix_i, v_iy_i, x_iy_i \,;\, 1 \le i \le n\}$

The ordinary labeling is



Define a function $f: V(G) \rightarrow \{k, k+1, k+2, ...k+q\}$ by

$f(u_i) = k + 5i - 3$, for $1 \le i \le n$

$f(v_i) = k + 5i - 2$, for $1 \le i \le n$

$f(x_i) = k + 5i - 5$, for $1 \le i \le n$

$f(y_i) = k + 5i - 4$, for $1 \le i \le n$

Then the induced edge labels are

$f^*(u_iu_{i+1}) = k + 5i - 1$, for $1 \le i \le n-1$

$f^*(u_iv_i) = k + 5i - 2$, for $1 \le i \le n$

$f^*(v_ix_i) = k + 5i - 4$, for $1 \le i \le n$.

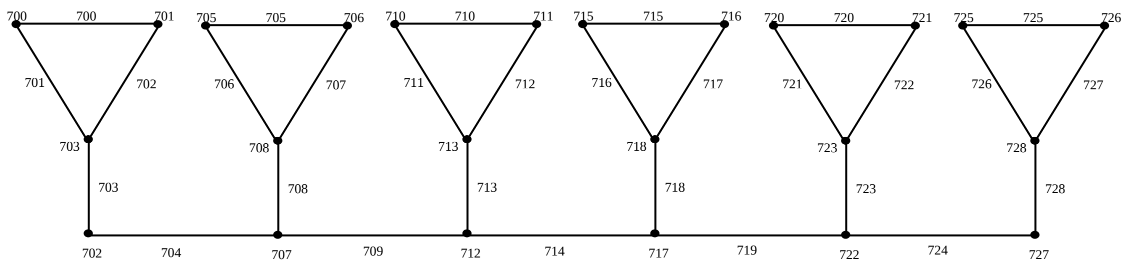
$f^*(v_iy_i) = k + 5i - 3$, for $1 \le i \le n$.

$f^*(x_iy_i) = k + 5i - 5$, for $1 \le i \le n$.

The above defined function f provides k- harmonic mean labeling of the graph.

Hence the graph G is k- harmonic mean graph.

Example 2.5:



700- harmonic mean labeling of G

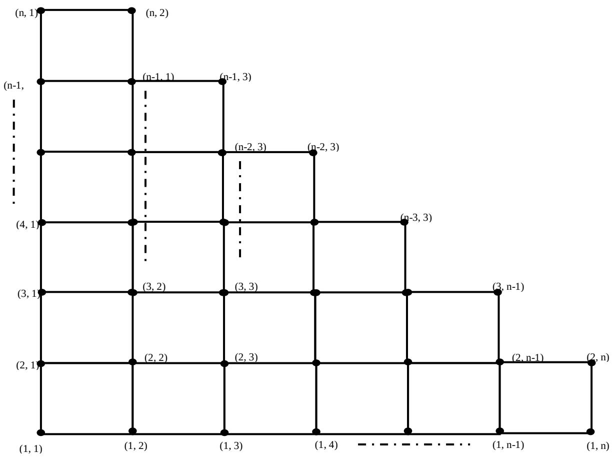
Theorem 2.6:

The Step ladder $S(T_n)$ is k – harmonic mean graph for all k.

Proof:

$V(S(T_n))$ = {(1, 1), (1, 2), ... (1, n), (2, 1), (2, 2),...(2, n), (3, 1), (3, 2), ... (3, n-1), ..., (n, 1), (n, 2)}

The ordinary labeling is



First we label vertices as follows

Define a function f: $V(S(T_n)) \to \{k, k+1, k+2, \ldots., k+q\}$ by

$f(i, 1) = k + n^2 + i - 2$, for $1 \le i \le n$

$f(1, j) = k + (n - j + 1)^2 - 1$, for $2 \le i \le n$

$f(i, j) = k + (n - j + 1)^2 + i - 2$, for $2 \le i \le n$, $2 \le j \le n-j+2$

Then the induced edge labels are

$f^*((i, 1), (i+1, 1)) = k + n^2 + i - 2$, for $1 \le i \le n - 1$

$f^*((1, j), (1, j+1)) = k + (n - j)(n - j + 1) + i - 1$, for $1 \le j \le n - 1$

$f^*((i, j), (i, j+1)) = k + (n - j)(n - j + 1) + i - 2$, for $2 \le i \le n$, $1 \le j \le n - j + 1$

$f^*((i, j), (i+1, j)) = k + (n - j + 1)^2 + i - 2$, for $2 \le j \le n$, $1 \le i \le n - j + 1$

The above defined function f provides k-harmonic mean labeling of the graph.

Hence $S(T_n)$ is k-harmonic mean graph.

Example2.6

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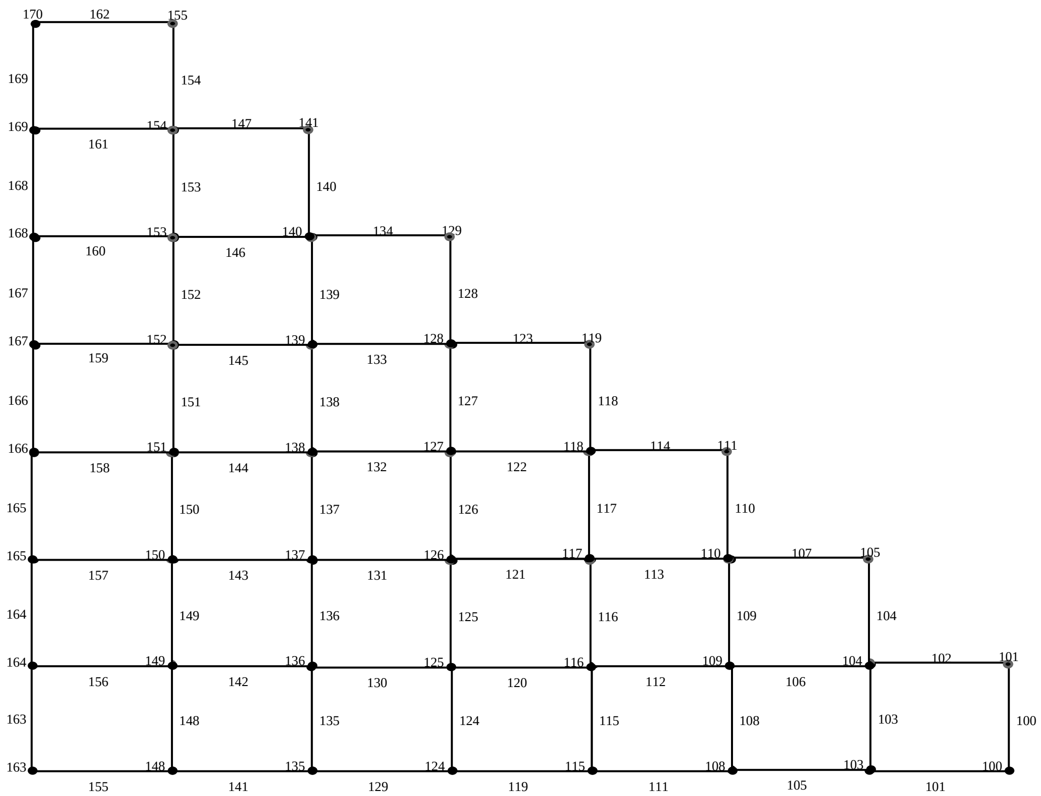

100 – harmonic mean labeling of $S(T_8)$

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