Discrete Heat Equation Model with 5-variables

G.Pavithra[1] , C.Glorireena [2], P.Thiruselvi[3].
Assistant Professors and Department of mathematics,
Adhiyaman Arts And Science college For Women,
Srinivasanagar ,Uthangarai (P.O & T.k),Krishnagiri (D.t).Pin Code: 635207.

ABSTRACT: *We investigate the generalized partial difference equation operator and propose a model of it in discrete heat equation in this paper. The diffusion of heat is studied by the application of newton's law of cooling in dimension up to five and several solutions are postulated for the same. Through numerical simulations solutions are validated and applications are derived.*

KEYWORD: *Generalized partial difference equation partial difference, operator and discrete heat.*

I. INTRODUCTION

In 1984 Jerzy popenda introduced the difference operator Δdefined on u(k) as

$\underset{\alpha}{\Delta} u(k) = u(k+1) - \propto u(k)$.In 1989, Miller and Rose introduced the discrete analogue of the Rieman-Liouville fractional derivative and proved some properties of the inverse fractional difference operator Δ_l^{-1} several formula on higher order partial sums on arithmetic geometric progressions and products of n-consecutive terms of arithmetic progression have been derived.

In the extended the definition of $\underset{\alpha}{\Delta}$ to $\underset{\alpha(l)}{\Delta}$ defined as $\underset{\alpha(l)}{\Delta} = v(k+l) - \alpha v(k)$ for the real valued function v(k)$l > 0$, In have applied q-difference operator different as $\Delta_q v(k) = v(qk) - v(k)$ and obtained finite series formula for logarithmic function. The difference operator $\underset{k(l)}{\Delta}$ with variable co-efficient defined equation $\underset{k(l)}{\Delta} v(k) = v(k+l) - kv(k)$ equation is established. Here we extend the operator $\underset{l}{\Delta}$ to a partial difference operator

Partial difference and differential equation play a vital role in heat equations. The generalized difference operator with n-variables.

$l = (l_1, l_2, l_3, \ldots . l_n) \neq 0$ on a real valued function $v(k): \xrightarrow{n}$ is defined as

$\underset{(l)}{\Delta} v(k) = v(k_1 + l_1, (k_2 + l_{2,} \ldots \ldots . k_n + l_{n,}) -$
$v(k_1, \ldots . . k_n)$......................(1)

The operator $\underset{(l)}{\Delta}$ becomes generalized partial difference operator if some $l_i = 0$. The equation involving $\underset{(l)}{\Delta}$with at least one $l_i = 0$is called generalized partial difference equation.

A liner generalized partial difference is of the form $\underset{(l)}{\Delta} v(k) = u(k)$. Then the inverse of generalized partial difference equation is

$$v(k) = \underset{(l)}{\overset{-1}{\Delta}} u(k) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where $\underset{(l)}{\Delta}$ is as given in (1)$l_i = 0$ and some i and $u(k): \xrightarrow{n}$ is given function.

A function $u(k): \xrightarrow{n}$ satisfying (2) is called a solution of equation (2) equation (2) has a numerical solution of the form

$v(k) - v(k-ml)\sum_{r=1}^{m} u(k-rl)$.................. (3)

Where $k = k_1, k_2 k_3, k_4, k_5$

$k - rl = k_1 - rl_1, k_2 - rl_2, \ldots, k_n - rl_n,$

m is any positive integer relation (3) is the basic inverse principle with respect to $\underset{(l)}{\Delta}$ [6]. Here, we form partial difference equation for the heat flow transmission in rod, plate and system and obtain its solution.

II. FORMATION OF 5 VARIABLE HEAT EQUATION OF ROD

Consider temperature distribution of a very long rod. Assume that the rod is so long that it can be laid on the top of the set R of real number. Let $v(k_1, k_2, k_3, k_4, k_5)$be the temperature at position(k_1, k_2, k_3). Real time k_4, and density (or pressure)k_5 . Assume that the diffusion rate α is constant throughout the rod Five variable$l > 0$

By the fouier law of cooling, the discrete heat equation of the rod is

$$\underset{(l_4, l_5)}{\Delta} v(k) = \alpha \left[\underset{(\pm l_1, 0)}{\Delta} v(k) + \underset{(\pm l_2, 0)}{\Delta} v(k) + \underset{(\pm l_3, 0)}{\Delta} v(k) \right]$$
.... (4)

Here we derive the temperature formula for $v(k_1, k_2, k_3, k_4, k_5)$ at the general position $(k_1, k_2, k_3, k_4, k_5)$

Theorem:2.1

Assume that there exists a positive integer m and a real number $l_4 > 0$ and $l_5 > 0$ suchthat $v(k_1, k_2, k_3, k_4 - ml_4, k_5 - ml_5)$ and $\underset{(\pm l_1, \pm l_2, \pm l_3)}{\Delta}$

$$v(k_1, k_2, k_3, k_4, k_5) = \underset{(\pm l_1, \pm l_2, \pm l_3)}{u} (k_1, k_2, k_3, k_4, k_5)$$

.... (5)

Proof:

$$\underset{(\pm l_1, \pm l_2, \pm l_3)}{\Delta} v(k_1, k_2, k_3, k_4, k_5) = \underset{(\pm l_1, \pm l_2, \pm l_3)}{u} (k_1, k_2, k_3, k_4, k_5)$$

$$v(k_1, k_2, k_3, k_4, k_5) = \alpha \underset{(l_4, l_5)}{\overset{-1}{\Delta}} \underset{(\pm l_1, \pm l_2, \pm l_3)}{u} (k_1, k_2, k_3, k_4, k_5)$$ ----------------- (6)

The proof of (5) follows by applying the inverse principle (3) in (6)

Example:2.2

From (2) we get,

Taking $v(k_1, k_2, k_3, k_4, k_5)$=$e^{k_1+k_2+k_3+k_4+k_5}$ in (6), (5)

$$\left(\frac{k_4+k_5}{l_4+l_5} - \frac{k_4-ml_5+k_5-ml_5}{l_4+l_5}\right) + \left(\frac{k_4}{l_4} - \frac{k_4-ml_4}{l_4}\right) + \left(\frac{k_5}{l_5} - \frac{k_5-ml_5}{l_5}\right) + \left(\frac{e^{k_4}}{e^{l_5-1}} - \frac{e^{k_5-ml_5}}{e^{l_5-1}}\right) + \left(\frac{e^{k_4+k_5}}{e^{l_4+l_5}} - \frac{e^{k_4-ml_5+k_5-ml_5}}{e^{l_4+l_5-1}}\right) = 1 + 1 + 1 + e^{k_4-rl_4} + e^{k_5-rl_5} + e^{k_4-rl_4+k_5-rl_5}$$ ------------------- (7)

The Numerical verification of $m = 1, k_4 = 3, k_5 = 2, l_4 = 4, l_5 = 1, r = 1$ in $(7) we$ get

$$7.086161269 = 7.086161269$$

Theorem: 2.3

Consider (4) and denote $v(k_1 \pm l_1, k_2, k_3, k_4$-$l_4, k_5 * l_5 = v(k_1 + l_1, k_2 * k_3) + v(k_1 - l_1, k_2 * k_3)$

Here *= k_3,k_4-l_4, $k_5 * l_5$

V$(k_1$,$k_2 \pm l_2$,k_3,$*$) =
V$(k_1, k_2 \pm l_2, k_3$,$*$)+V$(k_1$,$k_2 - l_2$,k_3,$*$)

V$(k_1$,k_2,$k_3 \pm l_3$,$*$)=V$(k_1$,k_2,$k_3 + l_3$,$*$)+V$(k_1$,k_2,$k_3 - l_3$,$*$)……………..(8)

Then the following four type's solutions of the equation (4) are equivalent.

a) $v(k_1, k_2, k_3, k_4, k_5)$=$\frac{\propto}{(1-6\propto)^m}$$[v(k_1, k_2, k_3 + l_3, k_4 + ml_4, k_5 + ml_5] -$ $\sum_{r=1}^{\propto} \frac{\propto}{(1-6\propto)^{r+1}}$[v($k_1 \pm l_1$,$*$) + $v(k_1, k_2 \pm l_2, *$) + $v(k_1, k_2, k_3 \pm l_3$ $*$)]-$\frac{\propto}{(1-6\propto)}$[v($k_1 \pm l_1$,$*$) + $v(k_1, k_2 \pm l_2, *)$ + $v(k_1, k_2, k_3 \pm l_3$ $*$)] ………………….(9)

b) $v(k_1, k_2, k_3, k_4, k_5)$ = $\frac{\propto}{(1-6\propto)}$(v($k_1$,k_2 , $k_3 + l_3$, $k_4 \pm l_4$,$k_5 \pm l_5$)-$\frac{\propto}{(1-6\propto)^{m+1}}$ $v(k_1 - l_1, k_2, k_3$,$k_4 \pm ml_4$, $k_5 \pm ml_5$)

$-\sum_{r=1}^{\propto}\frac{\propto^2}{(1-6\propto)^r}v$([v($k_1+l_1$, k_2, $k_3\pm l_3$*)+v(k_1-2_1, $k_2\pm l_2$, sk_3,*)-$\frac{\propto}{(1-6\propto)}$v($k_1+l_1$, k_2,k_3,*)+v(k_1 , $k_2\pm l_2$, k_3,*)+(v(k_1 , k_2 , k_3+l_3,*)]. ………………(10)

c) $v(k_1,k_2,k_3,k_4,k_5)=\frac{\propto}{(1-6\propto)}$[v($k_1$, k_2 , k_3+l_3, $k_4\pm l_4$, $k_5\pm l_5$)- $v(k_1-l_1,k_2,k_3,k_4,k_5)$]

- $\frac{\propto}{(1-6\propto)^{m+1}}v(k_1+l_1,k_2,k_3,k_4\pm ml_4,k_5\pm ml_5)$- $\sum_{r=1}^{\propto}\frac{\propto^2}{(1-6\propto)^r}$[v($k_1,k_2,k_3$,*)($k_1+l_1$, k_2,k_3,*)+v(k_1 , $k_2\pm l_2$, k_3,*)+(v(k_1 , k_2 , k_3+l_3,*)]-$\frac{\propto}{(1-6\propto)}$v($k_1$, $k_2\pm l_2$, k_3,*)+(v(k_1,k_2,k_3+l_3,*)] ……………(11)

d)$v(k_1,k_2,k_3,k_4,k_5)$=$\frac{\propto}{(1-6\propto)}$[v($k_1,k_2,k_3,k_4\pm l_4$, $k_5\pm l_5$)-v(k_1+l_1, k_2,k_3,*)-v(k_1 , $k_2\pm l_2$, k_3,*)+v(k_1 , k_2 , k_3+l_3,*)]-$\frac{\propto}{(1-6\propto)^{m+1}}$($k_1$, k_2-l_2, k_3, $k_{4+}ml_4$, $k_{5+}ml_5$)- $\sum_{r=1}^{\propto}\frac{\propto^2}{(1-6\propto)^r}$[v($k_1\pm l_1$, k_2-l_2, $k_{3,*}$)+v(k_1, k_2-2l_2, k_3,*)+v(k_1,k_2,k_3,*)+v(k_1,k_2-l_2,$k_3+l_{3,*}$)]. ………..(12)

e) $v(k_1,k_2,k_3,k_4,k_5)$=$\frac{\propto}{(1-6\propto)}$[v($k_1,k_2,k_3,k_4+l_4$, k_5+l_5)-v(k_1+l_1, k_2,k_3,*)+v(k_1 , k_2-l_2 , k_3,*)+(v(k_1 , k_2,k_3+l_3,*)- $\sum_{r=1}^{\propto}\frac{\propto^2}{(1-6\propto)^r}$[v($k_1\pm l_1$,$k_2+l_2$,$k_{3,*}$)+v$(k_1,k_2,k_3,*)$ +v (k_1,$k_2\pm l_2$,k_3,*)+v(k_1,k_2,k_3+l_3,*)]. ……………(13)

f)$\boldsymbol{v}(k_1,k_2,k_3,k_4,k_5)$ = $\frac{\propto}{\mathbf{(1-6\propto)}}$[v($k_1,k_2,k_3,k_4\pm \boldsymbol{l}_4$, $k_5\pm \boldsymbol{l}_5$)- v(k_1+l_1, k_2,k_3,*)- v(k_1 , $k_2\pm \boldsymbol{l}_2$, k_3,*)+v(k_1 , k_2 , $k_3+\boldsymbol{l}_3$,*)]-$\sum_{r=1}^{\propto}\frac{\propto^2}{(1-6\propto)^r}$[v($k_1$, k_2 , k_3-$\boldsymbol{l}_3$,$k_4\pm ml_4$, $k_5\pm ml_5$)-$\sum_{r=1}^{\propto}\frac{\propto^2}{(1-6\propto)^r}$[v($k_1\pm \boldsymbol{l}_1$, k_2, k_3-$l_{3,*}$)+v(k_1, $k_2\pm \boldsymbol{l}_2$, $k_3\pm \boldsymbol{l}_3$,*)] ....(14)

g).v(k_1,k_2,k_3,k_4,k_5)= $\frac{\propto}{(1-6\propto)}$[u($k_1$,$k_2$, k_3,k_4+l_4,k_5+l_5)- v($k_1\pm l_1$, k_2 , k_3)-v(k_1 , $k_2\pm l_2$, k_3)- v(k_1 , k_2 , $k_3\pm l_3$]- $\frac{\propto}{(1-6\propto)^{m+1}}$[u($k_1$, k_2 , k_3+l_3, $k_4\pm ml_4$, $k_5\pm ml_5$)- $\sum_{r=1}^{\propto}\frac{\propto^2}{(1-6\propto)^r}$[v($k_1\pm l_1$, $k_2\pm l_2$, $k_3\pm l_3$)+v(k_1, $k_2\pm l_2$, $k_3\pm l_3$)+v(k_1 , k_2 k_3+ $2l_3$,)+v(k_1 , k_2 , k_3)]

.... (15)

Proof

a) From (4) we arrive the relation v(k_1 , k_2 , k_3, k $_4$,k_5)=$\frac{\propto}{(1-6\propto)}$ v (k_1 , k_2 , k_3, k_4+l_4, k_5+l_5)-$\frac{\propto}{(1-6\propto)}$ v (k_1 , k_2 , k_3,∗)-(vC k_1 , $k_2\pm l_2$, k_3,∗)-v(k_1 , k_2 , k_3+l_3,∗)]………………(16)

By replacing k $_4$ by k_4+l_4 and k_5 by k_5+l_5 in (16) gives expressions for v($k_1\pm l_1$, k_2 , k_3, k_4+l_4, k_5+l_5) and k_1 , k_2 , k_3+l_3, $k_{4+2}l_4$, k_5+2l_5),v(k_1 , k_2 , $k_3\pm l_3$, $k_4\pm l_4$, $k_5\pm l_5$), v(k_1 , k_2 , $k_3\pm l_3$, k_4+l_4,k_5+l_5).

Now proof of (a) follows by applying all these values in(16).

b) The heat equation (4) directly derives the relation.

In equation (16) Replacing k_1 by k_1-l_1 in (16) and substituting corresponding values in (16)yields (b).

c) The proof of (c) follows by replacing k_1 by k_1+l_1 in (16)and substitution corresponing values in (16) v(k_1 , k_2 , k_3, k $_4$,k_5)=$\frac{\propto}{(1-6\propto)}$[u($k_1$, k_2 , k_3,k_4+l_4,k_5+l_5)-$\frac{\propto}{(1-6\propto)}$ [v (k_1-l_1 , k_2 , k_3,∗)+ v(k_1 , $k_2\pm l_2$, k_3,∗)]- $\frac{\propto}{(1-6\propto)^2}$ v(k_1 , k_2 , k_3,k_4+l_4,k_5+l_5)+$\frac{\propto^2}{(1-6\propto)^2}$ V[(k_1+l_1, $k_2\pm l_2$, k_3,∗)+(k_1+l_1, k_2, $k_3\pm l_3$,∗)].

d) The proof of (d) follows by replacing k_2 by k_2-l_2 in (16) and we get the proof (12).

e) The proof of (e) follows by replacing k_2 by k_2+l_2 in (16) and we get the proof (13).

f) The proof of (f) follows by replacing k_3 by k_3-l_3 in (16).and we get the proof (14).

g) The proof of (g) follows by replacing k_3 by k_3+l_3 in (16).and we get the proof (15).

Example:2.4

The following example shows that the diffusion rate of rod can be identified if the solution

v(k_1 , k_2 , k_3, k $_4$,k_5) of (4) is known and vice versa.

Suppose that v(k_1 , k_2 , k_3, k $_4$,k_5)=$2^{(k1+k2+k3+k\,4+k5)}$ is a closed from solution of (4) then we have the relation .

$$2^{k1+k2+k3+k4+l4+k5+l5}-2^{k1+k2+k3+k\,4+k5}=\propto[2^{k1-l1+k2+k3+k4+k5}+2^{k1+l1+k2+k3+k\,4+k5}+2^{k1+k2-l2+k3+k4+k5}+2^{k1+k2+k3+k\,4+k5}+2^{k1+k2+k3-l3+k4+k5}+2^{k1+k2+k3+l3+k\,4+k5}-62^{k1+k2+k3+k\,4+k5}]$$

Cancelling $2^{k1+k2+k3+k\,4+k5}$on both sides derives.

$$\propto=2^{l4+l5}-1$$

$=2^{-l1}+2^{l1}+2^{-l2}+2^{l2}+2^{-l3}+2^{l3}-6$

Taking $k_1=2$, $k_2=3$, $k_3=1$, $l_1=3$, $l_2=2$, $l_3=1$, $m=1$ in (a),(b),(c),(d),(e),(f),we get the solution 65536=65536.

III.CONCLUSION

The study of partial difference operator has wide applications in discrete fields and heat equation is one such theorem 2.1 & 2.3 provide the possibility of predicating the temperature either for the past of the future getting the know the temperature at few finite points at present terms. It is also show the nature of transmission of heat for the material study. Thus in conclusion, we can say that the above research help us in reducing any wastage of heat and also enables us in material of optimal choice of material (α).

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