Relativistic Magneto hydrodynamic Shocks In a Gyrotropic Plasma

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Abstract: *We have derived the general jump conditions for a relativistic magnetohydrodynamic shock in a gyrotropic plasma with an arbitrary magnetic field oriented toward the shock normal.*

Keywords: Tensor, magnetohydrodynamic (MHD), relativistic theory, shock waves

Introduction:

Colburn and Sonett [3], Kunkel and Brown [7], investigated the problem of collisionless shock generation theoretically as well as experimentally. Treumann and Jaroschak [12] presented that velocity anisotropic distributions (VADs) of plasma particles play an important role in the shock formation process. Spitkovsky [11], Sironi and Spitkovsky [10] studied the important role of shock-reflected charged particles in the shock formation process. Majorana & Anile [9] derived the jump condition and presented its solution numerically for fast and slow shocks. Double et al [4] presented the study of relativistic magnetohydrodynamic fast shocks to the case of anisotropic pressure in the down-stream region by using an approximate form for the adiabatic index.

We have derived the general jump conditions for a relativistic magnetohydrodynamic shock in a gyrotropic plasma with an arbitrary magnetic field oriented toward the shock normal.

Magnetohydrodynamic Jump Conditions:

Ideal relativistic magnetohydrodynamic equations for a gyrotropic plasma was presented by Cissoko [2]. The mass four-flow of a plasma reads.

$$M^k = \rho u^k \tag{1}$$

where ρ be the rest-mass density and u^i as a four velocity of the fluid normalised with the speed of light c

$$u^i = \gamma(1,\beta), \tag{2}$$

such that continuity equation

$$M^k{}_{;k} = 0, \tag{3}$$

Where $$\gamma = \left(1-\beta^2\right)^{-1/2} \tag{4}$$

as the Lorentz factor of the fluid velocity $\underset{\sim}{u}$ and $\underset{\sim}{\beta} = \frac{\underset{\sim}{u}}{c}$. .

In the case of perfect magnetohydrodynamic the dual of the electromagnetic tensor ${}^*F^{ik}$ reads

$${}^*F^{ik} = h^i u^k - h^k u^i, \tag{5}$$

Where $$h_k = u^i \, {}^*F_{ik}. \tag{6}$$

Hence, one may obtain

$$h^i = \left(-\gamma \underset{\sim}{\beta} . \underset{\sim}{B} - \underset{\sim}{B} - \frac{\gamma - 1}{\beta^2} \underset{\sim}{\beta} \left(\underset{\sim}{\beta} . \underset{\sim}{B} \right) \right). \qquad (7)$$

Therefore, the conservation of magnetic flux reads

$$^{*}F_{ik;k} = 0. \qquad (8)$$

Chew at al [1] investigated that the pressure tensor for an ideal collisionless nonrelativistic plasma in a strong parallel background magnetic field assumes a gyrotropic form in proper frame of plasma

$$\Pi^{\alpha\beta} = \left(P_{\parallel} - P_{\perp} \right) \frac{B_\alpha B_\beta}{B^2} + P_{\perp} \delta_{\alpha\beta}, \qquad (9)$$

where $\Pi^{\alpha\beta}$ be rotational symmetric with respect to the magnetic field vector, $P_{\parallel}$ and $P_{\perp}$ denote the pressure parallel and perpendicular to the magnetic field. This model was generalised by Tsikarishvili et al [14] for the special relativistic case with four pressure tensor

$$\Pi^{ik} = \left(P_{\parallel} - P_{\perp} \right) \frac{h^i h^k}{B^2} - P_{\perp} \left(\eta^{ik} - u^i u^k \right). \qquad (10)$$

Hence, the total energy momentum tensor for relativistic gyrotropic plasma assumes the form

$$T^{ik} = \rho u^i u^k + \Pi^{ik} + \frac{B^2}{4\pi} \left(u^i u^k - \frac{1}{2} \eta^{ik} \right) - \frac{h^i h^k}{4\pi}, \qquad (11)$$

where ρ be the total energy density, $\eta^{ik} = \text{diag } (1,-1,-1,-1)$ be the Minkowski metric for a flat spacetime,

$$B^2 = -h^i h_i \qquad (12)$$

i.e. square of the magnetic field strength.

The conservation of energy momentum tensor reads

$$T^{ik}{}_{;k} = 0. \qquad (13)$$

Double et al [4] presented the rest mass energy density $\in c^2$ is related to total energy density as

$$\rho = \frac{2P_{\perp} + P_{\parallel}}{3(k-1)} + \in c^2 \qquad (14)$$

where k denoted the adiatabic index of a perfect gyrotropic plasma.

The Jump Conditions for a relativistic Magnetohydrodynamic Shock in a gyrotropic Plasma:

It is obvious that all state variables of a plasma change in general across a shock front. In view of equations (3), (8) and (13) one may obtain the Jump conditions for a relativistic magnetohydrodynamic shock in a gyrotropic plasma as

$$\left[M^{k}\right]n_k = 0, \tag{15}$$

$$\left[T^{ik}\right]n_k = 0, \tag{16}$$

$$\left[{}^{*}F^{ik}\right]n_k = 0, \tag{17}$$

where $\left[Q\right] = Q_2 - Q_1$ represents the jump in the physical quantity Q across the shock $n_k = \left(n_o, \underset{\sim}{n}\right)$ is the shock normal. The sub-scripts 1 and 2 denote the upstream and downstream states respectively.

Let us consider a Cartesian coordinate system along the z-axis pointing toward the downstream. The upstream magnetic field has a components in the x and z-directions, without loss of generality. The system becomes plane shock which is expanded in the x-y plane in a steady state. Let us assume a perfect collisionless upstream plasma with infinite conductivity Erkaev et al $[5]$, Liu et al $[8]$investigated ansatz from the nonrelativistic shock theory that the downstream anisotropy $\alpha_2 = P_{\perp 2}/P_{\parallel 2}$ be a fixed quantity. But the downstream adiabatic index K_2 is not well known even for an isotropic plasma.

Parallel Shock:

Let us consider the case that the shock normal is parallel to the magnetic field i.e. transverse magnetic field vanishes. Therefore, the jump conditions for a parallel shock assumes the form

$$[\gamma\ \beta \in] = 0, \tag{18}$$

$$\left[\gamma^2 \beta \left(\rho + P_{\parallel}\right)\right] = 0, \tag{19}$$

$$\left[\gamma^2 \beta^2 \left(\rho + P_{\parallel}\right) + P_{\parallel}\right] = 0, \tag{20}$$

$$[B] = 0. \tag{21}$$

Let us define dimensionless parameters as,

$$r = \beta_1/\beta_2\,, \tag{22}$$

$$M_{A1} = \left(\frac{4\pi \in_1 \beta_1^2 c^2}{B_1^2}\right)^{1/2}, \tag{23}$$

$$M_{s1} = \left(\frac{\in_1 \beta_1^2 c^2}{K_1 P_1}\right)^{1/2}, \tag{24}$$

where r as the compression ratio, M_{A1} be the Alfven Mach number and M_{s1} as the sonic Mach number.

In view of eqs. (18) - (21), one obtains an analytic solution for the parallel downstream plasma beta

$$\xi_{\parallel 2} = \frac{3\xi_1}{2\alpha_1 + 1} + \gamma_1^2 \beta_1^2 \left(1 - \tfrac{1}{r}\right) \times \left[\left(\frac{1}{K_1 - 1} + \frac{3}{2\alpha_1 + 1}\right)\xi_1 + 2\frac{M_{A1}^2}{\beta_1^2}\right], \tag{25}$$

where $\quad \xi_1 = \left(\xi_{\parallel 1} + 2\xi_{\perp 1}\right)/3$

and the parallel and perpendicular plasma beta are

$$\xi_{\parallel} = 8\pi\, P_{\parallel} / B^2 , \tag{26}$$

$$\xi_{\perp} = 8\pi\, P_{\perp} / B^2 . \tag{27}$$

The downstream anisotropy reads

$$\alpha_2 = \left[\frac{\gamma_1^2}{r}\left(r^2 - \beta_1^2\right)\left(\frac{1}{K_1 - 1} + \frac{3}{2\alpha_1 + 1}\right)\xi_1 + \frac{2M_{A1}^2}{\beta_1^2}\right]$$
$$-\left[2\gamma_1\sqrt{r^2 - \beta_1^2}\,\frac{M_{A1}^2}{\beta_1^2} - \frac{\left(2k_2 - 2\right)}{3\left(K_2 - 1\right)}\xi_{\parallel 2}\right]\frac{3\left(K_2 - 1\right)}{2\xi_{\parallel 2}}, \tag{28}$$

where $\quad K_2 = \frac{1}{3}\left(4 + \frac{\sqrt{r^2 - \beta_1^2}}{\gamma_1\left(r - \beta_1^2\right)}\right).\qquad (29)$

Perpendicular Shock:

The jump conditions for perpendicular shock reads

$$[\gamma\ \beta\in] = 0, \tag{30}$$

$$\left[\gamma^2\beta\left(\rho + P_{\perp} + \frac{B^2}{4\pi}\right)\right] = 0, \quad (31) \qquad \left[\gamma^2\beta^2\left(\rho + P_{\perp} + \frac{B^2}{8\pi}\right) + P_{\perp} + \gamma^2\frac{B^2}{8\pi}\right] = 0, \quad (32)$$

$$[\gamma\ \beta\ B] = 0. \tag{33}$$

Solving above set of equations, one obtains the perpendicular downstream plasma beta

$$\xi_{\perp 2} = \frac{3\alpha_1}{2\alpha_1 + 1}\frac{\xi_1}{\gamma_1^2\left(r^2 - \beta_1^2\right)} - \frac{r^2 - 1}{r^2 - \beta_1^2} + \frac{r-1}{r\left(r^2 - \beta_1^2\right)} \times \left[\left(\frac{1}{K_1 - 1} + \frac{3\alpha_1}{2\alpha_1 + 1}\right)\xi_1\beta_1^2 + 2\left(M_{A1} + \beta_1^2\right)\right], \tag{34}$$

And the downstream anisotropy

as $\alpha_2 = \frac{\xi_{\perp 2}}{3\left(K_2 - 1\right)}\left[\frac{1}{r}\left(\left(\frac{1}{K_1 - 1} + \frac{3\alpha_1}{2\alpha_1 + 1}\right)\xi_1 + 2 + 2\frac{M_{A1}^2}{\beta_1^2}\right) - \frac{2M_{A1}^2}{\gamma_i\beta_1^2\sqrt{r^2 - \beta_1^2}} - 2 - \frac{3K_2 - 1}{3\left(K_2 - 1\right)}\xi_{\perp 2}\right]^{-1}. \quad (35)$

Concluding Remarks:

We have presented relativistic jump conditions in a magnetised anisotropic plasma for a parallel and perpendicular shock front. We have taken upstream flow velocity as a normal to the shock front. On the basis of the downstream anisotropy

parameter $\alpha_2 = P_{\perp 2} / P_{\parallel 2}$, we have obtained the jump conditions in a magnetised anisotropic plasma. Since the downstream anisotropy is commonly unknown but it was supposed as a parameter to get a closed system of equations and hence, we have followed the work of Erkaev et al [5] for non relativistic shock to use the onset conditions for plasma instabilities developing in the downstream medium to constrain the allowed value of the downstream anisotropy. Using the firehose instability condition $\alpha < \alpha_f \left(\alpha_f = 1 - 2/\xi_{\parallel}\right)$ and the mirror instability condition $\alpha > \alpha_m \left(\alpha_m = 1 + 1/\xi_{\perp}\right)$ combined with the solutions of jump conditions, resulted in thresholds for the downstream anisotropy.

References:

[1].Chew, G.F., Goldberger, M.L. & Low, F.E.(1956), Proc. R. Soc. A, 236, 112.

[2].Cissoko, M. (1975), Ann. Inst. Henri Poincare, 22, 1.

[3].Colburn, D.S. & Sonett, C.P. (1966), Space Sci. Rev., 5, 439.

[4].Double, G.P. et al (2004), ApJ, 600, 485.

[5].Erkaev, N.V. et al (2000), J. Plasma Phys. 64, 561.

[6].Erkaev, N.V., Vogl, D.F., & Biernat, H.K. (2000), J. Plasma Phys. 64, 561.

[7].Kunkel, W.B. and Brown, S.C. (1967), Phys. Today, 20, 88.

[8].Liu, Y., Richardson, J.D., Belcher, J.W. and Kasper, J.C. (2007), ApJ, 659, L65.

[9].Majorana, A. & Anile, A.M. (1987), Phys. Fluids,30,3045.

[10].Sironi, L. and Spitkovsky, A. (2009), ApJ, 707, L92.

[11] Spitkovsky, A. (2008), ApJ, 682, L5.

[12].Treumann, R.A. & Jaroschek, C.H. (2011), Physics of Collisionless Shocks, ed. A. Balogh, K.L. Klein, & R.A. Treumann (ISSI Scientific Report Ser. SR-10, New York: Springer), in press (arXiv: 0805-2132).

[13] Treumann, R.A. and Jarosckek, C.H. (2011), arXiv: 0805, 2132.

[14] Tsikarishvili, E.G., Lominadze, J.G., & Javakhishvili, J.I. (1994), Phys. Plasmas, 1, 150.