Changing And Unchanging Properties Of Single Chromatic Transversal Domination Number Of Graphs

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$G = (V, E)$ is called a dominating set if for every vertex $v \in V - D$, there exists a vertex $u \in D$ such that u and v are adjacent. The cardinality of a minimum dominating set is called the domination number of a graph and is denoted by $\gamma(G)$. A dominating set D is called a single chromatic transversal dominating set if D intersects every member (color class) of some chromatic partition, also called χ-partition of G.This set is called an std-set. The cardinality of a minimum std-set is called the single chromatic transversal domination number of a graph G and is denoted by $\gamma_{st}(G)$. The effect of the removal of an edge, a vertex or an addition of an edge of a graph over the γ_{st} value is studied.

Keywords - Domination number, single chromatic transversal domination number.

I. Introduction

By a graph $G = (V, E)$, we mean a finite, undirected graph with neither loops nor multiple edges. The order and size of G are denoted by n and m respectively. For graph theoretic terminology we refer to Gary Chatrand [5].

Coloring and domination are two areas in graph theory which have been extensively studied. Graph coloring deals with the fundamental problem of partitioning vertex set into classes according to certain rules. Time tabling, sequencing and scheduling problems in their many terms are basically of this nature. The fundamental parameter in graph coloring is the chromatic number $\chi(G)$ of a graph G which is defined to be the minimum number of colors required to color vertices of G receive the same color. A partition of $V(G)$ into $\chi(G)$ independent sets is called a chromatic partition or χ-partition of G.

Another fastest growing area in graph theory is the study of domination and related subset problems such as independence, covering and matching. A set $D \subseteq V(G)$ is said to be a dominating set of G if every vertex in $V(G) - D$ is adjacent to a vertex in D. The minimum cardinality of a dominating set is called the domination number of G and is denoted by $\gamma(G)$. A comprehensive treatment of the fundamentals of domination is given in the book by Haynes et al[6]. A survey of several advanced topics in domination can be seen in Haynes et al [7]. Benedict et al.[1] introduced the concept of chromatic transversal domination using the concept of graph coloring and domination. A dominating set D of a graph G is called a chromatic transversal dominating set (ctd-set) if D is a transversal of every χ-partition of G. That is, D has non-empty intersection with every color class of every χ-partition of G. The minimum cardinality of a std-set of G is called chromatic transversal domination number of G and is denoted by $\gamma_{ct}(G)$. Obviously $\chi(G) \leqslant \gamma_{ct}(G)$.

Restricting a dominating set to be a transversal of at least one χ-partition of G, a new domination parameter namely single chromatic transversal domination parameter was defined by Lawrence et al.[8]. Accordingly a dominating set D is called a single chromatic transversal dominating set if D intersects every member (color class) of some χ-partition of G. This set is called an std-set. The cardinality of a minimum std-set is called the single chromatic transversal domination number of a graph G and it is denoted by $\gamma_{st}(G)$.

The following are the exact values of the parameter $\gamma_{st}(G)$ for some standard graphs found in [1] and [8]

***Result* 1.1.**

1. $\gamma_{st}(P_n) = \lceil \frac{n}{3} \rceil$, for all $n \geqslant 4$.
2. $\gamma_{st}(C_n) = \lceil \frac{n}{3} \rceil$, for all $n \geqslant 6$.
3. $\gamma_{st}(W_n) = \begin{cases} 3 & \text{if } n \text{ is odd,} \\ 4 & \text{if } n \text{ is even.} \end{cases}$
   where W_n is a wheel with $(n-1)$ spokes.
4. For the Petersen graph P, $\gamma_{st}(P) = 4$, whereas $\gamma_{ct}(P) = 5$.

***Theorem* 1.2.** [1] *Let G be a connected bipartite graph G with bipartition (X, Y); $|X| \leqslant |Y|$ and $n \geqslant 3$. Then $\gamma_{st}(G) = \gamma(G) + 1$ if and only if every vertex in X has at least two neighbours which are pendant vertices. Here X is the unique γ set of G.*

For a bipartite graph G, $\gamma_{st}(G) = \gamma(G)$ or $\gamma(G)+1$. All connected bipartite graphs for which $\gamma_{st}(G) = \gamma(G) + 1$ are called Type-2 graphs and all other bipartite graphs are Type-1 graphs. Note that a disconnected bipartite graph is a Type-1 graph. Every path P_n and even cycle C_n are Type-1 bipartite graphs.

II. Main Theorems

In this section now we study the effect of the removal of an edge, a vertex or an addition of an edge of a graph over the γ_{st} value

We use the following terminology $C-$Changing, $U-$Unchanging, $V-$Vertex, $E-$Edge, $A-$Addition, $R-$Removal

***Definition* 2.1.** A graph G is said to be a

i. CVR if $\gamma_{st}(G-u) \neq \gamma_{st}(G)$ for each vertex $u \in V(G)$

ii. CER if $\gamma_{st}(G-e) \neq \gamma_{st}(G)$ for each edge $e \in E(G)$

iii. CEA if $\gamma_{st}(G+e) \neq \gamma_{st}(G)$ for each edge $e \in E(\overline{G})$

iv. UVR if $\gamma_{st}(G-u) = \gamma_{st}(G)$ for each vertex $u \in V(G)$

v. UER if $\gamma_{st}(G-e) \neq \gamma_{st}(G)$ for each edge $e \in E(G)$

vi. UEA if $\gamma_{st}(G+e) \neq \gamma_{st}(G)$ for each edge $e \in E(\overline{G})$

***Theorem* 2.2.** *All the odd cycles are CEA graph*

Proof. C_3 and C_5 are CEA graph. Let $n \geq 7$ be an odd integer. Let $C_n = v_1, v_2, ..., v_n, v_1$. n can be expressed in the form $3k$ or $3k+2$, where k is an odd integer and $3k+1$ where k is an even integer. By Result 1.1

$$\gamma_{st}(C_n) = \lceil \frac{n}{3} \rceil = \begin{cases} k & \text{if } n = 3k; \\ k+1 & \text{if } n = 3k+1 \text{ or } 3k+2. \end{cases}$$

Consider an edge $e \in E(\overline{C_n})$. Without loss of generality let $e = v_1v_i, 3 \leq i \leq n$. In $C_n + e$, the edge e divides C_n into two cycles. One is odd and the other is even. Let us assume that the cycle $v_,v_2, ..., v_iv_1$ is an odd cycle and $v_1v_iv_{i+1}, ..., v_nv_1$ is an even cycle in $C_n + e$.

Obviously $\chi(C_n + e) = 3$. Colour $C_n + e$ in such a way that v_1, v_2 and v_i are assigned three distinct colours and all other vertices are coloured using the two colours used for v_1 and v_i. Such a proper colouring is possible for $C_n + e$. This gives a $\chi-$partition for $C_n + e$. When $n = 3k$, take $S = \{v_2, v_5, ..., v_{3k}\}$ and $n = 3k+1$, take $S = \{v_2, v_5, ..., v_{3k-1}, v_{3k+1}\}$ and $n = 3k+2$ take $S = \{v_2, v_5, ..., v_{3k+1}, v_{3k+2}\}$. In the first case S has k elements and in all other cases S has $k+1$ elements. In each S, the first vertex and the last two vertices are assigned the three distinct colours and so S is a transversal of the $\chi-$ partition specified earlier. In each case S is a $\gamma-$ set. Hence S is a γ_{st} set of $C_n + e$. So $\gamma_{st}(G) = \gamma_{st}(G+e)$. □

***Remark*.** When a vertex v is removed from a graph G, $\gamma_{st}(G-v) > \gamma_{st}(G)$ or $\gamma_{st}(G-v) < \gamma_{st}(G)$ or $\gamma_{st}(G-v) = \gamma_{st}(G)$. For example when the centre vertex v of a star graph $K_{1,n}, n \geq 2$ is removed $\gamma_{st}(G-v) = n$ whereas $\gamma_{st}(G) = 2$. Similarly when an edge e is removed from a graph G, $\gamma_{st}(G-e) > \gamma(G)$ or $\gamma_{st}(G-e) < \gamma_{st}(G)$ or $\gamma_{st}(G-e) = \gamma_{st}(G)$.

For the graph given in Figure 1, $\gamma_{st}(G) = 4$, since $\{v_1, v_2, v_3, v_6\}$ is a γ_{st} set of G. Let $e = v_2v_4$. Then

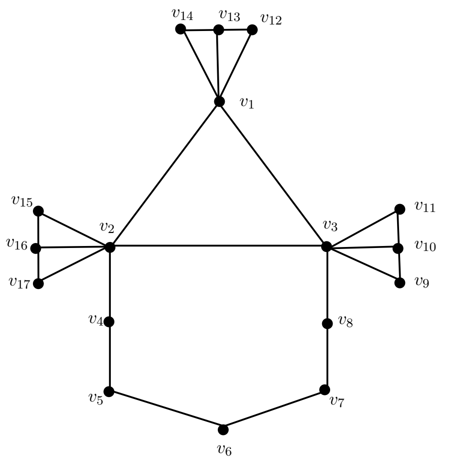

Figure 1.

$\gamma_{st}(G-e) = 5$, since we need an additional vertex to dominate the pendant vertex v_4 in $G-e$. Hence $\gamma_{st}(G-e) > \gamma_{st}(G)$.

Notation. We can partition of $V(G)$ as $V = V^0 \cup V^+ \cup V^-$ where

$$V^0 = \{v \in V / \gamma_{st}(G-v) = \gamma_{st}(G)\}$$

$$V^+ = \{v \in V / \gamma_{st}(G-v) > \gamma_{st}(G)\}$$

$$V^- = \{v \in V / \gamma_{st}(G-v) < \gamma_{st}(G)\}$$

Similarly we can partition of $E(G)$ as $E = E^0 \cup E^+ E^-$ where

$$E^0 = \{e \in E / \gamma_{st}(G-e) = \gamma_{st}(G)\}$$

$$E^+ = \{e \in E / \gamma_{st}(G-e) > \gamma_{st}(G)\}$$

$$E^- = \{e \in E / \gamma_{st}(G-e) < \gamma_{st}(G)\}$$

Theorem 2.3. *If G is a connected Type-2 bipartite graph, then $E^+ = \phi$. In fact E^- the set of all non pendant cut edges of G and $E^0 = E(G) - E^-$.*

Proof. By Theorem 1.2 G becomes a connected Type-II bipartite graph with the bipartition (X, Y) such that $|X| < |Y|$ and every vertex in X has at least two pendant neighbours.

Clearly $\gamma_{st}(G) = \gamma(G) + 1$. Let $e = uv$ be an edge of G. If e is a pendant edge such that $u \in X$ and $v \in Y$. Then v is a pendant vertex and $G-e$ is a disconnected bipartite graph for which $X \cup \{v\}$ is a $\gamma-$ set of G. Now $\gamma_{st}(G-e) = \gamma(G) + 1 = \gamma_{st}(G)$. Therefore $e \in E^0$.

Let $e = uv$ be not a pendant edge of G. Suppose $G-e$ is not disconnected. Then $G-e$ again becomes a connected Type-2 bipartite graph such that $\gamma_{st}(G-e) = \gamma_{st}(G)$. Clearly $e \in E^0$. Let $e = uv$ be a cut edge of G. Clearly $G-e$ is a disconnected graph so that $\gamma_{st}(G-e) = \gamma(G-e) = |X| = \gamma(G) < \gamma_{st}(G)$. So $e \in E^-$. □

Corollary 2.4. *For the star graph $K_{1,l}, l \geq 2, E^0 = E$ and $E^- = \phi$. Moreover $|E^0| = l$.*

Proof. $K_{1,l}$ is a Type-2 bipartite graph and all its edges are pendant edges. By Theorem 2.3 $E^- = \phi$ and $E^0 = E$. □

Corollary 2.5. *If T is a Type-2 tree with k pendant vertices, then E^- and E^0 are non empty sets. Moreover $|E^0| = k$ and $|E^-| = n - (k+1)$*

Proof. By Theorem 2.3

$E^0 =$ The collection of pendant edges of T.

$E^- =$ The collection of non - pendant edges.

Hence $|E^0| = k$ and $|E^-| = (n-1) - k = n - (k+1)$. □

Theorem 2.6. *If G is a connected Type-1 bipartite graph then $E^- = \phi$. In fact $E^+ =$ The set of all $\gamma-$ critical edges of G and $E^0 = E(G) - E^+$*

Proof. When G is a connected Type-1 graph, $G-e$ is also a Type-1 bipartite graph. Hence $\gamma(G) = \gamma_{st}(G)$ and $\gamma(G-e) = \gamma_{st}(G-e)$ for every edge $e \in E(G)$. The removal of an edge from G can not decrease γ value and can increase it by at most one. Hence $\gamma_{st}(G-e) = \gamma_{st}(G)$ or $\gamma_{st}(G) + 1$. Therefore $E^- = \phi$. If $e \in E(G)$ is a $\gamma-$ critical edge of G then $\gamma_{st}(G-e) = \gamma(G-e) > \gamma(G) = \gamma_{st}(G)$. Hence $e \in E^+$. If $e \in E(G)$ is not a $\gamma-$ critical edge of G then, $\gamma(G-e) = \gamma(G)$. This implies that $\gamma_{st}(G-e) = \gamma_{st}(G)$. Hence $e \in E^0$. □

Corollary 2.7. *If G is a hypercube Q_n, then $E^+ = \phi, E^0 = E(G)$ and $|E^0| = 2^{n-1}n$.*

Proof. Hyper cube Q_n is a Type-1 bipartite graph with $2^{n-1}n$ edges. All its edges are not γ - critical. By Theorem 2.6 $E^+ = \phi$ and $E^0 = E(Q_n)$. Hence $|E^0| = 2^{n-1}n$. □

Corollary 2.8. *Let $G = P_n$, $n = 3k+1, k > 0$ Then $E^+ = \phi, E^0 = E(G)$ and $|E^0| = n-1$.*

Proof. P_n is a Type-1 graph. $\gamma_{st}(P_n) = \lceil \frac{n}{3} \rceil = k+1$. Let e be an edge of P_n. $P_n - e = P_l \cup P_t$ where $l+t = n$ and $l, t \geq 1$.

$$\begin{aligned}\gamma_{st}(P_n - e) &= \gamma(P_l) + \gamma(P_t)\\ &= \lceil \frac{l}{3} \rceil + \lceil \frac{t}{3} \rceil\end{aligned}$$

When

$$\begin{aligned}l &\equiv 0 \pmod 3 \text{ we have } t \equiv 1 \pmod 3\\ l &\equiv 1 \pmod 3 \text{ we have } t \equiv 0 \pmod 3\\ l &\equiv 2 \pmod 3 \text{ we have } t \equiv 2 \pmod 3\end{aligned}$$

In all these cases $\lceil \frac{l}{3} \rceil + \lceil \frac{t}{3} \rceil = k+1$. Hence $\gamma_{st}(P_n - e) = \gamma_{st}(P_n)$. So $E^0 = E(G)$ and $E^+ = \phi$. □

Corollary 2.9. *Let $G = P_n, n \neq 3k+1, k \geq 1, E^0 \neq \phi$ and $E^+ \neq \phi$. Moreover $|E^+| = \lfloor \frac{n-1}{4} \rfloor$.*

Proof. Clearly $n = 3k$ or $3k+2$. Consider the case $n = 3k . \gamma_{st}(P_n) = k$.

Let e be an edge in P_n such that $P_n - e = P_l \cup P_t$, where $l + t = n$ and $l, t \geq 1$. Clearly $\gamma_{st}(P_n - e) = \lceil \frac{l}{3} \rceil + \lceil \frac{t}{3} \rceil$. When $l \equiv 0 \pmod 3$ we have $t \equiv 0 \pmod 3$. So $\gamma_{st}(P_n - e) = k$. There are $\lfloor \frac{n-1}{4} \rfloor$ edges in P_n satisfying $\gamma_{st}(P_n - e) = k$. When $l \equiv 1 \pmod 3$ we have $t \equiv 2 \pmod 3$. So $\gamma_{st}(P_n - e) = k+1$. When $l \equiv 2 \pmod 3$, we have $t \equiv 1 \pmod 3$. In this case also $\gamma_{st}(P_n - e) = k+1$. There are $(n-1) - \lfloor \frac{n-1}{4} \rfloor$ edges in P_n for which $\gamma_{st}(P_n - e) = k+1$. □

Corollary 2.10. *If $G = C_n, n \geq 6, E^0 = E(G)$*

Proof. When G is $C_n, n \geq 6, \gamma_{st}(C_n) = \lceil \frac{n}{3} \rceil$. $\gamma_{st}(C_n - e) = \gamma_{st}(P_n) = \lceil \frac{n}{3} \rceil$ for every edge e in C_n. Hence $E^0 = E(G)$ and $|E^0| = n$. □

Theorem 2.11. *Let T be a tree with $n \geq 3$. Then there exists a vertex $v \in V$ such that $\gamma_{st}(T - v) = \gamma_{st}(T)$. That is $V^0 \neq \phi$.*

Proof. Let T be a tree with $n \geq 3$. Clearly $\gamma_{st}(T)$ or $\gamma(T) + 1$.

Case 1. Let $\gamma_{st}(T) = \gamma(T) + 1$.

Let (X, Y) be the bipartition of G. Every vertex in X has atleast two pendent neighbours and $\gamma(T) = |X|$. Suppose X has a vertex v such that v has more than two pendant neighbours. Let w be one such pendant neighbour of v. $T - w$ is a Type-2 tree and $\gamma_{st}(T - w) = \gamma(T) + 1$. So $w \in V^0$. Assume that every vertex in X has exactly two pendant neighbours. Let v be one such vertex in X. $T - v$ is forest with two isolated vertices and hance $\gamma_{st}(T - v) = (|X| - 1) + 2 = \gamma(T) + 1$. Hence $v \in V^0$.

Case 2: Let $\gamma_{st}(T) = \gamma(T)$.

As $n \geq 3$ and T is not a star there exists a support vertex v with exactly one non pendant neighbour w. If v is adjacent to two or more pendant vertices say u_1 and u_2 , then v is in every γ set of T and $\gamma(T - u_1) = \gamma(T) = \gamma_{st}(T)$. Let S be a γ_{st} set of T. As $\gamma(T) = \gamma_{st}(T)$, S is also a γ set of T that contains v and therefore $u_1, u_2 \notin S$. This implies that S is a γ set of $T - \{u_1\}$. Also S is a transversal of $T - \{u_1\}$. Hence S is a γ_{st} set of $T - \{u_1\}$ and $\gamma_{st}(T - u_1) = |S| = \gamma_{st}(T)$. If v is adjacent to exactly one pendant vertex u, then $deg\ v = 2$. Let $T' = T - u - v, \gamma_(T') \leq \gamma(T - u) \leq \gamma(T)$. However $\gamma(T') \geq \gamma(T) - 1$. If $\gamma(T') = \gamma(T) - 1$, then $\gamma(T) = \gamma(T - v)$. Otherwise $\gamma(T') = \gamma(T) = \gamma(T - u)$.

Subcase 1:

Let $\gamma_{st}(T) = \gamma(T) = \gamma(T - v)$. T' is a tree (Type-1 or Type-2). If $\gamma_{st}(T') = \gamma(T' + 1)$, then by Theorem 1.2, there exists a χ- partition (X', Y') with $|X'| < |Y'|$. Here X' is a unique γ set of T'. So $X' \cup \{v\}$ is a γ set for T. If $w \in X'$, then $X' \cup \{v\}$ is a γ_{st} set for $T - \{u\}$. So $\gamma_{st}(T - u) = \gamma(T) = \gamma_{st}(u)$. If $w \notin X'$, then $X' \cup \{w\}$ is a γ_{st} set for $T - u$. So $\gamma_{st}(T - u) = \gamma_{st}(T)$. If T' is a Type-1 graph, that is $\gamma_{st}(T') = \gamma(T')$, then $\gamma_{st}(T - v) = 1 + \gamma_{st}(T') = 1 + \gamma(T')$. Since $\gamma(T - v) = 1 + \gamma(T')$, we have $\gamma_{st}(T - v) = \gamma(T - v) = \gamma_{st}(T)$.

Subcase 2:

Let $\gamma_{st}(T) = \gamma(T) = \gamma(T') = \gamma(T - u)$. Now $\gamma_{st}(T) = \gamma(T - u) \leq \gamma_{st}(T - u)$. Let S be a γ_{st} set of T. S is also a γ set of T. Suppose $u \in S$, then $S - \{u\}$ is a dominating set for T' , contradicting the fact that $\gamma(T) = \gamma(T')$. So $u \notin S$. Hence S is an std-set for $T - u$. $\gamma_{st}(T - u) \leq |S| = \gamma(T) = \gamma_{st}(T)$. Hence$\gamma_{st}(T - u) = \gamma_{st}(T)$. So in all the cases we are able to obtain a vertex satisfying the required condition. Hence $V^0 \neq \phi$. □

For any connected unicyclic graph G, $\gamma_{st}(G) = \gamma(G)$ or $\gamma(G)+1$ or $\gamma(G)+2$. A classification theorem is available in the following form

***Definition* 2.12.** $\mathcal{G}$= The set of graphs obtained by attaching l pendant vertices to a vertex of C_3, where l is a non negative integer.

***Definition* 2.13.** $\mathcal{G}'$= The set of graphs obtained from C_3 by attaching k and l pendant vertices respectively to two distinct vertices of C_3, where k and l are any two positive integers

***Definition* 2.14.** $\mathcal{G}''$= The set of graphs obtained from C_5 by attaching k and l pendant vertices respectively to two non-adjacent vertices of C_5, where k and l are any two non negative integers.

***Definition* 2.15.** $\mathcal{G}_r$= The set of graphs obtained by joining the centres of $K_{1,l}$ to a vertex of C_3 by a path of length r, where l is a positive integer.

Note that if $G \in \mathcal{G}$, then $\gamma_{st}(G) = \gamma(G) + 2$ and if $G \in \mathcal{G}' \cup \mathcal{G}'' \cup \mathcal{G}_1 \cup \mathcal{G}_2, \gamma_{st}(G) = \gamma(G) + 1$.

***Definition* 2.16.** A connected unicyclic graph is classified as a class-1, class-2,class-3 according as its $\gamma_{st}(G)$ value is $\gamma(G)$ or $\gamma(G)+1$ or $\gamma(G)+2$. In fact G is a class-3 graph if $G \in \mathcal{G}$, G is a class-1 graph if G is either Type-2 bipartite graph or $G \in \mathcal{G}_1 \cup \mathcal{G}_2 \cup \mathcal{G}' \cup \mathcal{G}''$

***Theorem* 2.17.** [3] For a connected unicyclic graph with the odd cycle

$$\gamma_{st}(G) = \begin{cases} \gamma(G)+2 & \text{if } G \in \mathcal{G}, \\ \gamma(G)+1 & \text{if } G \in \mathcal{G}_1 \cup \mathcal{G}_2 \cup \mathcal{G}' \cup \mathcal{G}'' , \\ \gamma(G) & \text{otherwise.} \end{cases}$$

***Corollary* 2.18.** [3] For any connected unicyclic graph G

$$\gamma_{st}(G) = \begin{cases} \gamma(G)+2 & \text{if } G \in \mathcal{G}, \\ \gamma(G)+1 & \text{if } G \text{ is a Type-2 connected} \\ & \text{bipartite graph} \\ & \text{(or) } G \in \mathcal{G}_1 \cup \mathcal{G}_2 \cup \mathcal{G}' \cup \mathcal{G}'' , \\ \gamma(G) & \text{otherwise.} \end{cases}$$

***Theorem* 2.19.** *For any connected unicyclic graph G, $E^- = \phi$ or $E^+ = \phi$.*

Proof. Let G be a connected unicyclic graph. Let G be a class-2 or class-3 graph. By the Corollary 2.18 $G \in \mathcal{G} \cup \mathcal{G}_1 \cup \mathcal{G}_2 \cup \mathcal{G}' \cup \mathcal{G}''$ or G is a Type-2 connected bipartite graph. If G is a connected Type-2 bipartite graph by Theorem 2.2 $E^+ = \phi$. Let $G \in \mathcal{G} \cup \mathcal{G}_1 \cup \mathcal{G}_2 \cup \mathcal{G}' \cup \mathcal{G}''$, $\gamma_{st}(G) = 3$. Let e be an edge in G that does not lie in the odd cycle. This is a cut edge of G. Hence $G-e$ is disconnected graph containing an odd cycle. By simple verification $\gamma_{st}(G-e) = 3$. If e is an edge in the odd cycle, then again $\gamma_{st}(G-e) = 2$ or 3. Hence $E^+ = \phi$.

Let G be a class-1 graph. $\gamma_{st}(G) = \gamma(G)$. We know that $\gamma(G-e) = \gamma(G)$ or $\gamma(G)+1$. Hence $\gamma_{st}(G-e) \geq \gamma(G-e) \geq \gamma(G) = \gamma_{st}(G)$. So $\gamma_{st}(G-e) \geq \gamma_{st}(G)$ implies that $E^- = \phi$. □

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